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INTEGRATED OPTIMIZATION OF TRAIN ROUTING AND TIMETABLE OF RAIL TRANSIT NETWORK WITH TIME-VARYING PASSENGER DEMAND

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***Abstract*:**

Urban rail transit is operated with network scenario mostly. Train routing determines spatial service range on the network, which will affect whether passengers transfer during the trip. Timetable determines arrival and departure time of each train at each station. To achieve balance between demand and supply and enhance service quality, simultaneous optimization of train routing and timetable is of significant importance for rail companies with the network. This will reduce the waiting time of passengers effectively, and the stations where train serve will be convenient for most passengers. However, existing studies in train operation planning with time-varying passenger demand haven't addressed this aspect adequately. To bridge this gap, we develop an integrated optimization model for the Train Routing and Timetable Problem (TRTP) within a rail transit network, considering the dynamic of passenger demand and passenger path choice. Our objective is to minimize the operating costs for companies and to reduce the total waiting time experienced by passengers. The constrains of TRTP include time constraint, path constraint and train constraint. What's more, the compatibility of headways on different routings is the focus when modeling. To verify the effectiveness of our proposed model, we conduct a numerical experiment with CPLEX. After that, the results of integrated optimization and staged optimization are compared to show the beneficial of integration with train routing and timetable. Moreover, we devise an algorithm tailored to this problem to solve the model, and a network case study is presented. As a case study, we apply the model to Guangzhou Metro Line 3 to validate its performance further, which is a Y-type line, the most common example of operation with network of urban rail transit in China. The results of our study demonstrate that the proposed integrated optimization model can achieve a reduction in operating costs and total waiting time for passengers effectively, providing convenience for passenger travel.

Keywords: *Rail transit network, Time-varying passenger demand, Train timetable, Train routing plan, Enumeration algorithm, Pareto optimality*

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1. Introduction

Urban rail transit has undergone a significant shift towards high-density networks, moving away from conventional linear structures to more intricate configurations characterized by intersecting and non-intersecting lines. Previously, due to traditional design and planning approaches, rail transit lines operated independently, lacking connectivity. However, as passenger demands have increased, the independent operation model has revealed drawbacks such as inefficient utilization of transport capacity resources and inconvenience for passengers during transfers. If a passenger needs to get off a train on one line and transfer to another train to get on a different line, this is called "cross-line", which leads to the emergence of an interconnected transportation organization mode, with the Y-type line being a prominent example of such connectivity.

As urban rail transit networks continue to expand, operating costs have risen, making cost reduction a crucial concern for rail companies. At the same time, passengers have higher expectations for service quality and train routing may affect the transfer in the network. Passenger demand exhibits networking and spatial-temporal imbalances in urban rail transit. Existing train operation plans often fail to meet these demands effectively, resulting in increased passenger transfer frequency and prolonged waiting times during transfers. Integrated optimization of train routing and timetable with time-varying passenger demand can reduce the number of transfers. Therefore, optimizing both train routing and timetable with passenger demand is of utmost importance. Train routing refers to the spatial service range of the train on the network, which can be represented as a set containing multiple stations. This corresponds to strategic level in planning process for urban rail transit. While timetable determines arrival and departure time of each train at each station on the basis of routing, which belongs to tactical level during planning (Wang et al., 2018). Currently, the optimization of train routing and timetable is carried out sequentially, limiting overall optimality, particularly within the scenario of a network. Hence, the optimization of both plans, considering passenger demand, holds immense significance. By prioritizing passenger demand as a foundational element, the efficiency and effectiveness of train operation plans can be enhanced significantly, leading to improved service quality and cost reduction ultimately.

The primary objective of this study is to achieve simultaneous optimization for both train timetables and train routing plans within a rail transit network, considering dynamic of passenger demand over time. This approach aims to align transportation service within network under the evolving needs of passengers effectively. Consequently, this integrated optimization ensures the simultaneous fulfillment of interests for both rail companies and passengers. Furthermore, this integrated approach overcomes potential challenges encountered when transitioning from the plan compiling to train operating. With passenger demand, this approach ensures a more realistic and feasible implementation of train routing plan, eliminating potential discrepancies and enhancing operational efficiency overall.

The rest of this paper is organized as follows. Section 2 provides a comprehensive review of recent research on train timetable and routing plan, pointing out the contributions of this research. Section 3 describes the research scenario and basic parameters of TRTP. The proposed model is presented in Section 4. The results of numerical experiment and real case study analysis are discussed in Section 5 and 6. Finally, in Section 7, we summarize the key conclusions of this paper.

2. Literature review

As rail transit networks expand continuously, the focus of train timetable studies needs to shift from optimizing single line to a more comprehensive approach. Collaborative optimization of train timetables with multiple rail lines becomes crucial for satisfying passenger demand by considering interplay and coordination among different lines. Additionally, the routing plan plays a vital role in determining the operational aspects of train services, encompassing such as operating section, turnaround point and frequency. It serves as a blueprint for organizing and managing train operations, ensuring smooth and efficient service delivery throughout the rail transit network.

2.1. Related research on train timetable

The complexity of model may affect solution efficiency, constructing reasonable and concise model will can reduce the problem scale. Yin et al. (2017) focused on the problem of metro scheduling with two directions, minimizing operating costs and passenger waiting time. They transformed the complex train scheduling problem with dynamic demand into a linear optimization model using space-time networks. Similarly, Meng et al. (2020) transformed the complex train scheduling problem with dynamic demand into a linear optimization model. Zhou et al. (2019) combined the train schedule with passenger's reservation based on passenger demand. By establishing a two-level programming model, the departure and arrival time of each train is optimized firstly to reduce the total travel cost of passengers. Gong et al. (2021) developed a variable neighborhood search algorithm for large-scale problems, which is suitable for dynamic and randomness of passenger demand.

The compilation of demand-driven timetables needs to balance the interests and costs for rail companies while considering passenger behavior. Robenek et al. (2016) proposed cyclical and non-periodic timetable problem based on passenger satisfaction and operating company revenue, formulating it as a mixed-integer linear programming (MILP) model to maximize profit and passenger satisfaction. Polinder et al. (2021) focused on decision-making in the strategic phase of public transportation planning, ensuring regular connections between passengers' origins and destinations. Cats and Haverkamp (2018) proposed an optimized model for automated on-demand rail-bound transit systems, considering passengers, equipment and operating costs. Dong et al. (2020) improved train schedule quality by optimizing stop plans and timetables under time-dependent passenger demand. Wang et al. (2020) addressed passenger transfers in urban rail transit, minimizing total waiting time and the number of passengers unable to transfer using a MILP. Su et al. (2021) put forward a concept of dwell time supplement, which was optimized considering passenger demand in peak hours or off-peak hours. Train timetable and energy consumption can be adjusted in this way. Cacchiani et al. (2020) addressed uncertain demand using a MILP model with desired protection levels. Mo et al. (2019) optimized service-oriented train timetables for inhomogeneous passenger demands in two directions. Tian and Niu (2020) proposed a novel approach to demand-oriented timetabling, eliminating indivisibility by replacing two-station-dependent dual variables.

Special circumstances such as passenger delays and operation interruptions have also been considered. Yin et al. (2021) aimed to minimize station crowdedness during peak hours by generating optimal coordinated train timetables synchronously. Zhu and Goverde (2019) tried to minimize passenger delays with three other dispatching measures: retiming, reordering and cancelling. Furthermore, a MILP model has been proposed, which includes dispatching measures of retiming, reordering, cancelling, adding stops and flexible short-turning. This can reschedule the timetable in case of multiple disruptions that occur at different geographic locations but have overlapping periods and are pairwise connected by at least one train line (Zhu and Goverde, 2021). Shakibayifar et al. (2018) proposed an optimization framework based on simulation, generating timetable with the minimum total delay of trains at destinations on average. Gao et al. (2020) considered train unit maintenance requirements and periodicity characteristics of trip sequences. Li et al. (2021) proposed a feeder-vehicle routing and high-speed train assignment model with time windows to minimize the average of passengers' delaying time at station by optimizing feeder-vehicle routes and passengers' assignments to trains.

Existing research indicates that passengers prioritize waiting time, travel costs, satisfaction levels and less transfers, while rail companies consider operating costs, profitability and train load rates. Striking a balance between service level and operating expenses is crucial. Table 1 provides a comparison of optimized contents proposed in this study with representative literature works, considering factors such as objective function, passenger transfer, passenger demand input, train capacity and solution methods.

2.2. Related research on train routing plan

Several recent studies have made efforts to train scheduling and routing problem for railway systems. In terms of train routing plan optimization on network, Claessens et al. (1998) established a nonlinear programming model with the objective of minimum operating cost, which can optimize the train operation plan of the railway network, solved by the branch and bound method. Goossens et al. (2006) established the optimization model of network train operation plan with the objective of minimizing the operating cost of companies, using CPLEX to solve it. Szeto and Wu (2011) optimized bus line routing on a network and departure frequency to minimize passenger travel time and transfers by genetic algorithm with neighborhood search.

Table 1. Comparison of timetable synchronization optimization models

Literature	Objective function	Consider both the main line and cross-line pssenger demand	Form of Passenger demand input	Consider train capacity	Solved algorithm
Ceder et al. (2001)	Maximize the number of synchronizations	No	No	No	Heuristic algorithm
Wong et al. (2008)	Minimize passenger waiting time	No	Static	No	CPLEX
Ibarra-Rojas and Rios-Solis (2012)	Maximize the number of synchronizations	No	No	No	Neighborhood search algorithm
Kang et al. (2016)	Maximize synchronization times and minimize transfer connection time	No	No	No	Genetic algorithm
Wu et al. (2015)	Minimize the maximum transfer waiting time	No	Static	No	Genetic algorithm
Niu et al. (2015)	Minimize passenger waiting time and minimize the negative effect of passenger congestion	Yes	Time-varying	Crowded negative utility	Dynamic programming, genetic algorithm
Wu et al. (2016)	Maximize the number of successful transfers and minimize the deviation from the existing timetable	No	Static	No	Non-dominated sorting genetic algorithm
Shang et al. (2018)	Minimize passenger travel time	Yes	Time-varying	No	Genetic algorithm
This research	Minimize business operating costs and total passenger waiting time	Yes	Time-varying	Yes	CPLEX, enumeration algorithm

Sun et al. (2014) considered the interests of both companies and passengers, established train operation plan and timetable optimization model of high-speed rail network, verified by an improved genetic algorithm. Zhao et al. (2016) focused on train routing plans for Y-type lines, minimizing passenger travel time and company-operated train travel distance with a genetic algorithm. Canca et al. (2016) studied network train routing plans, optimizing departure frequency for each routing to minimize corporate costs and average passenger travel time with an extended cutting plane algorithm.

Train routing is closely tied to train scheduling and requires further research. Table 2 provides a comparison of optimization content related to this study with representative literature works. Factors such as objective function, passenger demand input and solution methods. This analysis highlights the similarities and differences between optimization approaches. The study aims to contribute to existing knowledge by considering various factors, employing accurate solving methods, and addressing scalability concerns.

Table 2. Comparison of train routing plan optimization models

Literature	Objective function	Form of passenger demand input	Solved algorithm
Goossens et al. (2006)	Minimize operating costs	No	CPLEX
Zhao et al. (2016)	Minimize passenger travel time and travel distance of company-operated trains	Static	Genetic algorithm
Canca et al. (2016)	Minimize companies operating costs and average passenger travel time	Static	Extended cutting plane algorithm
This research	Minimize companies operating costs and total passenger waiting time	Time-varying	CPLEX, enumeration algorithm

Through a comprehensive comparison with existing literature, it becomes evident that the train timetable studies up to now have seldom taken into account both main line and cross-line passenger demand. Consequently, the corresponding train timetables fail to meet the diverse passenger demand adequately. Moreover, the consideration of train capacity in these studies has been limited, leading to discrepancies between the generated train timetables and actual operational conditions. Additionally, the majority of solution algorithms employed in these studies are heuristic in nature, which prevents attainment of optimal solutions through precise solving algorithms.

In the context of train routing plan research, the existing studies use static passenger demand input forms predominantly. Consequently, the obtained train routing plans don't cater to the dynamic passenger demand effectively. Furthermore, while there have been numerous studies focusing on the integration of TRTP for single lines, the integration of these plans for multiple lines remains relatively unexplored.

Overall, existing literature gaps necessitate more comprehensive and integrated approaches considering passenger demand, train capacity, accurate solving algorithms and dynamic passenger demand. The integration of TRTP for multiple lines requires further exploration.

