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Application of Interval Interpolation for the Description of Compression-Ignition Engine Performance Characteristics

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Abstract

In this paper is presented a calculation method based on Lagrange's interpolation formula which has been used for mathematical description of the performance characteristics of a compression-ignition engine of the 359 type of the Polish production. Its application allows simplification of experimental tests through preservation of a minimum number of measuring points and estimation of other data analytically. In order to minimise the interpolation error occurring with polynomials of high degrees and constant node distances, the characteristics were approximated by spline functions with both solutions being shown comparatively in the graphical form. As calculation examples, the curves of specific fuel consumption and infrared radiation absorption coefficient were chosen, which had been obtained during examinations on engine test bench for a drive unit fuelled with four types of fuel. In addition, results of the experiment required for their determination were tabulated. The presented method may be used in further tests of a given engine as well as on other experimental benches, aiding long-lasting and expensive optimisation of operating parameters when using fuels of plant origin. Description of any performance characteristics by means of interval interpolation is convenient from the practical side and does not cause greater calculation problems since polynomials of low degrees are being used in the procedure.

Keywords: polynomial interpolation, performance characteristics, specific fuel consumption, exhaust gas smokiness

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1. Introduction

Interpolation belongs to basic numerical methods which allow finding a function in a given interval assuming a priori set values in specific points, so called nodes. In calculation practice, it is most frequently used for the replacement of a specific curve determined experimentally with an appropriate polynomial to reproduce fairly exactly the course of function in the quantitative and the qualitative aspect [3]. In case of a small number of nodes and constant distances between them, Lagrange's interpolation formula is being frequently used [1]:

$$W_n(x) = y_0\frac{(x-x_1)(x-x_2)...(x-x_n)}{(x_0-x_1)...(x_0-x_n)} + y_1\frac{(x-x_0)(x-x_2)...(x-x_n)}{(x_1-x_0)(x_1-x_2)...(x_1-x_n)} + \\ +... + y_n\frac{(x-x_0)(x-x_1)...(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)...(x_n-x_{n-1})} \quad (1)$$

where:

$W_n(x)$ – Lagrange's interpolation polynomial of the *n*-th degree,

$x_0, x_1, x_2, \ldots, x_n$ – measuring points (nodes),

$y_0=f(x_0)$, $y_1=f(x_1)$, $y_2=f(x_2)$, …, $y_n=f(x_n)$ – function's values in this points.

It should be stressed, however, that increasing the number of measuring points leads in a quick way to the complexity of calculations being performed, which results from a high degree of the polynomial obtained. Furthermore, it does not ensure a more exact description of the curve in question since analytically determined values of the function outside main nodes may be encumbered with a large error and the same do not reproduce physical aspects of the experiment. Deterioration of the interpolation results occurs usually at the ends of the interval in question, whereas this effect is being determined with Runge's phenomenon [1].

Necessary condition for obtaining the correct course of performance characteristic is determination of minimum six measuring points [2]. Their larger number may be troublesome with experimental cycles repeated many times, e.g. when changing engine adjustment settings, fuelling it with different types of fuels, interfering in the parameters on the inlet side, etc. This is connected both with an increase in the time consumption of research process and multiplication of operating costs. In addition, it is recommended to concentrate measuring points near the extremes and other places of the curves important from the point of view of experiment. With this end in view, a calculation procedure was developed which allows analytical estimation of other data basing on their minimum required number. Having in mind the minimisation of errors occurring when using Lagrange's interpolation formula (1), the characteristics for respective fuels were described comparatively in two ways: by interval interpolation, i.e. with the second and the third degree polynomial, and by a single polynomial of the fifth degree.

