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OPTIMAL DRIVER SCHEDULING WITH EVOLUTIONARY APPROACH IN URBAN PUBLIC TRANSPORT

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Abstract:

The optimal vehicle (VSP) and crew (CSP) scheduling problems solving in urban public transportation involves assigning timetabled trips to daily vehicle tasks, called blocks, and next to daily driver tasks, called duties. The aim is to minimize the number of vehicles and drivers. This maximizes usage of the both assets, makes the schedules more convenient for vehicle operations and drivers, and as a result mitigates possible delays, and fits into the legal requirements for the drivers' working time. The scheduling can be done either sequentially or simultaneously. In the former, vehicles are scheduled first, then drivers based on an already constructed vehicle blocks. Whereas in the latter, vehicles and drivers are scheduled at the same time, thus integrated (VCSP). As for the solution methods for the VCSP the Evolutionary Algorithms (EA) play a crucial role in addressing this complex challenge. Moreover, recent publications point out that metaheuristics, in particular the EA, are widely and successfully used in both the sequential and simultaneous solution approaches. The integrated scheduling of vehicles and rivers requires much longer computational time, which often cannot be accepted in practice for the VSP and CSP are usually solved separately by different institutions, i.e., the public transport authority (PTA) and the public transport operator (PTO), respectively. Alternatively, in the sequential approach, that is less computationally demanding, the influence of a solution of the VSP on a solution of the CSP can also be included. To solve sequentially the CSP taking into account an already solved the VSP and its characteristics (e.g., multiple depots, electric vehicles (EVs)), a specific and original EA is proposed in this paper. The algorithm divides vehicle blocks into pieces of work to be assigned to drivers' duties. The pieces of work can be composed of the entire vehicle blocks as well as their parts, for the algorithm can divide them according to a set of proposed rules. The algorithm was tested on a real-world databases of public transport systems of the three large Polish cities. The results were compared with those obtained manually by a team of experienced transport planners. The crew schedules obtained using the algorithm were comparable or even better than those prepared manually and reached within significantly shorter time.

Keywords: *bus public transport, crew scheduling problem, heuristics*

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1. Introduction

Providing bus transport services requires solving several planning problems such as network design, line planning, timetabling, vehicle scheduling, crew scheduling, and crew rostering. Problems are on all three levels of management. And in practice, these problems are solved in a sequential manner since solving them in one integrated step is too complex (Perumal et al., 2022). The two of the listed above problems are of scheduling nature. Optimal scheduling in urban public transport, specifically the vehicle (VSP) and crew scheduling (CSP), involves efficiently assigning trips to daily vehicle tasks, called blocks, and based on this assigning tasks or pieces of work to daily driver tasks, called duties. This two scheduling problems are solved either sequentially – first vehicles then crews based on an already constructed vehicle tasks, or simultaneously as partially or fully integrated vehicle and crew scheduling problem (VCSP). Both approaches have practical background in urban transport planning process, however, the sequential approach applied to improve operational efficiency is pointed out to be one of the promising ways to solve the VCSP (Pan et al., 2021). Evolutionary algorithms (EAs) play a crucial role in addressing this complex challenge, as seen in various research papers (Kisielewski, 2019; Mertens et al., 2023). Recent publications show that metaheuristics, in particular the EAs, can be successfully implemented to solve the sequential or simultaneous formulations of the scheduling problems.
The complexity of the listed above planning problems, including the VSP, CSP and especially VCSP, increases with fast-growing public transport sector undergoing rapid technological changes, e.g., the inclusion into fleets of an alternative powertrain vehicles, especially electric ones (EVs), and organizational ones too, e.g., global trends such as zero emission or smart cities, but also shortage of drivers on the labor market. Also rapidly growing driver salaries in developed and developing countries plays a role here. As a result, the scheduling problems become harder to solve especially for real-world instances. The VSP when considering electric vehicles (EVSP), in contrast to the traditional, internal combustion ones, has to cover such aspects as the much more limited range or required recharging time. Here, the type of batteries used place a role. If they are high energy ones (e.g., NMC or LFP), which are charged primarily at depots with plug-in chargers, but allow for longer ranges, or low energy batteries (e.g., LTO), which require overnight plug-in charging, allow for shorter ranges, but gives opportunity to recharge on the route via pantographs. This further influences the CSP that in turn has to cover such aspects as breaks within duties that have to stay in accordance with the drivers' working time law regulations or transport providers/operators' internal restrictions, and have to be planned in so called social points (e.g. equipped with toilets), but also adjusted to charging times (e.g., minimum required or long enough time to recharge traction batteries up to expected level), and charging points (location of charging stations).
The above-mentioned problems require an increasing effort from transport organizers and operators to solve them, e.g., optimally plan vehicle and crew schedules sequentially or simultaneously. The VCSP is the subject of a lot of research, and in the recent years gained a significant interest. The VCSP is an *NP*-hard combinatorial optimization problem, and in the case of real-world instances, the problem is even more complex. The real-world instances of the problem cover many thousands of line trips and a few thousands of driver duties to be scheduled on a daily basis. Moreover, in the real-world public transport systems there is a large number of decision variables and various constraints. All of that makes manual scheduling of vehicles and crews very difficult and time-consuming. In this paper we consider the sequential formulation of the VCSP with numerous raised in practice constraints, especially those concerning breaks in vehicle schedules influencing further the breaks in driver duties. Such formulation can handle the real-world instances of the VCSP, including the MD-VCSP.
The objective function of the VCSP should take into account both vehicle and crew aspects, especially the economical ones. Thus, the overall cost of using the two assets, i.e., vehicles and drivers, is minimized (Pan et al., 2021) leading to the efficiency of the entire public transport systems in which the minimum number of vehicles and drivers required is essential. And, as mentioned above, there are a large number of scientific works devoted to the VCSP, a significant part of them presents solutions that were tested on small transport networks with at most a few hundred line trips or driver duties (Ge et al., 2024). Meanwhile, in practice, even in a medium size cities, the number of line trips can reach several