This research makes the following significant contributions. Firstly, it proposes an integrated optimization approach for TRTP within a rail transit network. This approach considers both strategic level and tactical level with integration, as well as train capacity, leading to a more comprehensive optimization process. Secondly, this research addresses nonlinear constraints in the optimization model by converting them into linear constraints, enabling more efficient solving. The proposed model's correctness and effectiveness are verified using a precise solver, providing confidence in the results obtained. Additionally, an enumeration algorithm is proposed to handle network cases, offering a practical solution for complex scenarios and effective application to real-world situations. These contributions enhance the field of TRTP optimization by considering various factors, using accurate solving methods and addressing scalability concerns. The research provides valuable insights and solutions for improving the efficiency and effectiveness of rail transit networks.

3. Problem statement of TRTP

3.1. Research scenario and passenger path

The integrated optimization model of TRTP based on passenger demand aims to optimize the train routing, train arrival and departure times and train departure headways for each routing while meeting the goals of the rail companies and passengers. The objective is to determine a unique routing for each train line in timetable that ensures an overall optimal result.

Consider the rail transit line shown in Figure 1, which consists of S stations with a transfer station Tr. There are 3 alternative routings, namely, routing 1, 2 and 3. Among them, in routing 1, trains run between station 1 and station m. In routing 2, trains run between station Tr and station S. In routing 3, trains run from station 1 to station S. In particular, the line between station 1 and station m is defined as the main line, and the line between station Tr and station S is defined as the branch line.

The research period is T, which is discretized into the time interval t with the same length σ. Passenger demand is inputted in the form of origin-destination-time. That means the passenger demand between each OD varies with time t. The following is a description of the integrated optimization of the TRTP from the perspective of train routing plan, train timetable and passenger path choice.

3.1.1. Train routing plan in TRTP

In a rail transit network with complex routing forms and turnaround stations, there are numerous possible routing options. However, specific technical conditions must be considered (e.g., availability of additional turnaround tracks) for a train routing to be established. By adhering to such principles, all feasible routings can be obtained through enumeration, reducing the solution space and computation time significantly.

The determination of train routing plan is based on actual cross-section passenger demand of the line. From a given set of alternative routings, routing selections are made to form a feasible routing plan, leading to the ultimate determination of the optimal train routing plan.

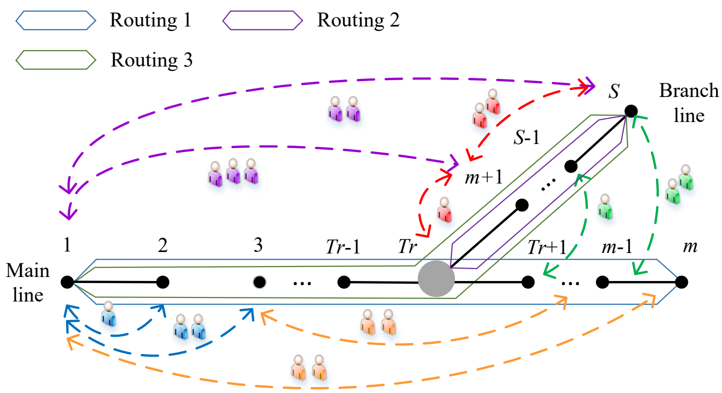

Fig. 1. Schematic diagram of rail transit lines and the set of train routing

3.1.2. Train timetable in TRTP

In rail transit system, trains on each routing operate with spaced departures evenly within a certain period (e.g., peak or off-peak hours), resulting in relatively fixed train departure headways. This practice facilitates traffic management. Rail companies usually determine the frequency of departures per hour based on passenger volume, increasing train frequency during peak hours and reducing it during off-peak hours.

The departure time of each train at the initial station varies within a certain range. The specific arrival and departure times of each train can be determined by considering passenger demand and routing plan. Additionally, to achieve a balance between passenger demand and the interests of rail companies, the number of operating trains in this model is not fixed, making more realistic. By assuming a slightly larger total number of trains, a binary variable is used to determine the number of trains used based on the departure headway.

3.1.3. Passenger path choice in TRTP

Taking Figure 1 as an example, the path choice by passenger is briefly analyzed. The term "path" refers to the route which passenger takes from their origin to destination, while "train routing" refers to the track where trains operate back and forth between the origin and turnaround station.

In Figure 1, passengers are classified into five categories based on the locations of their origin and destination.

Blue passengers: Both the origin and destination stations are on the main line, located on the left side of the transfer station (between station 1 and station *Tr*). These passengers have two path options, taking trains on routing 1 or routing 3.

Orange passengers: Both the origin and destination stations are on the main line. The origin stations are between station 1 and station *m*-1, while the destination stations are between station *Tr*+1 and station *m*, or vice versa. These passengers can only select one travel path, taking the train on routing 1 to the destination.

Red passengers: The origin and destination stations are on the branch line (station *Tr* or between station *m*+1 and station *S*). For these passengers, there are two path options, taking the train on routing 2 or routing 3.

Green passengers: The origin and destination stations are on the main line and the branch line, respectively, located on the right side of the transfer station. The origin stations are between station *Tr*+1 and station *m*, and the destination stations are between station *m*+1 and station *S*, or vice versa. These passengers have two path options, taking the train on routing 1 (or 2) to the transfer station, then transferring to the train on routing 2 (or 1); or taking the train on routing 1 (or 3) to the transfer station, then transferring to the train on routing 3 (or 1).

Purple passengers: The origin and destination stations are on the main line and the branch line, respectively, and are located on opposite sides of the transfer station. The origin stations are between station 1 and station *Tr*, while the destination stations are between station *m*+1 and station *S*, or vice versa. These passengers also have two path options, taking the train on routing 1 (or 2) to the transfer station and then transferring to the train on routing 2 (or 1); or taking the train on routing 3 directly to the destination.

Additionally, considering train capacity in this model, it may exceed the maximum train capacity when passenger demand is very high, leading to overcrowding. This can result in passengers not being able to board trains within the specified time period, and this scenario is also considered in the optimization model.

3.2. Assumptions proposed in TRTP

This problem involves train operation and passenger path choice and the relationship between them is intricate. To simplify the modeling, the following assumptions are made considering with the reality of rail transit operations.

For passengers, transfer can occur at any station where two routings overlap. To simplify passenger travel paths, it is assumed that these passengers transfer to trains on other routing only at terminus stations at both ends of the routing overlap, and the time required for transfers is relatively consistent.

For rail companies, train operating time in each section and dwell time at each station are fixed values under normal circumstances. Additionally, for the ease of organizing train operate from one line to another, each routing adopts a balanced departure pattern (Lv et al., 2019). That is, the headway between different routings on the network can vary, but it remains the same for adjacent trains on the same routing.

3.3. Parameters and variables

The related parameters and variables are defined as follows.

Parameters:

L the set of alternative train routings, $l \in L$

S the set of stations, $m^{'} \in S$

OD the set of passenger origin-destination pairs, $od \in OD$

P_{od} the set of paths that can be selected by passengers whose origin-destination is *od*, $p \in P_{od}$

L_p the set of routings on path *p*, $l_a, l_b \in L_p$

N_l the set of available trains on routing *l*, $i \in N_l$

S_l the set of stations covered by routing *l*, $m \in S_l$

Tr the set of transfer stations, $tr_{l_a,l_b} \in Tr$, $l_a, l_b \in L_p$

T research period and the end time is the last time when passenger arrives at the station

t discretized the period *T*, *t* represents the discretized time interval, in minutes, $t = 1,2,3,\cdots,T$

σ the length of each time interval

$n_{od}(t)$ the number of passengers whose origin-destination is *od* during the time interval *t*

$w_{l,i}^{m}$ dwell time of train *i* at station *m* on routing *l*

$r_{l,i}^{m}$ operating time of train *i* in section *m* on routing *l*

h_{min}^{l} minimum departure headway of trains on routing *l*

h_{max}^{l} maximum departure headway of trains on routing *l*

hd safety departure headway of trains

e walking time for passengers to transfer

M a large positive integer

$C_{l,i}$ maximum number of passengers that train *i* on routing *l* can accommodate

$\varphi_{l}^{m^{'}}$ if routing *l* covers station m', $\varphi_{l}^{m'} = 1$, otherwise $\varphi_{l}^{m^{'}} = 0$

l_{max} maximum number of train routings for a plan

ε additional penalty time for each transfer

α additional penalty time for unserved passengers

β penalty for transfer waiting time

γ unit of account for operating cost of the train, CNY / (train · min)

$s_{l,m}^{l^{'},m^{'}}$ relationship between the station *m* on routing *l* and the station m' on routing l', if station *m* and station m' are the same station, $s_{l,m}^{l',m'} = 1$, otherwise $s_{l,m}^{l^{'},m^{'}} = 0$

$r_{od,p}^{l}$ relationship between the path *p* that can be selected by passengers whose origin-destination is *od* and the routing *l*, if path *p* contains the portion of routing *l*, $r_{od,p}^{l} = 1$, otherwise $r_{od,p}^{l} = 0$

$k_{od,p}^{l,m}$ relationship between the path *p* that can be selected by passengers whose origin-destination is *od* and the station *m* on the routing *l*, if path *p* contains station *m* and station *m* is not the destination of the path p, $k_{od,p}^{l,m}=1$, otherwise $k_{od,p}^{l,m}=0$

Decision variables:

ρ_l if routing l is selected, $\rho_l=1$, otherwise $\rho_l=0$

λ_i^l if train i on routing l is used, $\lambda_i^l=1$, otherwise $\lambda_i^l=0$

$d_{l,i}^m$ departure time of train i on routing l at station m

$a_{l,i}^m$ arrival time of train i on routing l at station m

h_l train departure headway on routing l

$q_{l,i,l',i'}^{m,m'}$ running sequence of trains, if the train i on routing l and the train i' on routing l' pass through the station m and m' respectively, and the station m and m' are the same station, and the train i departs before the train i', $q_{l,i,l',i'}^{m,m'}=1$, otherwise $q_{l,i,l',i'}^{m,m'}=0$

$z_{od}^p(t)$ if passengers whose origin-destination is od choose path p in the time interval t, $z_{od}^p(t)=1$, otherwise $z_{od}^p(t)=0$

$L_{od}^{p,l,i}(t)$ if passengers whose origin-destination is od and choose path p in the time interval t take train i on routing l, $L_{od}^{p,l,i}(t)=1$, otherwise $L_{od}^{p,l,i}(t)=0$

$Num_{l,i}^m$ the number of passengers in the train after departure of train i on routing l from station m

$\tau_{od}(t)$ waiting time of passengers whose origin-destination is od in the time interval t

$wt_{od}^p(t)$ waiting time at the origin station when the passengers with origin-destination of od choose path p in the time interval t

$tt_{od}^p(t)$ actual waiting time for transfer at the transfer station when the passenger with the origin-destination of od choose path p in the time interval t

$tt_{od}^{p,l_a}(t)$ actual waiting time for transfer at the transfer station when the passengers with the origin-destination of od choose path p and transfer out from the routing l_a in the time interval t

4. Integrated optimization model of TRTP

4.1. Objectives from supply and demand

The integration of TRTP in rail transit network is relevant to the balance between rail companies and passenger satisfaction. The optimization model aims to minimize operating costs for the companies while reducing passenger waiting time. By considering both aspects, the model needs to find a solution that benefits both parties. It optimizes train routing, departure times and headways to reduce costs for rail companies, while ensuring efficient transfers and minimizing waiting times for passengers. The optimization model discussed in this section aims to strike a balance between the interests of companies and satisfaction of passengers by achieving cost savings without compromising service quality.

a. operating costs of the company

The operating cost considered in this model is only train operating cost, which can be obtained by multiplying total operating distance of the train by unit operating cost. The total operating distance is equal to the length of each routing multiplied by the number of operating trains on the routing. Since the operating time in each section is fixed, the length of the train travel distance can be expressed by the train operating time. The calculation of company operating cost is shown in equation (1).

$$Z_1=\gamma\cdot\sum_{l=1}^{L}\left(\sum_{i=1}^{N_l}\sum_{m=1}^{S_l}\lambda_i^l\cdot r_{l,i}^m\right) \tag{1}$$

b. waiting time of passengers

Passengers who reached their destination successfully are called serviced passengers, and passengers who failed to reach their destination are called unserved passengers. For the served passengers, total waiting time of each consists of two components: the waiting time of passengers after swiping their card at the origin station $wt_{od}^p(t)$ and the transfer waiting time for passengers with transfer demands at the transfer station $tt_{od}^p(t)$. At this time, $z_{od}^p(t)=1$, served passenger waiting time is calculated by formula (2). Considering that the difference in transfer waiting time will affect the passenger's path choice, the transfer waiting time is multiplied by the corresponding penalty factor β.