2. Research Methods

Test bed measurements were carried out on a standard engine test bench being part of laboratory facilities of the Department of Automotive Vehicles Operation, Western Pomeranian University of Technology in Szczecin. The study object was an unsupercharged four-stroke engine of the 359 type of domestic production. It is a six-cylinder compression-ignition drive unit with direct fuel injection system (Table 1) [5]. The test bench was equipped additionally with hydraulic brake HWZ-3 together with a complete control system, gravimetric fuel gauge and light obscuration smokemeter MDO 2 type. In respective measuring cycles, the engine was fuelled with fuels having different physicochemical properties, i.e. summer grade diesel oil (ON), low-sulphur diesel oil (EKODIESEL PLUS 50), rapeseed oil (OR), and rapeseed methyl ester (RME). The obtained test results allowed determination of a number of performance characteristics of the engine while preserving its factory settings [4]. Because the paper is focused on a mathematical description of the characteristic curves of specific fuel consumption (g_e) and infrared radiation absorption coefficient (k), their values are compiled in Table 2 according to the type of applied fuel.

Table 1

Base technical data of engine by 359 type [5]

Description	Technical data
Arrangement of cylinders	vertical, in-line
Number of cylinders	6
Cylinder diameter	0,11 m
Piston stroke	0,12 m
Engine displacement	6,846 dm^3
Compression ratio	17
Maximum output	110 kW at 2800 1/min
Maximum torque	432 Nm at 1800÷2100 1/min
Minimal specific fuel consumption	223 g/kWh
Firing sequence	1-5-3-6-2-4
Delivery angle (static)	18,5 degrees of crankshaft rotation before upper dead center (U.D.C.)
Injection system	direct

3. Calculation Methods

During the examinations, readings were made in six measuring points, with [1/s] being adopted as a unit of engine crankshaft rotational speed due to the simplification of calculation procedure. In this connection, the following nodes with constant

Table 2

The values of specific fuel consumption and exhaust gas smokiness for the examined fuels

n [1/min]	g_e [g/kWh]				k [1/m]			
	ON	EKODIESEL PLUS 50	OR	RME	ON	EKODIESEL PLUS 50	OR	RME
1200	222,83	225,68	269,29	251,44	1,57	1,83	0,82	0,88
1500	227,56	221,84	271,71	249,97	1,78	2,05	1,01	1,09
1800	219,40	219,86	268,60	245,09	0,76	0,81	0,64	0,67
2100	226,59	224,77	273,53	248,21	0,85	0,78	0,46	0,49
2400	231,48	229,97	277,80	252,63	0,83	0,81	0,35	0,41
2700	238,22	234,06	286,27	264,03	0,88	0,84	0,32	0,44

distances of arguments were obtained: x_0=20, x_1=25, x_2=30, x_3=35, x_4=40, and x_5=45. According to formula (1), a general equation of the function describing any external characteristic of the engine under examination may be presented as follows:

$$D(x) = \sum_{k=0}^{5} f(x_k) \cdot d_k(x) \tag{2}$$

where:

$$d_k(x) = \frac{(x - x_0)...(x - x_{k-1})(x - x_{k+1})...(x - x_5)}{(x_k - x_0)...(x_k - x_{k-1})(x_k - x_{k+1})...(x_k - x_5)} \tag{3}$$

The above expressions are correct when applying a single Lagrange's polynomial of the fifth degree for {k=0, 1, 2, 3, 4, 5}. Interpolation with spline functions forces division of the area in question into smaller parts hence calculations were performed separately for two intervals. By substituting measuring points and reducing successive to a common denominator, the following was obtained:

$$d_0(x) = \frac{(x-25)(x-30)}{(-5)(-10)} = \frac{15 \cdot S_0(x)}{750} \tag{4}$$

$$d_1(x) = \frac{(x-20)(x-30)}{(5)(-5)} = \frac{-30 \cdot S_1(x)}{750} \tag{5}$$

$$d_{2a}(x) = \frac{(x-20)(x-25)}{(10)(5)} = \frac{15 \cdot S_{2a}(x)}{750} \tag{6}$$

as well as:

$$d_{2b}(x) = \frac{(x-35)(x-40)(x-45)}{(-5)(-10)(-15)} = \frac{-S_{2b}(x)}{750} \tag{7}$$

$$d_3(x) = \frac{(x-30)(x-40)(x-45)}{(5)(-5)(-10)} = \frac{3 \cdot S_3(x)}{750} \tag{8}$$

$$d_4(x) = \frac{(x-30)(x-35)(x-45)}{(10)(5)(-5)} = \frac{-3 \cdot S_4(x)}{750} \tag{9}$$

$$d_5(x) = \frac{(x-30)(x-35)(x-40)}{(15)(10)(5)} = \frac{S_5(x)}{750} \tag{10}$$

where:

$S_0(x)\ldots S_5(x)$ – has define the working function (partial), which necessary for calculation of final interpolation polynomials, and obtained their by multiplication of numerators of several definition $d_0(x)\ldots d_5(x)$.