thousand per day. Practical issues also require taking into account factors such as more than one depot, the possibility of changing lines within vehicle blocks and between blocks within drivers duties considering the given predefined preferences, fleet heterogeneity, and limitations related to electric vehicle charging infrastructure. In the literature such a problem is known as Multi Depot- Multi Type Vehicle Scheduling Problem (MD-MTVSP).

In addition, most authors consider only one or two optimization criteria in the objective function. Whereas an algorithm, the evolutionary one, proposed in this paper uses a multi-criteria objective function to assess particular solutions, parent and offspring ones, in the consecutive populations, i.e., in the selection process. There are six criteria including the number of driver duties, overall duration of breaks between line trips, number of line trips not included into driver duties, number of so called degenerated (short) duties, deviation of the actual duration of duties in relation to the expected one, and number of vehicle changes within a single duty. The listed criteria or aspects of the VCSP are also covered by the proposed mathematical model of the CSP solved as a secondary to the VSP (the sequential formulation). The developed heuristic makes it possible to obtain a solution in the form of a set of driver duties in an acceptable by practitioners time, even for very large instances. The developed algorithm was verified and validated using three real urban transport databases. The test results showed that the algorithm can be an effective tool for planning public transport even in large urban agglomerations.

The remainder of the paper is organized as follows. In the next section, related works from the literature are briefly summarized and research gaps are highlighted. In Section 3, the research problem is defined and binary liner model formulated. Then, the proposal of the heuristic algorithm is presented in Section 4. The description of real-world instances used for computational experiments and the obtained results are provided in Section 5. Finally, in Section 6, conclusions are drawn and proposals of future works are given.

2. Literature Review

The vehicle and crew scheduling problems in public transport can be approached separately (VSP and CSP, accordingly) or jointly (VCSP) through sequential (meaning combined to same degree, i.e., partially) and integrated optimization (meaning fully combined) (Shen and Li, 2023). Each offering unique advantages, but also having significant drawbacks. Both approaches, i.e., the sequential and integrated one, aim to enhance overall operational efficiency and service quality, yet they differ in flexibility, execution time and outcomes. Moreover, both aim to minimize operational fixed and variable costs of using the two assets, i.e., vehicles and drivers (crews), while maintaining expected service quality (Mertens et al., 2024). Integrated methods often achieve the above by considering multiple planning tasks simultaneously, leading to solutions reducing vehicle and crew costs at the same time (Mertens et al., 2024; Pan et al., 2021; Shen and Li, 2023). Such solutions show so called Pareto-optimality feature (Liu et al., 2023), which leads to the multicriteria optimization of the VCSP (Kisielewski, 2019), and the VSP and CSP as well (Wang et al., 2022). In the latter, a bi-level multi-objective programming model solved by a multi-objective particle swarm algorithm based on the ε-constraint processing mechanism is presented. Authors call it a collaborative scheduling of vehicles (first level) and drivers (second level). As for the service quality, both approaches ensure that schedules are reliable, and meet passengers' needs but also those of transport providers/operators, as well. As for the latter they have to fulfill strict law as well as company-specific regulations concerning drivers working time when scheduling their tusks. It includes breaks to be incorporated into shifts, as well. Mertens et al. (2023) state that legal, union-related, and company-defined regulations vary significantly regarding each problem specification. The detailed considerations on the drivers' working and break time constraints in the CSP can be found in works of Kletzander and Musliu (2020), and Boyer et al. (2018). But, however, in both researches the constraints in question are analyzed very deeply, the proposed solution methods have been applied to very limited instances of the CSP, i.e., only 12 shifts and 25 lines (1150 line trips), respectively. Integrated approaches, however, tend to enhance the service quality aspect more effectively taking into account interdependencies among tasks (Mertens et al., 2024). Both approaches also address the inherent complexity of public transport planning (Mertens et al., 2024; Shen and Li, 2023). While sequential approaches break down tasks, integrated ones manage complexity through advanced algorithms, such as evolutionary ones, which allow planning all tasks at once (Mertens et al., 2024). Despite these similarities, integrated optimization often yields superior solutions by leveraging synergy between vehicle and crew scheduling (Mertens et al., 2024). Whereas sequential optimization, may struggle with efficiency (a significantly lower computational time) in larger public transport networks, especially when applied in practice (Perumal et al., 2021; Perumal et al., 2022). The two approaches can also be viewed as quite flexible (Steinzen et al., 2010). The integrated approach when considering mathematical modelling (Pan et al., 2021), whereas the sequential one, when application stage is taken into account. Thus, while the integrated optimization can provide superior solutions by considering interdependencies among vehicle and crew tasks, sequential one remains valuable for their simplicity and ease of implementation. And even though an integrated consideration of multiple tasks leads to an overall improved solutions, i.e., significantly reducing operational costs while maintaining high service quality, it further increases the complexity of an already hard-to-solve planning problems (Mertens et al., 2024). The detailed and comprehensive comparison of the VSP, CSP and VCSP including their integrated and sequential formulations is presented in (Perumal et al., 2022; Shen and Li, 2023). Perumal et al. (2021) also state that the first complete integration of the VCSP for practical size instances and comparison between the integrated and sequential approaches were given by Freling et al. (2003).