$$\begin{gathered}\tau_{od}(t)\geq wt_{od}^p(t)+\beta\cdot tt_{od}^p(t)-\\ -M\cdot\left[1-z_{od}^p(t)\right]\\ od=1,2,\cdots,OD;p=1,2,\cdots,P_{od};t=\\ 1,2,\cdots,T\end{gathered} \tag{2}$$

For the unserved passengers, $\sum_{p=1}^{P_{od}} z_{od}^{p}(t) = 0$, the waiting time should be calculated by a certain penalty time, as shown in formula (3).

$$\tau_{od}(t) \geq \alpha \cdot \left[1 - \sum_{p=1}^{P_{od}} z_{od}^{p}(t)\right] \quad od = 1,2,\cdots,OD; t = 1,2,\cdots,T \tag{3}$$

After clarifying how the waiting time is calculated, the objective of the total waiting time of passengers can be shown in equation (4), including total waiting time of served passengers and total penalty time of unserved passengers.

$$Z_2 = \sum_{od=1}^{OD} \sum_{t=1}^{T} \tau_{od}(t) \cdot n_{od}(t) \tag{4}$$

Considering the two objectives, it is necessary to transform the multi-objective optimization model into a single-objective optimization model for solution. In addition, the dimensions of company operating cost and passenger waiting time are different. The two cannot be directly added as the objective function. Therefore, the linear weighting method is used to transform the multi-objective model into a single-objective model.
The objective function after linear weighting is shown in equation (5). By adjusting the value of the weight coefficient, company operating cost and passenger waiting time can be treated in the same dimension. It can also reflect the different emphasis between the two parties.

$$min\,Z = min(\omega_1 \cdot Z_1 + \omega_2 \cdot Z_2) \tag{5}$$

4.2. Constraints of TRTP

In this model, there are three types of constraints, including time constraint, path constraint and train constraint.

a. time constraint

(1) departure time constraint for the first train
Since the trains depart with equal headway, the first train on each selected routing must depart within the first departure headway of this routing. If the routing is not selected, the departure time of the first train on this routing is 0.

$$0 \leq d_{l,1}^{1} \leq h_l \cdot \rho_l \quad l = 1,2,\cdots L \tag{6}$$

(2) train operating time constraint
The arrival time of train i at station $m+1$ on routing l is equal to the departure time at station m plus the operating time in section $(m, m+1)$. If the routing l is not selected, the arrival time of the train on this routing at the next station is set to be 0.

$$a_{l,i}^{m+1} - d_{l,i}^{m} = r_{l,i}^{m} \cdot \rho_l \; l = 1,2,\cdots L; \quad i = 1,2,\cdots N_l; m = 1,2,\cdots S_l - 1 \tag{7}$$

(3) train dwell time constraint
In order to ensure that passengers have enough time to get on and off the train, the train has a certain dwell time at each station. If the routing l is not selected, the departure time of the train on this routing at this station is set to be 0.

$$d_{l,i}^{m} - a_{l,i}^{m} = w_{l,i}^{m} \cdot \rho_l \quad l = 1,2,\cdots L; \quad i = 1,2,\cdots N_l; m = 2,\cdots S_l - 1 \tag{8}$$

(4) safety departure headway time constraints
There are upper and lower limits for the departure headway of trains on each separate routing in all sections. If the routing l is not selected, the departure time of all trains on the routing is 0.

$$d_{l,i+1}^{1} - d_{l,i}^{1} = h_l \cdot \rho_l \quad l = 1,2,\cdots L; \quad i = 1,2,\cdots N_l - 1 \tag{9}$$

$$h \quad min_{l_{l_{max}}^{l}}^{l=1,2,\cdots L} \tag{10}$$

If there is no overlapping section between a certain routing and other routings, the departure headways in all sections on this routing shall meet the above constraints. If routing l and routing l' pass through the same section, the minimum safety departure headway is expressed as formula (11) and (12). When either routing l or routing l' is not selected, then formula (11) and (12) are always true. When both routing l and routing l' are selected, a constraint arises from equation (13). If station m and station m' are the same station, and the departure time of train i at station m is earlier than that of train i' at station m', then $q_{l,i,l',i'}^{m,m'} = 1$. As a result, formula (11) can be transformed into $d_{l,i}^{m} - d_{l',i'}^{m'} \geq hd$, which ensures the minimum departure headway constraint. The same applies vice versa. In addition, the departure headway of each routing has its upper and lower limits. Therefore, when two routings pass through the same section, the maximum departure headway of this section will certainly not exceed the upper limit of the given departure headway of each routing.

$$d_{l,i}^{m} - d_{l',i'}^{m'} \geq hd + M \cdot \left(q_{l,i,l',i'}^{m,m'} + \rho_l + \rho_{l'} - 3\right) \quad (11)$$
$$l, l' = 1,2,\cdots,L;\ i = 1,2,\cdots, N_l - 1; i' = 1,2,\cdots,N_{l'} - 1; m \in S_l; m' \in S_{l'}$$

$$d_{l',i'}^{m'} - d_{l,i}^{m} \geq hd + M \cdot \left(q_{l',i',l,i}^{m',m} + \rho_l + \rho_{l'} - 3\right) \quad (12)$$
$$l, l' = 1,2,\cdots,L; i = 1,2,\cdots, N_l - 1; i' = 1,2,\cdots,N_{l'} - 1; m \in S_l; m' \in S_{l'}$$

$$q_{l,i,l',i'}^{m,m'} + q_{l',i',l,i}^{m',m} = s_{l,m}^{l',m'} \quad (13)$$
$$l, l' = 1,2,\cdots,L; i = 1,2,\cdots,N_l - 1; i' = 1,2,\cdots,N_{l'} - 1$$

b. path constraint

(5) passenger path selection constraints

For each passenger from origin to destination, only when routing l on path p is selected, the passenger can select the path p to travel, as shown in formula (14). And the passenger can only choose one path at most, as shown in formula (15), for the serviced passengers, $\sum_{p=1}^{P_{od}} z_{od}^{p}(t) = 1$. For the unserved passengers, $\sum_{p=1}^{P_{od}} z_{od}^{p}(t) = 0$.

$$z_{od}^{p}(t) \leq \rho_l \quad (14)$$
$$od = 1,2,\cdots,OD; p = 1,2,\cdots,P_{od}; l = 1,2,\cdots,L_p; t = 1,2,\cdots,T$$

$$\sum_{p=1}^{P_{od}} z_{od}^{p}(t) \leq 1 \quad (15)$$
$$od = 1,2,\cdots,OD; p = 1,2,\cdots,P_{od}; t = 1,2,\cdots,T$$

(6) passenger boarding constraints

When the path$p$is selected, passengers must take a train on each routing on path$p$, as shown in equation (16). And only when a train is used, passengers can take this train, which can be described by formula (17).

$$\sum_{i=1}^{N_l} L_{od}^{p,l,i}(t) = z_{od}^{p}(t) \cdot r_{od,p}^{l} \quad (16)$$
$$od = 1,2,\cdots,OD; p = 1,2,\cdots,P_{od}; l = 1,2,\cdots,L_p; t = 1,2,\cdots,T$$

$$L_{od}^{p,l,i}(t) \leq z_{od}^{p}(t) \cdot r_{od,p}^{l} \cdot \lambda_i^l \quad (17)$$
$$od = 1,2,\cdots,OD; p = 1,2,\cdots,P_{od}; l = 1,2,\cdots,L_p; t = 1,2,\cdots,T; i = 1,2,\cdots,N_l$$

(7) relationship between passenger waiting time and path selection

The waiting time of the passenger at the origin station refers to the difference between passenger's boarding time and arrival time, satisfying equation (18).

$$wt_{od}^{p}(t) = \left(d_{l_1,i}^{o} - t\right) \cdot L_{od}^{p,l_a,i}(t) \quad (18)$$
$$od = 1,2,\cdots,OD; p = 1,2,\cdots,P_{od}; t = 1,2,\cdots,T; l_1 \in L_p, i = 1,2,\cdots,N_l$$

Waiting time for passenger during transfer is equal to the departure time at the transfer station minus the time to arrive at the transfer station and transfer walking time, shown as equation (19).

$$tt_{od}^{p,l_a}(t) = \left(d_{l_b,i'}^{tr_{l_a l_b}} - a_{l_a,i}^{tr_{l_a l_b}} - e\right) \cdot L_{od}^{p,l_a,i}(t) \cdot \cdot L_{od}^{p,l_b,i'}(t) \quad (19)$$
$$od = 1,\cdots,OD; p = 1,\cdots,P_{od}; t = 1,\cdots,T; l_a, l_b \in L_p; l_b > 1; l_a = l_b - 1; i = 1,\cdots,N_{l_a}; i' = 1,\cdots,N_{l_b}$$

Passengers may encounter more than one transfer during travel, so the total transfer waiting time is equal to the sum of the transfer waiting time of each transfer station in the selected path. Furthermore, the choice of different paths can result in varying transfer times during travel, ultimately impacting the level of travel comfort experienced by passengers. Therefore, the transfer times is multiplied by an additional penalty time and added to the transfer waiting time, as shown in equation (20).

$$tt_{od}^{p}(t) = \sum_{l_a=1}^{L_p} tt_{od}^{p,l_a}(t) + \left(L_p - 1\right) \cdot \varepsilon \quad (20)$$
$$od = 1,2,\cdots,OD; p = 1,2,\cdots,P_{od}; t = 1,2,\cdots,T$$

(8) routing selection constraints

Too many routings will make the operation organization more complex, so the total number of routings has upper limit requirements, as shown in formula (21). At the same time, it is also necessary to ensure that all stations and sections can be covered by at least one routing to meet the travel needs of all origin-destination demands, as shown in formula (22).

$$\sum_{l=1}^{L} \rho_l \leq l_{max} \quad (21)$$

$$\sum_{l=1}^{L} \rho_l \cdot \varphi_l^{m'} \geq 1 \quad m' = 1,2,\cdots,S \tag{22}$$

c. train constraint

(9) determination of the number of operating trains

When a certain routing is not selected, the trains on that routing will not be used, as shown in formula (23).

When the departure time of a train exceeds the research time period, the previous train of that is the last train used, as shown in formula (24) and (25). When$\rho_l = 0$, then $\lambda_i^l = 0$,$d_{l,i}^1 = 0$, (24) and (25) are always true. When$\rho_l = 1$, if $d_{l,i}^1 \leq T$, then$\lambda_i^l = 1$, otherwise $\lambda_i^l = 0$.

In addition, when train i is used, train $i-1$ must be used, as shown in formula (26).

$$\lambda_i^l \leq \rho_l \quad l = 1,2\cdots,L; i = 1,2,\cdots,N_l \tag{23}$$

$$T - d_{l,i}^1 \geq M \cdot (\rho_l + \lambda_i^l - 2) \quad l = 1,2\cdots,L; i = 1,2,\cdots,N_l \tag{24}$$

$$T - d_{l,i}^1 < M \cdot (1 - \rho_l + \lambda_i^l) \quad l = 1,2\cdots,L; i = 1,2,\cdots,N_l \tag{25}$$

$$\lambda_i^l \leq \lambda_{i-1}^l \quad l = 1,2\cdots,L; i = 1,2,\cdots,N_l \tag{26}$$

(10) train capacity constraints

The number of passengers on the train is equal to the sum of the number of passengers for each origin-destination who choose the train, as shown in equation (27). The number of passengers on each train must be less than or equal to the capacity of the train, which is train capacity constraint shown in formula (28).

$$Num_{l,i}^m = \sum_{o=1}^{OD} \sum_{t=1}^{T} \sum_{p=1}^{P_{od}} L_{od}^{p,l,i}(t) \cdot n_{od}(t) \cdot k_{od,p}^{l,m} \quad l = 1,2\cdots,L; i = 1,2,\cdots,N_l; m = 1,2,\cdots,S_l - 1 \tag{27}$$

$$Num_{l,i}^m \leq C_{l,i} \quad l = 1,2\cdots,L; i = 1,2,\cdots,N_l; m = 1,2,\cdots,S_l - 1 \tag{28}$$

4.3. Compatibility of headways on different routings

In a feasible train routing plan, there may be two routings covering the same section. The minimum safe departure headway time has more stringent requirements on the departure headway of these train lines. Therefore, this section discusses the compatibility of departure headways covering the same section. If given departure headways meet the train safety departure headway time, they are compatible, otherwise they are incompatible. The discussion results of departure headway compatibility of different routings can help to predict before solving, avoiding the generation of some solutions that don't meet the conditions and speeding up the solution process. In addition, solution space of the integrated model can be further reduced by combining with the compatibility analysis of departure headway of different routings.