Therefore, a general equation of the function describing the performance characteristic of the engine may be presented as follows:

$$D_1(x) = f(x_0) \cdot d_0(x) + f(x_1) \cdot d_1(x) + f(x_2) \cdot d_{2a}(x) \tag{11}$$

$$D_2(x) = f(x_2) \cdot d_{2b}(x) + f(x_3) \cdot d_3(x) + f(x_4) \cdot d_4(x) + f(x_5) \cdot d_5(x) \tag{12}$$

As a calculation example, specific fuel consumption curve obtained when fuelling the engine with traditional summer grade diesel oil (ON) was chosen for examination. Having in mind the values presented in Table 1, functions $D_{ON1}(x)$ and $D_{ON2}(x)$ describing a given performance characteristic curve will have the following form:

$$D_{ON1}(x) = (1/750)[3342,45 \cdot S_0(x) - 6826,80 \cdot S_1(x) + 3291,00 \cdot S_{2a}(x)]$$

$$D_{ON2}(x) = (1/750)[-219,40 \cdot S_{2b}(x) + 679,77 \cdot S_3(x) - 694,44 \cdot S_4(x) + 238,22 \cdot S_5(x)]$$

In order to present final Lagrange's interpolation polynomials, its respective components were determined:

$$S_0(x) = x^2 - 55x + 750 \tag{13}$$

$$S_1(x) = x^2 - 50x + 600 \tag{14}$$

$$S_{2a}(x) = x^2 - 45x + 500 \tag{15}$$

as well as:

$$S_{2b}(x) = x^3 - 120x^2 + 4775x - 63000 \tag{16}$$

$$S_3(x) = x^3 - 115x^2 + 4350x - 54000 \tag{17}$$

$$S_4(x) = x^3 - 110x^2 + 3975x - 47250 \tag{18}$$

$$S_5(x) = x^3 - 105x^2 + 3650x - 42000 \tag{19}$$

Finally, the spline functions describing the performance characteristic of specific fuel consumption when fuelling the engine with summer grade diesel oil (ON) assume the following form:

$$D_{ON1}(x) = (1/750)[-193,35x^2 + 9410,25x - 56257,50]$$

$$D_{ON2}(x) = (1/750)[4,15x^3 - 470,25x^2 + 18468,50x - 78330,00]$$

Interpolation polynomials of other curves were determined in a similar way, whereas their cumulative comparison and graphical interpretation are presented in Table 3 and Figure 1, respectively. For comparison, also fifth degree functions obtained without division of the area in question into two parts are presented. Because all tests were carried out for identical measuring points, there is possibility of a direct use of the calculation procedure for description of other performance characteristics of a given type engine. This is because the form of polynomials of components (13), (14), (15), (16), (17), (18) and (19) remains unchanged, while determination of the functions for other operating parameters is only limited to the replacement and multiplication of applied gradients in the formula (2). An example can be interpolating polynomials of exhaust gas smokiness curves, being expressed by absorption coefficient of infrared radiation (Figure 2, Table 4).

Table 3

Interpolation of the cumulative performance characteristic of specific fuel consumption