Based on the discussion of compatibility in reference (Canca et al., 2016), here makes some modifications for the differences of this research. The original problem studies train routing plan and the decision variable is the departure frequency. However, proposed model in this research involves specific train timetable except for train routing plan, and decision variables include departure headway and departure time of the first train at the initial station. Therefore, the compatibility discussion of departure frequency in the original problem can be modified to the compatibility discussion of departure headway, which is more suitable for TRTP in this study.

Set the departure headway of one routing as h_l and the departure headway of the other routing covering the same section as $h_{l'}$, the compatibility matrix ζ is used to show whether the departure headways of the two routings are compatible. If $\zeta(h_l, h_{l'}) = 1$, they are compatible. If$\zeta(h_l, h_{l'}) = 0$, they are incompatible.

The compatibility of departure headway h_l and $h_{l'}$ shall meet the following conditions.

(a) $hd \cdot (\lceil T/h_l \rceil + \lceil T/h_{l'} \rceil) \leq T$

(b) $\forall i \in [0, \lceil T/h_l \rceil], j \in [0, \lceil T/h_{l'} \rceil], a \in [0, min\{h_l, h_{l'}\})$, $i, j, a \in Z$

$|a + h_l \times i - h_{l'} \times j| \geq hd$, where α is the allowable time adjustment amount.

Condition (a) ensures that departure headway should large enough to meet the requirements of safe departure headway time, which has eliminated many combinations of departure headways. For example, if $T = 60min$, $hd = 2min$, $h_1 = 4min$, $h_2 = 3min$ and routing 1 and routing 2 cover the same section, then

$hd \cdot (\lceil T/h_1 \rceil + \lceil T/h_2 \rceil) = 70 > 60$

The condition (a) is not met, so the departure headways are not compatible. This is because when the safety departure headway time is 2 minutes, assuming that train i on routing 1 and train j on routing 2 are two adjacent trains from the station, train i needs to depart 2 minutes after train j departs. However, the departure headway between train j+1 and train j is 3 minutes. The departure headway between train j+1 and train i is only 1 minute, which does not meet the safety departure headway time.

When condition (a) is satisfied, it is necessary to judge whether condition (b) is satisfied. An example is given, $T = 30$ min, $hd = 2$ min, $h_l = 10$ min, $h_{l'} = 5$min, both routings pass through section A-B. Figure 2 (a) shows the train departure headway of two routings. In Figure 2 (b), considering the safety departure headway time (represented by small rectangle), the compatibility of departure headway of two routings can be satisfied by adjusting the departure time of trains on both routings. It can be observed that this relationship can be obtained when the headway of one routing is a multiple of another.

When the departure headways of two routings are large enough, they may be compatible even though the departure headways are not an integer multiple. For example, $T = 30min$, $hd = 2min$, $h_l = 10min$, $h_{l'} = 15min$, both routings pass through section A-B. Figure 3(a) shows the departure headway of two routings. In Figure 3(b), considering the safety departure headway time (represented by small rectangle), the compatibility of departure headway of two routings can be satisfied by adjusting the departure time of trains on both routings.

Finally, an example is given to discuss the incompatibility of departure headways. $T = 30min$, $hd = 2min$, $h_l = 5min$, $h_{l'} = 6min$, both routings pass through section A-B. Although condition (a) is met, condition (b) cannot be satisfied. Figure 4 (a) shows the departure headway of two routings and Figure 4 (b)-(e) show all the adjustments. The departure headway in circle does not meet the train safety departure headway time. It can be seen that the departure headways of two routings are incompatible in any case.

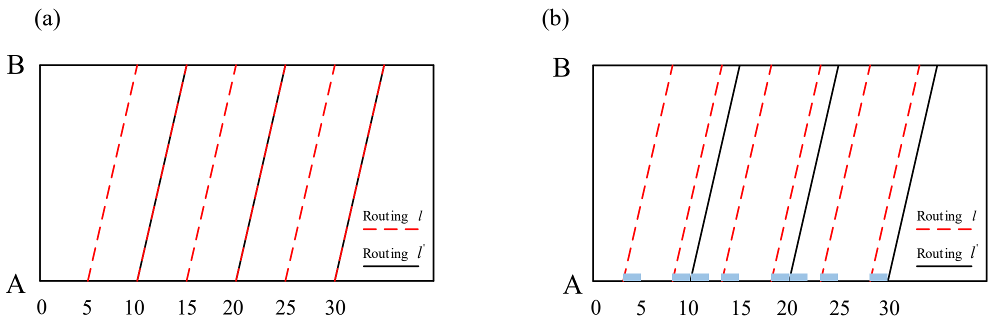

Fig. 2. Compatibility between departure headway of 10 min and 5 min

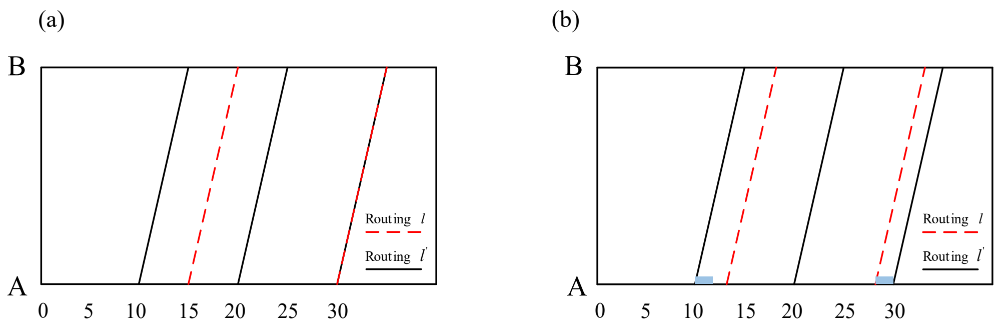

Fig. 3. Compatibility between departure headway of 10 min and 15 min

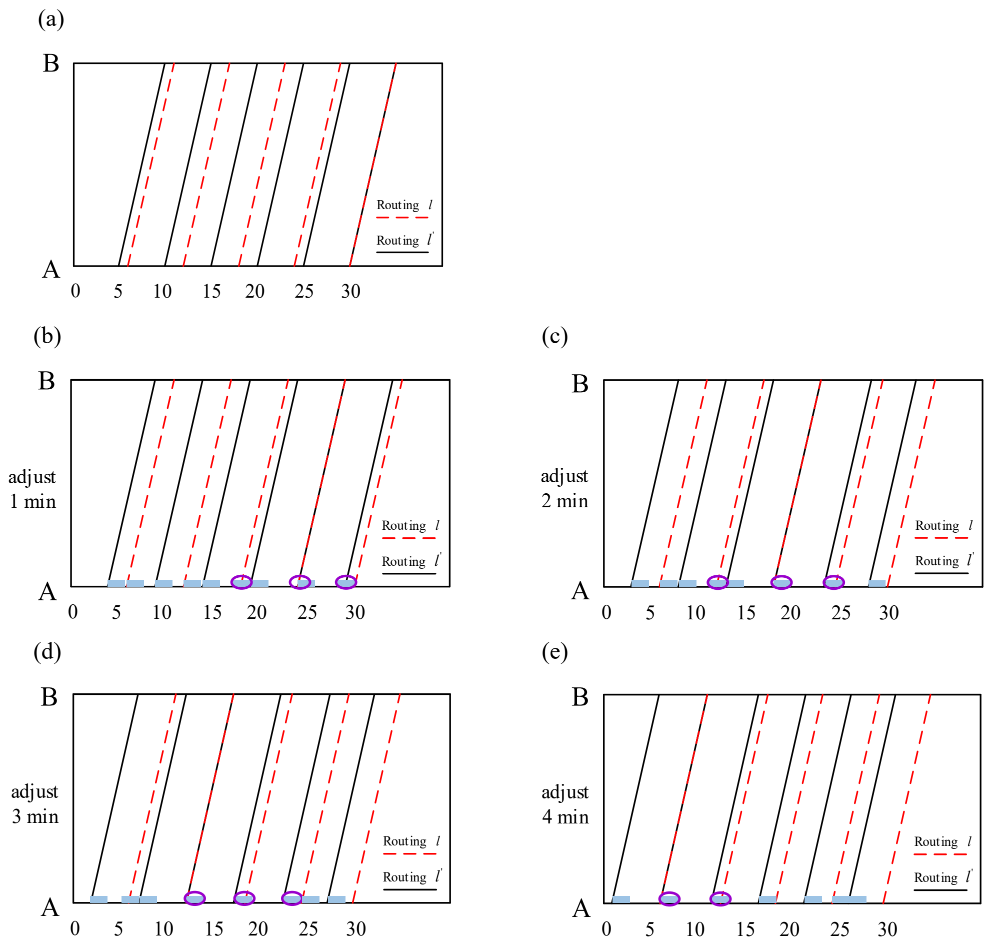

Fig. 4. Compatibility between departure headway of 5 min and 6 min

Taking one hour as an example, assuming the safety departure headway is 2 min, the compatibility matrix of some typical departure headways is given as follows.

$$\zeta = \begin{pmatrix} & h_l = 2 & h_l = 3 & h_l = 4 & h_l = 5 & h_l = 6 & h_l = 10 & h_l = 12 & h_l = 15 & h_l = 20 & h_l = 30 \\ h_{l'} = 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ h_{l'} = 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ h_{l'} = 4 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ h_{l'} = 5 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ h_{l'} = 6 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ h_{l'} = 10 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ h_{l'} = 12 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ h_{l'} = 15 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ h_{l'} = 20 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ h_{l'} = 30 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

5. Numerical experiment

5.1. Experiment description and result

As shown in Figure 5, there are four stations in the rail network. The main line and branch line are interconnected at station 2, with facilities and equipment conditions required for cross-line operation. There are four alternative routings, i.e. routing 1 (operating section is 1-3), routing 2 (operating section is 1-4), routing 3 (operating section is 2-3) and routing 4 (operating section is 2-4). Parameters required for solution are shown in Table 3 and the passenger demand with a granularity of 1 min is shown in Figure 6.

In the example, routing 1 and 2, routing 1 and 3, routing 2 and 4 cover the same section. Based on the discussion of departure headway compatibility and the parameter values, the compatibility matrix ζ of departure headway is obtained as follows.

$$\zeta = \begin{pmatrix} & h_l = 5 & h_l = 6 & h_l = 7 & h_l = 8 \\ h_{l'} = 5 & 1 & 0 & 0 & 0 \\ h_{l'} = 6 & 0 & 1 & 0 & 0 \\ h_{l'} = 7 & 0 & 0 & 1 & 0 \\ h_{l'} = 8 & 0 & 0 & 0 & 1 \end{pmatrix}$$

After the analysis and classification, CPLEX is used to solve the model. EPGAP is set to 5%. The computer processor is Intel (R) core (TM) i3-8100 @ 3.60 GHz 3.60 GHz and the internal storage is 8G.

EPGAP is a parameter for CPLEX, representing the ratio of difference between the upper and lower bounds of the objective function to the upper bound. This parameter has a significant impact on calculating quality and time. By general, CPLEX can seek the optimal solution, i.e., a solution with EPGAP equals to 0. When EPGAP is set to a certain value, CPLEX terminates the solution search while meeting the EPGAP and outputs the optimal solution. When EPGAP is no more than 5%, the difference between the upper and lower bounds of the objective function is not significant, but there is a significant difference in calculating time. Therefore, for this numerical experiment, to balance the calculating time and quality, EPGAP is set to 5%, just to verify that the optimization result of this model is convergent (Qi et al., 2021).

In this example, there are 7 combination schemes of routings. Among them, train departure headway of each routing in the same scheme shall meet the compatibility conditions. The combination schemes of routings with train departure headway and the optimized results can be calculated.

Among all the schemes, the optimal solution is achieved when routing 1 and routing 2 are selected. For this scheme, the departure headway between trains is set at 7 minutes, and a total of 7 trains are utilized. The objective function value for this optimal solution is 7155. The results of the integrated model are shown in Figure 7.

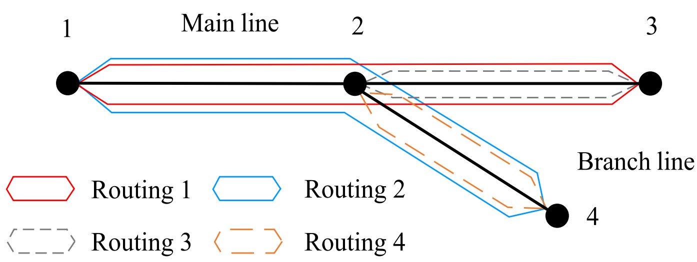

Fig. 5. Schematic diagram of experiment line and alternative routings

Table 3. Parameters of the numerical experiment

Parameters	Value	Parameters	Value	Parameters	Value
T	50 min	h_{max}^{l}	8 min	$r_{l,i}^{(2-3)}$	9 min
N_l	10	α	50 min/person	$r_{l,i}^{(2-4)}$	10 min
hd	2 min	β	1	e	4 min
h_{min}^{l}	5 min	$w_{l,i}^{2}$	1 min	ε	5 min/time
h_{max}^{l}	8 min	$r_{l,i}^{(1-2)}$	5 min	$C_{l,i}$	50 people/ train

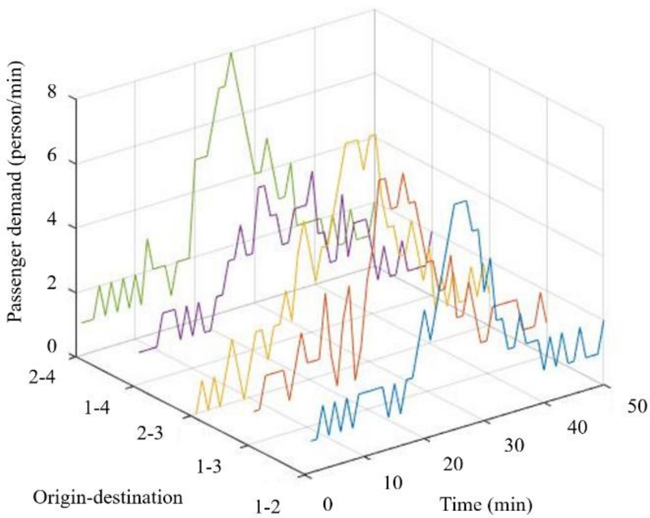

Fig. 6. Passenger demand of the numerical experiment

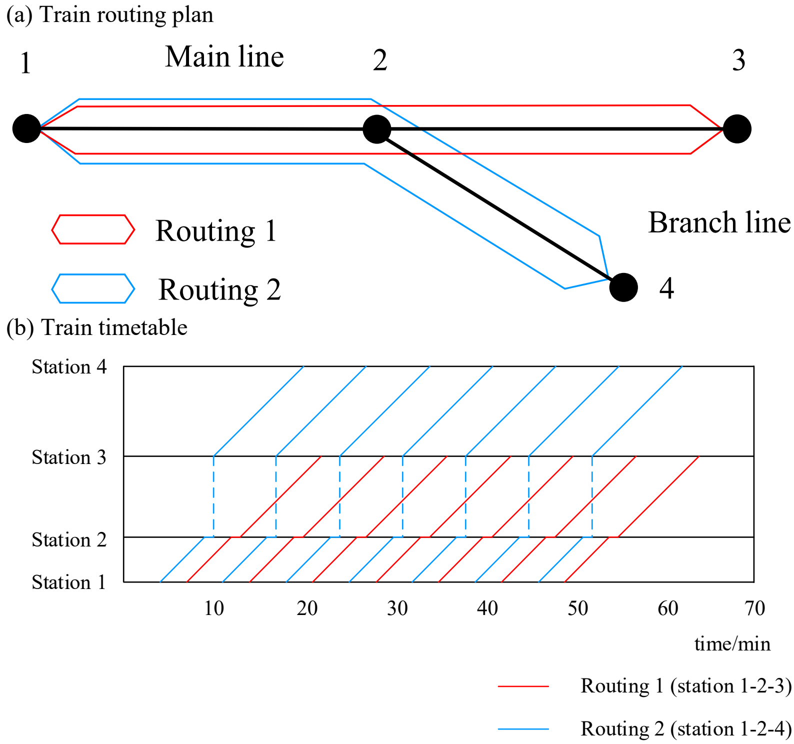

Fig. 7. Solution obtained by integrated optimization

5.2. Comparison with staged model

5.2.1. Optimization of TRTP

In order to analyze the optimization effect of this model further, the above result is compared with that of the staged optimization. Staged optimization refers to the optimization of the train routing plan based on static passenger demand data. After that, train timetable can be optimized based on the optimal train routing plan. The results of the staged optimization model are shown in Figure 8. The comparison of optimized results between staged model and integrated model are shown in Table 4.

The integrated optimized train operation plan achieves a 22.95% reduction in the objective function value compared to the staged model. However, it is observed that company operation cost in the integrated optimized plan has increased by 9.14% due to longer train running times resulting from the integration. Despite the increase in operating costs, the total passenger waiting time is reduced by 44.39%, indicating that the integrated optimized plan meets passenger demand better.

In terms of passenger waiting time, the integrated optimized plan shows significant improvements. The waiting time at origin station is reduced by 307 minutes, while transfer waiting time and its penalty are reduced by 814 minutes. This reduction is attributed to passengers with an origin-destination of 1-4 being able to reach their destination without transferring, saving waiting and penalty time significantly. Additionally, penalty time for unserved passengers is reduced by 1350 minutes, as the integrated and optimized plan accommodates more passengers, minimizing the number of unserved individuals.

Therefore, the result validates the effectiveness and accuracy of the proposed integrated optimization model. This model can reduce total passenger waiting time significantly while maintaining a minimal increase in the company operating cost. This solution can achieve a balance between passenger satisfaction and operational efficiency.

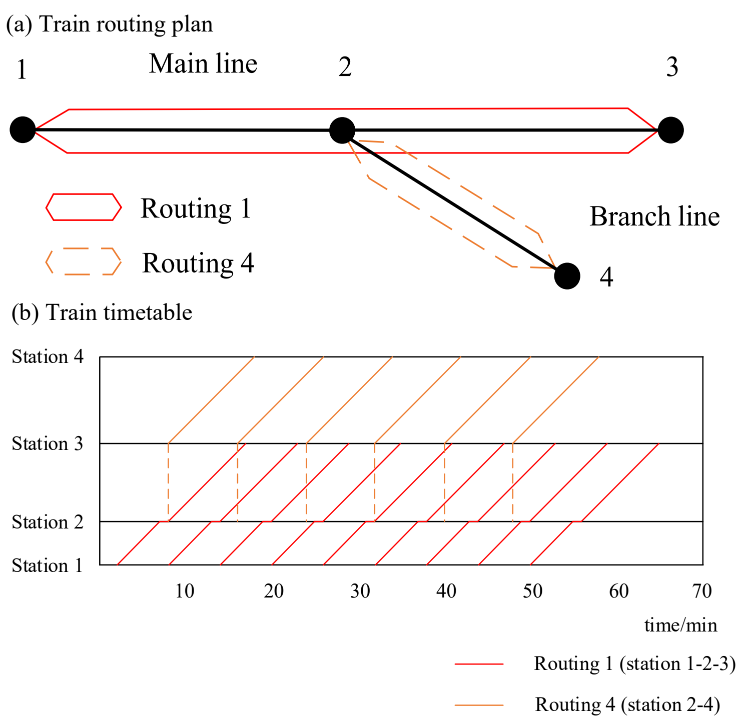

Fig. 8. Solution obtained by staged optimization

Table 4. Comparison of optimization results of the numerical experiment

	Value of objective function	Company operating cost/CNY	Total passenger waiting time/min	Waiting time at origin station/min	Waiting time for transfer and penalty/min	Penalty time for unserved passengers/min
Staged model	9286	3720	5566	3002	814	1750
Integrated model	7155	4060	3095	2695	0	400

Note: The value of EPGAP is 5%

5.2.2. Number of people and occupancy rate of each train

Table 5 and Table 6 show the number of trains in each section and the occupancy rate of trains under the integrated optimized model and staged model respectively. The data demonstrates a trend where the number of passengers on trains in each section and the occupancy rate of trains generally increase at first and then decrease. This pattern aligns with the observed passenger demand trend shown in Figure 6, where the demand increases and then decreases over time. Due to the constraints of train capacity, occupancy rate of all trains is less than or equal to 100%, which can meet passenger demand without overcrowding in the carriage, improving the comfort during travel.

In addition, the number of passengers in each section under the integrated optimization changes smoothly than the staged optimization. The average number of passengers is also smaller than staged optimization plan. Occupancy rate of each train is also more balanced.

Table 5. The number of passengers on trains and occupancy rate with optimization in stage

			train 1	train 2	train 3	train 4	train 5	train 6	train 7	train 8	train 9
Routing 1	Section 1-2	Number of passengers on trains/person	6	29	33	49	50	50	50	50	43
		Occupancy rate	12%	58%	66%	98%	100%	100%	100%	100%	86%
	Section 2-3	Number of passengers on trains/person	8	19	35	44	45	49	14	50	21
		Occupancy rate	16%	38%	70%	88%	90%	98%	28%	100%	42%
			train1	train 2	train 3	train 4	train 5	train 6			
Routing 4	Section 2-4	Number of passengers on trains/person	11	16	50	50	50	48			
		Occupancy rate	22%	32%	100%	100%	100%	96%			

Table 6. The number of passengers on trains and occupancy rate with integrated optimization

			train 1	train 2	train 3	train 4	train 5	train 6	train 7
Routing 1	Section 1-2	Number of passengers on trains/person	16	22	30	45	41	28	14
		Occupancy rate	32%	44%	60%	90%	82%	56%	28%
	Section 2-3	Number of passengers on trains/person	23	39	50	50	50	50	22
		Occupancy rate	46%	78%	100%	100%	100%	100%	44%
			train1	train 2	train 3	train 4	train 5	train 6	train 7
Routing 2	Section 1-2	Number of passengers on trains/person	9	19	23	39	47	35	16
		Occupancy rate	18%	38%	46%	78%	94%	70%	32%
	Section 2-4	Number of passengers on trains/person	18	25	48	50	50	47	17
		Occupancy rate	36%	50%	96%	100%	100%	94%	34%

5.3. Sensitivity analysis

5.3.1. Impact of weight coefficient

To facilitate the use of CPLEX to solve the model, it is necessary to convert the multi-objective optimization model into a single objective model. The weight coefficients of each objective are different and the impact on results are also different. Therefore, sensitivity analysis of the weight coefficients ought to be carried out.

In order to study the influence of different weight coefficients on optimization results, letω_1 take the values 1, 0.5, 0 and$\omega_2$take the values 0, 0.5, 1, forming a total of 5 weight coefficient combinations (1,0), (1,0.5), (1,1), (0.5, 1), (0,1). The optimal train operation plan under different combinations is shown in Table 7, and the corresponding company operating cost and total passenger waiting time are shown in Figure 9.

As shown in Figure 9, with the decrease of$\omega_1$and the increase of ω_2, the total waiting time of passengers is getting lower and the operating cost of companies is increasing. As the optimization objective becomes more focused on minimizing the total waiting time of passengers, the operating cost of companies is gradually neglected. To minimize passenger waiting time, more frequent service and selected routings ought to be considered, leading to an increase in the operating cost.

From sensitivity analysis of multiple experiments on weight coefficient, it can be observed that the coefficient combinations of (0,1) and (1,0) result in a significant disparity between the two objectives. The optimization of this model aims for better coordination between the supply side and demand side, which is contradictory with considering only one objective evidently. When the coefficient combination changes from (1,0) to (1,0.5), the ratio of decrease in total waiting time of passengers to increase in operating costs of companies is the highest. This indicates that the company can achieve a significant reduction in total passenger waiting time by increasing costs modestly, thus yielding the most optimal return on investment. For the supply side, this is undoubtedly the most cost-effective coefficient combination, which is suitable for scenarios with low demand and moderate profitability levels in rail transit operations.