Fuel	Form of interpolating polynomials
ON	$D_{ON1}(x) = (1/750)[-193,35x^2 + 9410,25x - 56257,50]$ $D_{ON2}(x) = (1/750)[4,15x^3 - 470,25x^2 + 18468,50x - 78330,00]$
	$D_{ON}(x) = (1/375000)[67,69x^5 - 11300,75x^4 + 741066,25x^3 - 23804406,25x^2 + 373788662,00x - 2207467500,00]$
EKODIESEL PLUS 50	$D_{EP1}(x) = (1/750)[27,90x^2 - 1831,50x - 194730,00]$ $D_{EP2}(x) = (1/750)[-1,40x^3 + 151,35x^2 - 4656,25x + 206167,50]$
	$D_{EP}(x) = (1/375000)[16,83x^5 - 2815,25x^4 + 183863,25x^3 - 5830543,70x^2 + 89447762,00x - 446433750,00]$
OR	$D_{OR1}(x) = (1/750)[-82,95x^2 + 4095,75x - 153232,50]$ $D_{OR2}(x) = (1/750)[4,86x^3 - 520,20x^2 + 19122,00x - 35250,00]$
	$D_{OR}(x) = (1/375000)[35,83x^5 - 5931,25x^4 + 386018,75x^3 - 12312818,70x^2 + 192109387,00x - 1069882500,00]$
RME	$D_{RME1}(x) = (1/750)[-51,15x^2 + 2076,25x + 167415,00]$ $D_{RME2}(x) = (1/750)[5,68x^3 - 576,90x^2 + 19932,50x - 48307,50]$
	$D_{RME}(x) = (1/375000)[30,49x^5 - 5026,25x^4 + 326106,25x^3 - 10368793,70x^2 + 160972037,00x - 879851250,00]$

Experimental verification of both cases in question showed that better approximation was obtained when using interval interpolation, i.e. with polynomials of the second and the third degree. For example, when fuelling the engine with low-sulphur diesel oil (EKODIESEL PLUS 50), the level of exhaust gas smokiness at intermediate point x_i=22.5 1/s amounted to 2.04 1/m which gives, in relation to the value calculated with spline function method 2.13 1/m, an error of 4.41%. Deterioration of the results of interpolation with a single polynomial is significant since the largest deviation 25.98% was obtained at a result of 2.57 1/m. For other curves, errors are not so large whereas the course of most of them has almost identical character, which is particularly visible for the functions describing specific

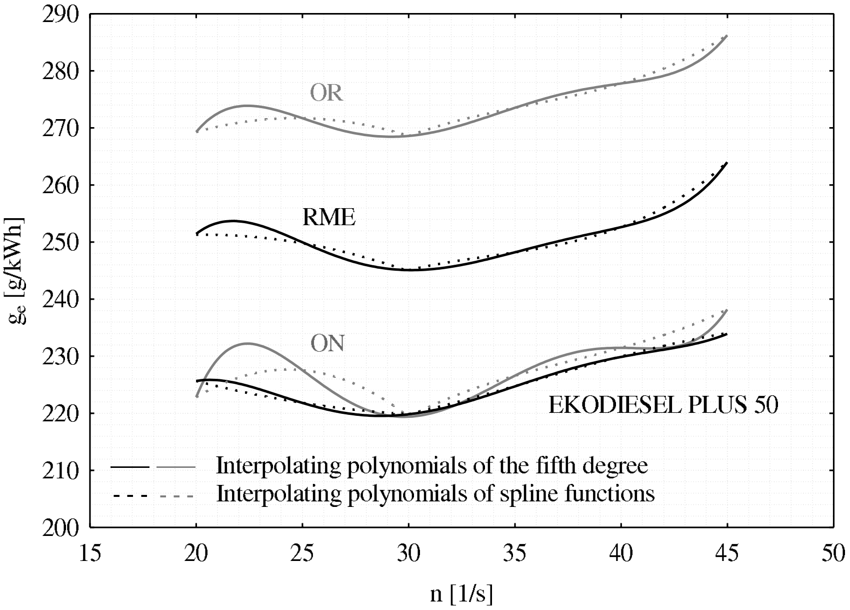

Fig. 1. Graphical interpretation of the interpolating polynomials of specific fuel consumption