Table 7. Optimal solution with different weight coefficients

ω_1, ω_2	Routing combinations	Number of trains used	Departure headway	Departure time of the first train	Value of objective function
1, 0	1-4	$N_1 = 6, N_4 = 6$	$h_1 = 8, h_4 = 8$	$d_{1,1}^1 = 8, d_{4,1}^1 = 8$	12050
1, 0.5	1-2	$N_1 = 7, N_2 = 7$	$h_1 = 7, h_2 = 7$	$d_{1,1}^1 = 7, d_{2,1}^1 = 5$	7221
1, 1	1-2	$N_1 = 7, N_2 = 7$	$h_1 = 7, h_2 = 7$	$d_{1,1}^1 = 7, d_{2,1}^1 = 4$	7155
0.5, 1	1-2	$N_1 = 10, N_2 = 10$	$h_1 = 5, h_2 = 5$	$d_{1,1}^1 = 3, d_{2,1}^1 = 5$	7292
0, 1	1-2-3	$N_1 = 10, N_2 = 10$ $N_3 = 10$	$h_1 = 5, h_2 = 5, h_3 = 5$	$d_{1,1}^1 = 3, d_{2,1}^1 = 5$ $d_{3,1}^1 = 2$	8815

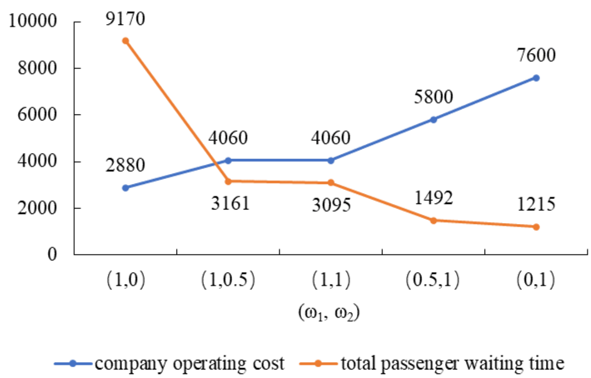

Fig. 9. Objective function with different weight coefficient combinations

However, if the company intends to enhance service quality further with increasing operational costs, the suitable coefficient combination will transition to (0.5,1). In this case, the company can still see returns on operating cost investment, although the return rate is not as high as transitioning from (1,0) to (1,0.5). Nevertheless, there is a noticeable compression effect on total waiting time of passengers, which can improve service quality effectively. This result is suitable for scenarios with high demand and optimistic profitability levels in rail transit operations.

The coefficient combination of (0,1) is not recommended because it increases operating costs significantly with minimal effect on compressing total waiting time. This is not a reasonable business strategy absolutely.

5.3.2. Impact of penalty coefficient of transfer waiting time

The difference in transfer waiting time will affect the passengers' choice of path. Therefore, sensitivity analysis of penalty coefficient β of transfer waiting time also ought to be carried out. To analyze the impact of different penalty coefficients on passenger path selection, we examine the case where β takes the values of 0, 0.2, 0.4, 0.6, 0.8 and 1. Specifically, we will focus on passengers with an origin-destination corresponding to 1-3 when the operation routing is 1-2-3 and the departure headway for each routing is 6 minutes. These passengers have two paths to choose. One is to take the direct train on routing 1 and the other is to take the train on routing 2 to station 2 and then transfer to the train on routing 3. The result of passenger path choice is shown in Figure 10.

With the increase of penalty coefficient, more passengers choose the direct path, while fewer passengers choose the path with transfer. As penalty coefficient increases, the waiting time for transfers perceived by passengers increases. To ensure the speed and convenience of travel, passengers will avoid transfer and are willing to choose direct path.

For potential transfer behavior that may occur, take Figure 1 as an example. For the fourth (green) type of passengers, no matter which path they choose, they must transfer. Therefore, they will take the first train that arrives at the origin station, and then take the first train on other routing after arriving at the transfer station. In the case of the fifth (purple) type of passengers, considering the up direction, if the first train arriving at the origin station is on routing 3, they will choose to board that undoubtedly. This choice is made to minimize their waiting time at the origin station and eliminate the need for any additional waiting time during transfers. If the first train arriving at the origin station is on routing 1, they will make a choice. In this research, choice behavior of passengers is as follows. If the arrival time of the next train on routing 3 and the train on routing 1 arriving at this time differs by less than or equal to a certain fixed value η, considering the speed and comfort of travel, they choose to wait for the direct train on routing 3. If not, they board the train on routing 1, travel to the transfer station and then transfer to the train on routing 2. This choice is made to optimize the overall travel time by taking advantage of the available train timetable and transfer opportunities.

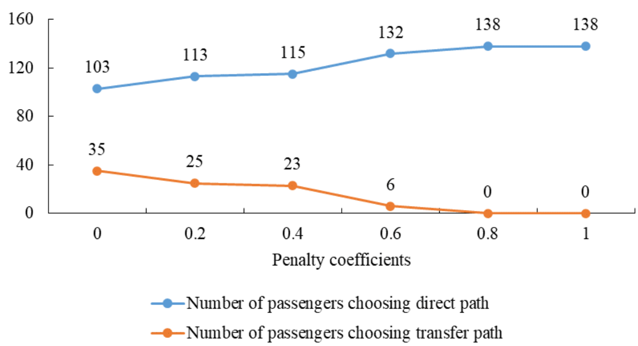

Fig. 10. The result of passenger path choice with different penalty coefficients for transfer

Figure 11 illustrates the scenario. In this case, the blue trains represent the direct trains on routing 3 without transfer. The yellow train is the non-direct train on routing 1, needing to transfer before they reach their destination. When the passenger arrives at the station, the last direct train (train *a*) has departed and the first train that arrived is a non-direct train on routing 1 (train *b*). In Figure 11(a), the arrival time gap between the next direct train (train *d*) and train *b* is 5 min. In Figure 11(b), the arrival time gap between the next direct train (train *d*) and train *b* is 3 min. If $\eta = 4min$, passenger in Figure 11(a) chooses to take train *b*, while passenger in Figure 11(b) choose to take train *d*.

6. Enumeration algorithm adapted to TRTP with network case study

Combining with research scenario, it can be seen that TRTP needs to consider the compatibility of headways on different routing covering the same section, and not each station has turnaround facilities and equipment conditions. Therefore, a large number of infeasible solutions can be eliminated from the candidate set, reducing the solution space. To address this, an enumeration algorithm adapted to TRTP is proposed.

Based on the above content, the process of designed enumeration algorithm is as follows.

Step1: Determine the set of alternative intersection.
Step2: Determine the feasible train routing plan.
Step2.1: Determine the departure headway of each routing under current train routing plan.
Step2.1.1: Determine the departure time of the first train at initial station of each routing under the current departure headway and train routing plan.
Step2.1.2: Calculate the objective function value of feasible scheme and record it.
Step2.1.3: Judge whether all the departure time of the first train at initial station of each routing under the current departure headway and train routing plan have been enumerated. If it is, turn to Step2.2. Else, turn to Step2.1.1.
Step2.2: Judge whether all the departure headway of each routing under current train routing plan have been enumerated. If it is, turn to Step3. Else, turn to Step2.1.
Step3: Judge whether all the feasible train routing plan have been enumerated. If it is, turn to Step4. Else, turn to Step2.
Step4: Compare all the values of objective function of all feasible schemes.
Step5: Output the set of optimal solution and end.

To demonstrate the applicability of proposed optimization model and enumeration algorithm, a case is tested on a railway network with a few lines. As shown in Figure 12, this railway network includes 13 stations and passenger can transfer at stations 2 and 5. Train operating time in each section has also included and train dwell time at stations is 2 min for each. There are 12 feasible routings in this case, and the specific operating path is detailed in Table 8. Figure 13 shows the passenger demand within each section of the railway network. Train operation parameters and other necessary parameters for resolution are provided in Table 9.

In this case, to cover all the passenger demands from each station, there are a total of 13 feasible combination schemes of routings. The headway of trains on each routing should satisfy the compatibility matrix mentioned in the previous section. Based on the proposed enumeration algorithm, the departure time of the first train from the initial station on each routing is determined sequentially, the objective function value can be computed, obtaining the optimization result.

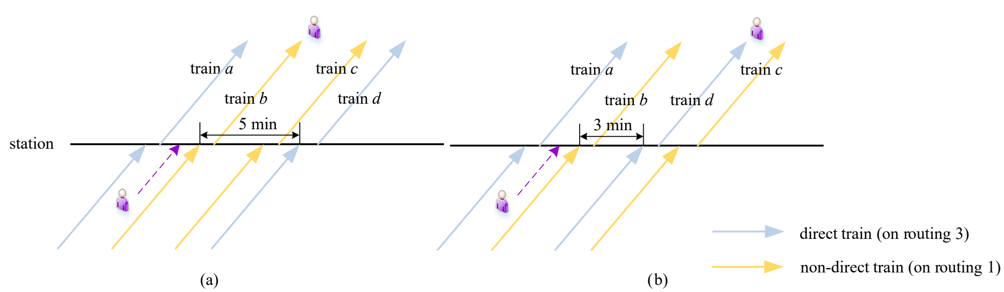

Fig. 11. Schematic diagram of passenger path choice behavior

Table 8. Set of alternative routings of the network case

Routing number	Train operating path	Routing number	Train operating path	Routing number	Train operating path
1	1-2-3-4-5	5	1-2-7-8-9-10-11-5	9	2-7-8-9-10-11-5
2	1-2-3-4-5-6	6	1-2-7-8-9-10-11-5-12-13	10	2-7-8-9-10-11-5-6
3	1-2-3-4-5-12-13	7	2-3-4-5-6	11	2-7-8-9-10-11-5-12-13
4	1-2-7-8-9-10	8	2-3-4-5-12-13	12	10-11-5-12-13

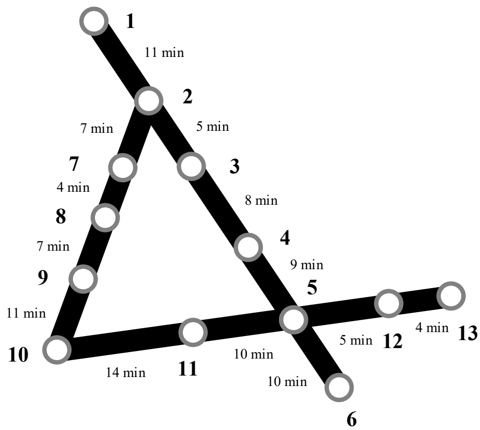

Fig. 12. Schematic diagram of the network case

Taking the weight coefficient combinations (1,1) represent (ω_1, ω_2) by example, among all the alternative schemes, the combination of routing 2, routing 9 and routing 12 yields the optimal solution. In this scheme, the headway on routing 2 is 6 minutes with a total of 10 trains operated, while routing 9 and routing 12 both have a headway of 8 minutes with 8 trains operated each. The value of objective function is 81576, with the company operating cost 44720 CNY and total passenger waiting time 36856 min. The corresponding train routing and timetable is shown in Figure 14. From the result, it can be observed that this solution caters to the significant passenger demand between station 10 and station 5. Routing 2 and routing 9 both have interconnected operation at station 2 and station 5, thereby avoiding transfer for some passengers.

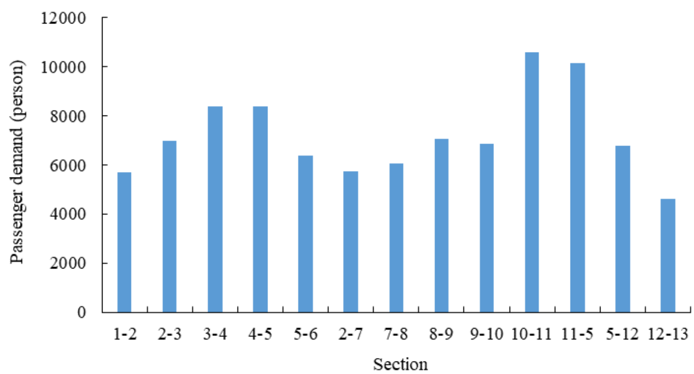

Fig. 13. Section passenger demand of the network case

Table 9. Parameters of the network case.

Parameters	Value	Parameters	Value
T	60 min	σ	2 min
hd	2 min	η	2 min
h_{min}^{l}	2 min	l_{max}	3
h_{max}^{l}	10 min	e	5 min
α	40 min/person	ε	3 min/time
β	1	$C_{l,i}$	900 people/train
γ	40 CNY / (train · min)		

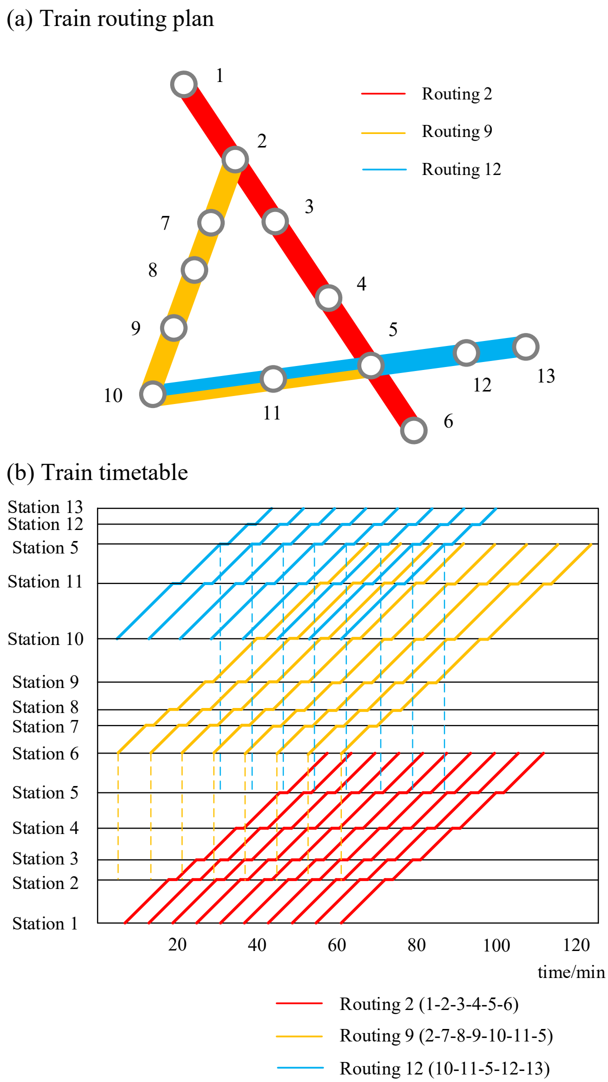

Fig. 14. Optimal solution of network case

7. Real case study of TRTP

7.1. Case description in practice

In this research, Guangzhou Metro Line 3 is selected to verify the effectiveness of the proposed integrated optimization model of TRTP in practical cases. The schematic diagram of Guangzhou Metro Line 3 is shown in Figure 15. The main line starts from Tianhe Coach Teriminal station and ends at Panyu Square Station and the north extension section starts from Tiyu Xilu station and ends at Airport North station. The total length of Line 3 is 64.41 km, with a total of 30 stations.

The research period of this case is 8:00-9:00. The train dwell time at stations and the section operation time are obtained from the actual train diagram. Other parameter settings are shown in Table 10.

The time-varying passenger demand data can be obtained from the Automatic Fare Collection (AFC) system of Guangzhou Metro, which is shown in Figure 16.

In Guangzhou Metro Line 3, Panyu Square, Dashi, Tiyu Xilu, Tonghe, Airport North and Tianhe Coach Teriminal have turnaround facilities and equipment conditions. According to the enumeration algorithm and operation rule, there are 8 feasible train routing for Line 3, as shown in Table 11.

Fig. 15. Schematic diagram of Guangzhou Metro Line 3

Table 10. Parameters of the case study

Parameters	Value	Parameters	Value
T	60 min	σ	1 min
hd	2 min	η	2 min
h_{min}^{l}	2 min	l_{max}	3
h_{max}^{l}	8 min	e	2 min
α	30 min/person	ε	0 min/time
β	1	$C_{l,i}$	1350 people/train
γ	60 CNY / (train · min)		

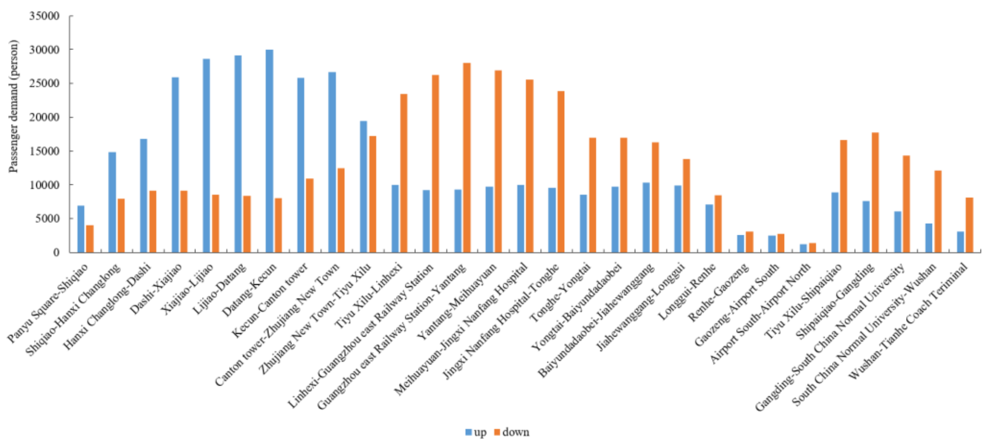

Fig. 16. Section passenger demand of Guangzhou Metro Line 3

Table 11. Set of alternative routings of the case study

Routing number	Train operating path	Routing number	Train operating path
1	Panyu Square -Tiyu Xilu	5	Dashi - Tonghe
2	Panyu Square -Tonghe	6	Dashi - Airport North
3	Panyu Square - Airport North	7	Dashi - Tianhe Coach Terminal
4	Panyu Square - Tianhe Coach Terminal	8	Tiyu Xilu - Airport North

7.2. Result of real case

According to the enumeration algorithm proposed, Python is used to solve the integrated optimization model. The computer processor is Intel (R) core (TM) i3-8100 @ 3.60 GHz 3.60 GHz, and the memory is 8G. When solving multi-objective model, optimizing one objective usually leads to the degradation of another. Obtaining only one specific result cannot achieve the purpose of multi-objective optimization. Therefore, multiple optimal solutions can be obtained, consisting the Pareto optimal solution set. The corresponding Pareto optimal frontier is shown in Figure 17 and all the optimization schemes contained with the Pareto optimal solution set are shown in Table 12.

The Pareto frontier demonstrates a clear relationship between company operating costs, total waiting time of passengers and the number of unserved passengers. As company operating costs increase, there is a consistent decrease in the total waiting time of passengers. Simultaneously, the number of unserved passengers also shows a declining trend. This can be attributed to the increase in the number of trains and the corresponding decrease in train departure headway. The greater number of trains allows for more frequent departures, thereby reducing passenger waiting time. Additionally, the decrease in train departure headway optimizes train resource utilization, resulting in further reductions in passenger waiting time. When decision makers pay special attention to the interests of company, they can choose Scheme 1. Scheme 20 may be chosen when decision makers place particular emphasis on passenger interests. When the decision maker has no obvious subjective preference, other alternatives can be chosen. Considering large passenger demand in Guangzhou Metro Line 3, a weight coefficients combination (ω_1, ω_2) of (0.5, 1) is suitable for this case with the sensitivity analysis of numerical experiments. According to this combination, the optimal result is scheme 7 of Table 12. The corresponding train timetable is shown in Figure 18.

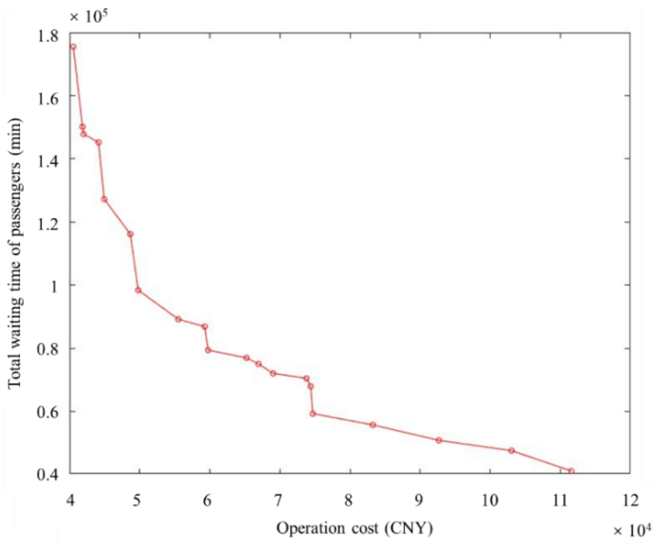

Fig. 17. Schematic diagram of Pareto optimal solution set

Table 12. Optimization schemes and results of the integrated model

Scheme number	Routing combination	Departure headway	Departure time of the first train	Company operating cost/CNY	Total passenger waiting time/ min	Actual waiting time of passengers/ min	Number of unserved passengers
1	1-3-4	$h_1 = 9, h_3 = 9, h_4 = 9$	$d_{1,1}^1 = 4, d_{3,1}^1 = 9, d_{4,1}^1 = 6$	40530	175512	139872	1188
2	1-3-4	$h_1 = 9, h_3 = 9, h_4 = 9$	$d_{1,1}^1 = 4, d_{3,1}^1 = 6, d_{4,1}^1 = 9$	41850	150136.5	139336.5	360
3	1-4-6	$h_1 = 8, h_4 = 8, h_6 = 8$	$d_{1,1}^1 = 4, d_{4,1}^1 = 8, d_{6,1}^1 = 6$	42000	147835	125245	753
4	1-4-8	$h_1 = 6, h_4 = 6, h_8 = 8$	$d_{1,1}^1 = 6, d_{4,1}^1 = 3, d_{8,1}^1 = 8$	44130	145210.5	127510.5	590
5	1-3-4	$h_1 = 8, h_3 = 8, h_4 = 8$	$d_{1,1}^1 = 4, d_{3,1}^1 = 8, d_{4,1}^1 = 6$	44940	127246	113626	454
6	1-3-4	$h_1 = 8, h_3 = 8, h_4 = 8$	$d_{1,1}^1 = 8, d_{3,1}^1 = 4, d_{4,1}^1 = 2$	48660	116199.5	116199.5	85
7	3-4	$h_3 = 6, h_4 = 6$	$d_{3,1}^1 = 6, d_{4,1}^1 = 3$	49800	98413.5	94513.5	130
8	4-8	$h_4 = 3, h_8 = 6$	$d_{4,1}^1 = 3, d_{8,1}^1 = 4$	55500	89230.5	84610.5	154
9	4-8	$h_4 = 3, h_8 = 5$	$d_{4,1}^1 = 3, d_{8,1}^1 = 3$	59280	86866.5	82246.5	154
10	3-4	$h_3 = 5, h_4 = 5$	$d_{3,1}^1 = 5, d_{4,1}^1 = 3$	59760	79446.5	78396.5	35
11	1-4-8	$h_1 = 4, h_4 = 4, h_8 = 6$	$d_{1,1}^1 = 2, d_{4,1}^1 = 4, d_{8,1}^1 = 5$	65250	76973	74033	98
12	2-4-8	$h_2 = 5, h_4 = 5, h_8 = 5$	$d_{2,1}^1 = 5, d_{4,1}^1 = 2, d_{8,1}^1 = 3$	66960	75065	72125	98
13	1-4-8	$h_1 = 4, h_4 = 4, h_8 = 5$	$d_{1,1}^1 = 2, d_{4,1}^1 = 4, d_{8,1}^1 = 4$	69030	72084	69144	98
14	4-8	$h_4 = 2, h_8 = 6$	$d_{4,1}^1 = 2, d_{8,1}^1 = 5$	73800	70513	67573	98
15	4-8	$h_4 = 3, h_8 = 3$	$d_{4,1}^1 = 3, d_{8,1}^1 = 1$	74400	67993.5	63373.5	154
16	3-4	$h_3 = 4, h_4 = 4$	$d_{3,1}^1 = 2, d_{4,1}^1 = 4$	74700	59343	58893	15
17	4-8	$h_4 = 2, h_8 = 4$	$d_{4,1}^1 = 2, d_{8,1}^1 = 3$	83250	55773	52833	98
18	4-8	$h_4 = 2, h_8 = 3$	$d_{4,1}^1 = 2, d_{8,1}^1 = 2$	92700	50875	47935	98
19	1-4-8	$h_1 = 4, h_4 = 4, h_8 = 2$	$d_{1,1}^1 = 2, d_{4,1}^1 = 4, d_{8,1}^1 = 1$	103050	47613	44673	98
20	4-8	$h_4 = 2, h_8 = 2$	$d_{4,1}^1 = 2, d_{8,1}^1 = 1$	111600	41153	38213	98

In order to verify the optimization effect of this model, the staged optimization of TRTP and the only optimization of train timetable under the current train routing plan are carried out respectively. The comparison is shown in Figure 19. Since the train routing scheme and departure frequency is given in the current train routing, only one optimal solution can be obtained when only the train timetable is optimized under the current routing plan.