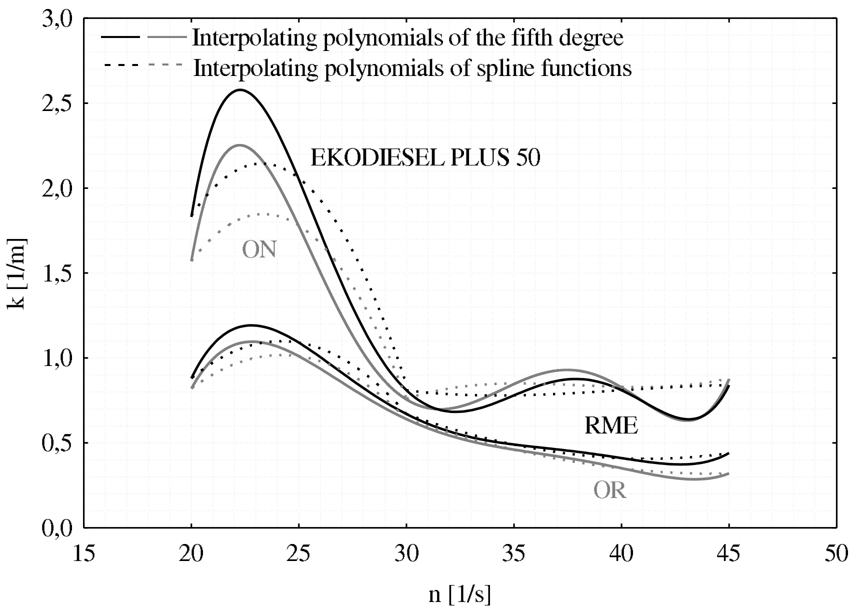

Fig. 2. Graphical interpretation of the interpolating polynomials of exhaust gas smokiness

fuel consumption (Figure 1). Nonetheless, it is possible to observe their increase at the ends of the analysed interval on both diagrams. This confirms the occurrence of so called Runge's effect for polynomials of the fifth degree with constant node distances and the appearance of extremes in interpolating functions. In such a case, better results may be obtained by applying so called spline functions, dividing the examined interval into smaller partitions.

Table 4

Interpolation of the cumulative performance characteristic of exhaust gas smokiness

Fuel	Form of interpolating polynomials
ON	$D_{ON1}(x) = (1/750)[-18,45x^2 + 861,75x - 8677,50]$ $D_{ON2}(x) = (1/750)[0,18x^3 - 20,55x^2 + 777,75x - 9127,50]$
	$D_{ON}(x) = (1/375000)[4,96x^5 - 833,00x^4 + 54980,00x^3 -$ $-1778650,00x^2 + 28119525,00x - 172773750,00]$
EKODIESEL PLUS 50	$D_{EP1}(x) = (1/750)[-21,55x^2 + 1003,00x - 10062,50]$ $D_{EP2}(x) = (1/750)[-0,06x^3 + 7,20x^2 - 282,00x + 4207,50]$
	$D_{EP}(x) = (1/375000)[4,91x^5 - 832,00x^4 + 55416,25x^3 -$ $-1808900,00x^2 + 28835775,00x - 178391250,00]$
OR	$D_{OR1}(x) = (1/750)[-8,40x^2 + 406,50x - 4155,00]$ $D_{OR2}(x) = (1/750)[0,01x^3 - 58,75x + 1972,50]$
	$D_{OR}(x) = (1/375000)[x^5 - 171,75x^4 + 11642,50x^3 -$ $-388406,25x^2 + 6346562,50x - 40121250,00]$
RME	$D_{RME1}(x) = (1/750)[-9,45x^2 + 456,75x - 4695,00]$ $D_{RME2}(x) = (1/750)[0,01x^3 + 0,45x^2 - 88,00x + 2467,50]$
	$D_{RME}(x) = (1/375000)[1,16x^5 - 199,25x^4 + 13507,50x^3 -$ $-450493,75x^2 + 7356212,50x - 46488750,00]$

4. Conclusions

The use of the calculation method presented allows simple description of any performance characteristic of a given engine as well as, after small modification, other drive units in tests with a similar profile. Its application allows simplification of experimental procedure since estimation of the values for rotational speeds not being interpolation nodes may be carried out analytically. It should be stressed, however, that more favourable results are being obtained through interpolation with spline functions, dividing the area in question into smaller parts. Owing to low polynomials of low degrees, calculations do not present greater problems and they can be carried out without complex algorithms, obtaining a satisfactory degree of approximation. Furthermore, oscillation of single interpolating functions is avoided at the extremes of the interval under examination, which falsify the physical aspect of experiment.

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