The dominant solution in optimal set is achieved through integrated optimization and staged optimization of TRTP. This indicates that compared to optimizing train timetable solely, both integrated optimization and staged optimization can result in reducing company operating cost and total waiting time of passengers.

By comparing the Pareto front of integrated optimization and staged optimization, it can be found that the Pareto front of staged optimization is on the top right of the Pareto front of integrated optimization. In the Pareto optimal solution set of integrated optimization, there are dominant solutions of Pareto optimal solution set of staged optimization. While in the Pareto optimal solution set of staged optimization, there are no dominant solutions of Pareto optimal solution set of integrated optimization. This shows that the result of integrated optimization is no worse than that of staged optimization in any case. It also shows that the integrated optimization model proposed in this research can reduce the company operating cost and the total waiting time of passengers effectively.

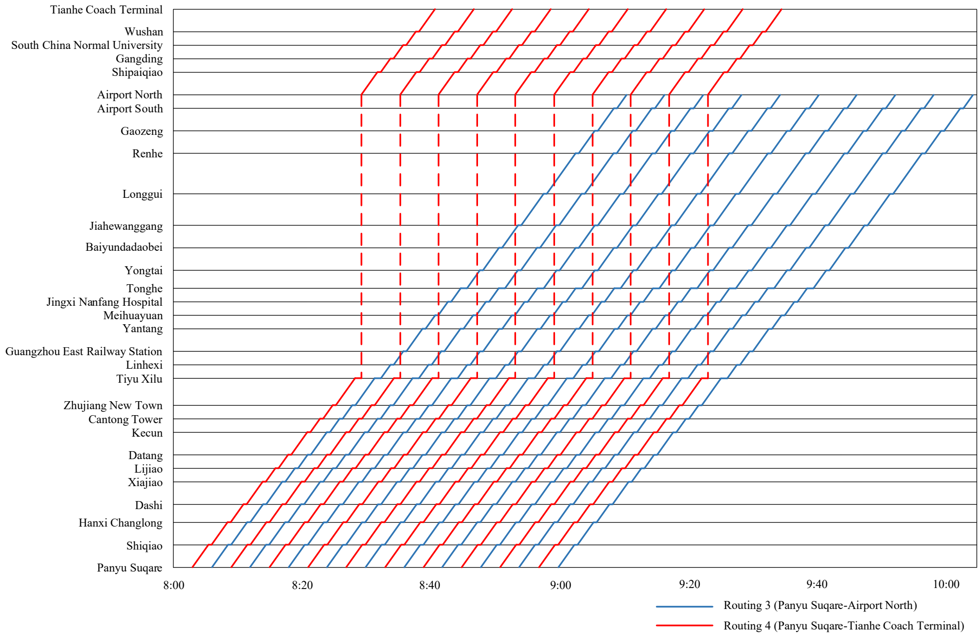

Fig. 18. Optimized train timetable of scheme 7 for Guangzhou Metro Line 3

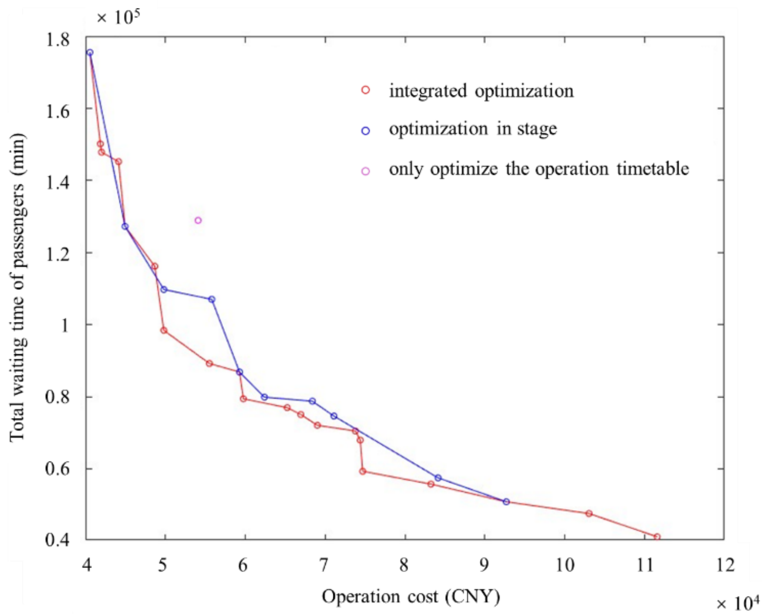

Fig. 19. Comparison of optimization results in different ways

7.3. Sensitivity analysis

7.3.1. Penalty time coefficient for unserved passengers

In the model of TRTP, the penalty time coefficient α for unserved passengers reflects the importance of passenger served or not. In order to analyze the influence of the penalty coefficient α on the optimization results, the values of α are set to 10, 20, 30, 40, and 50 respectively, and the Pareto frontier obtained is shown in Figure 20.

As shown in Figure 20, as α increases, the total waiting time of passengers generally increases, and the Pareto frontier changes. This is because when the value of α increases, in order to avoid unserved passengers, TRTP will be adjusted accordingly, resulting in a change in the optimal solution set. Therefore, the total waiting time of passengers is most sensitive to the change of the penalty time coefficient α of unserved passengers, and company operating cost is slightly sensitive to α.

7.3.2. Penalty coefficient for transfer waiting time

The penalty coefficient β of the transfer waiting time reflects the passenger's care about the transfer. In order to analyze the influence of the penalty coefficient β on the optimization results, the values of β is set to 0, 0.5, 1, 1.5 and 2, respectively, and the obtained Pareto frontier is shown in Figure 21.

As shown in Figure 21, an increase in the value ofβ leads to a general increase in the total waiting time of passengers, while company operating cost may remain unchanged or exhibit slight changes. This results in a shift in the Pareto frontier. The reason behind this observation is that asβ increases, there is a greater emphasis on reducing passenger transfers and minimizing transfer waiting times. To achieve this, TRTP is adjusted accordingly, leading to changes in the optimal solution set. Therefore, the total waiting time of passengers is most sensitive to the change of the penalty coefficient β for transfer waiting time, and company operating cost is slightly sensitive to β.

7.3.3. Unit operation cost of company

Changes in the unit operation cost of company will cause changes in the total operating cost, which will affect the decision-making of operators in turn. In order to analyze the impact of unit operation cost γ on the optimization results, the values of γ is set to 20, 40, 60, 80, and 100 respectively, and the obtained Pareto frontier is shown in Figure 22.

As shown in Figure 22, with the increase of γ, the company operating cost increases, the total waiting time of passengers remains unchanged, and the Pareto frontier simply shifts. When the value of γ increases, it results in a proportional change in the operating cost of all feasible solutions, while the total waiting time of passengers remains constant. Consequently, the optimal solution set is not significantly affected by this increase. Therefore, the company operating cost is sensitive to the change of unit operation cost, while the total waiting time of passengers is not.

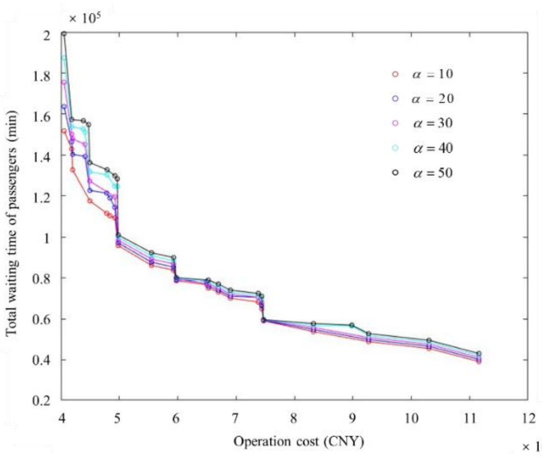

Fig. 20. Optimization results with different values of penalty coefficient for unserved passengers

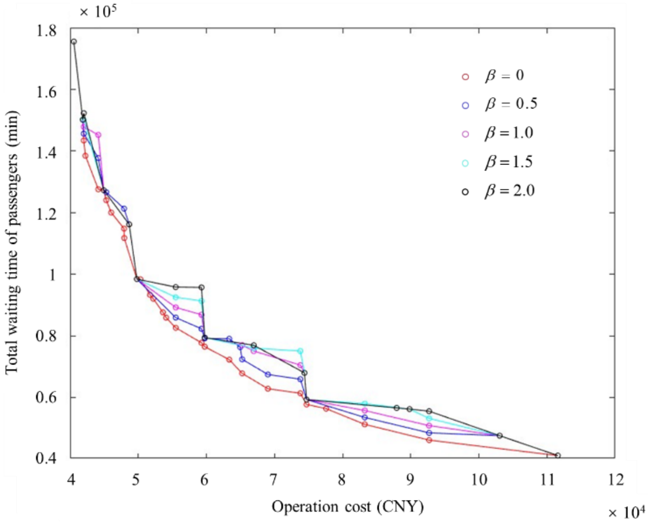

Fig. 21. Optimization results with different values of penalty coefficient for transfer

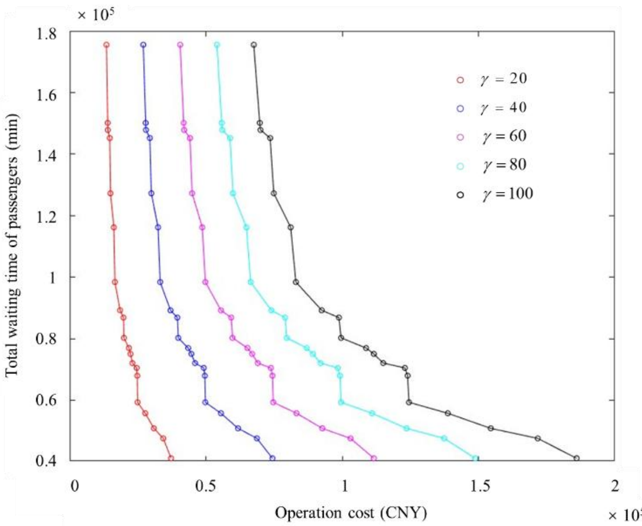

Fig. 22. Optimization results with different unit operation cost of company.

In summary, by comparing the optimization results of the proposed integrated optimization model of TRTP with results of the only optimizes train timetable in current routing plan and the staged optimization, it can be found that the proposed integrated model can reduce company operation cost and total waiting time of passengers effectively.

The sensitivity analysis reveals that the total waiting time of passengers is highly sensitive to the penalty time coefficient for unserved passengers and the penalty coefficient for transfer waiting time. Changes in these coefficients have a significant impact on the total waiting time experienced by passengers. On the other hand, the unit operation cost of the company does not have a substantial effect on the total waiting time of passengers.

In contrast, the company operating cost is most sensitive to the unit operation cost of the company. Variations in this cost parameter have a significant influence on the operating cost incurred by the company. Additionally, the company operating cost demonstrates a slight sensitivity to the penalty time coefficient for unserved passengers and the penalty coefficient for transfer waiting time.

These findings indicate that different factors have varying degrees of impact on the total waiting time of passengers and company operating cost. By understanding these sensitivities, decision-makers can focus their efforts on optimizing the parameters that have the greatest influence on the desired outcome.

8. Conclusion

An optimal train operation plan can balance the capacity and passenger demand of rail transit, reducing operating costs. At the same time, it can adapt to the passenger demand better, reducing waiting time of passengers. This research proposes an integrated optimization model of TRTP, attempting to solve the train routing and timetabling with time-varying passenger demand. The model aims at minimizing the company operating cost and total waiting time of passengers, and considers the constraints of the number of trains, train operation, passenger boarding, transfer relationship and train capacity. The model is verified by CPLEX with numerical experiment. The optimization results show that compared with staged optimization, the proposed integrated model is more effective. Thus it is necessary to integrate both the strategic and tactical level in the process of planning for urban rail transit network. Taking Guangzhou Metro Line 3 as an example, the integrated optimization model of TRTP is applied to a real case study, and the Pareto optimal solution set is obtained with proposed enumeration algorithm. The optimization results are obvious. In the future, other elements of the train operation plan (e.g. train stopping plan, train formation, etc.) can also be integrated into this model, cooperating to optimize all aspects of train operation in rail transit network.

Acknowledgement

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