The solution methods (algorithms solving mathematical models of the VSP, CSP and VCSP) can be exact or approximate (Peña et al., 2022). For instance, Shen and Li (2023) employ minimum cost flow formulation and integer linear programming to achieve exact optimal solutions of the electric vehicle and crew scheduling problem (EVCSP). The two another applications of the exact methods to solve the CSP are those of Esquivel-González et al. (2023) and Öztop et al. (2017). In the former, the graph based binary linear programming (BLP) is used to model the problem and the general purpose solver to find its solution. The instance of the problem solved is 5500 line trips, which is quite large. But the trips are clustered first into eight groups each for one particular depot, and further for vehicle types giving 49 classes in total. The problem is solved separately for each class, which reduces its instance significantly. Whereas in the latter, the problem is formulated as a BLP too, but solved using proposed by the authors the iterative valid inequality generation scheme and one of the general purpose solvers. Authors conclude that the proposed approach is quite effective in terms of computational time for instances with up to 120 tasks. As a result, however, the exact approaches are computationally intensive, thus, typically only feasible for smaller problem instances, as highlighted in (Mertens et al., 2024). However, Ge et al. (2024) point out that an integrated problems in public transport, that years ago had to be tackled by means of specialized heuristics due to their inherent problem complexity, now can be solved to optimality by means of standard solvers.

As for the approximate approaches heuristic and metaheuristic algorithms are involved. For instance, mentioned above Mertens et al. (2024) propose the mutation-based evolutionary scheme, which balances computational efficiency with solution quality, effectively handling problems even for large transport networks. In turn, Sistig and Sauer (2023) focus on metaheuristic approaches, particularly the Adaptive Large Neighborhood Search (ALNS), to address the integrated EVCSP and emphasize the importance of crew scheduling constraints influence on an operational costs.

And finally, Amberg and Amberg (2023) state that for vehicle and crew scheduling the two concepts are intensively studied to reduce resource usage and minimize operational costs. They are integrated scheduling of vehicles and drivers, and scheduling of this two resources with simultaneous trip shifting. The latter can lead to the further reduction of the resource usage comparing with pure cost-efficient integrated scheduling. And the trip shifting is not the only one extinction of the VCSP proposed in the literature. Mertens et al. (2024), who propose mentioned above an adaptive modular evolutionary extendable scheme to solve bus timetabling and scheduling problems, integrate them with crew scheduling as a third planning step. Thus, introduce a threefold integration. A possibility of a such extension confirm Ge et al. (2024), who point out that nowadays additional features can be incorporated into vehicle and crew scheduling problem formulations to make them richer.

Summing up the above analysis of the previous works on the VSP, CSP, and especially VCSP, it can

be stated that while the exact algorithms provide precise solutions, their complexity limits the applicability. In contrast, the approximate, heuristic and metaheuristic methods offer flexibility and efficiency, making them suitable for larger and more complex sets of tasks to be scheduled (Peña et al., 2022). Moreover, experimental results show that the accuracy of heuristic methods can be very high. The can lead to close to the optimum solutions, while capturing all the problem's characteristics and the same time (Andrade-Michel et al., 2021).
And despite the commonly recognized fact that vehicle schedules, including those of electric ones, are essential for the crew arrangement (Tang et al., 2019), and the large number of publications related to the sequential optimization of the VCSP, there is still a lack of papers which tackle the real-world instances of the problem taking into account its complexity. The complexity covering constraints which represent expectations of transport organizers and providers/operators, and including drivers and electric buses aspects as well. And at the solution stage algorithms dedicated to instances covering a few thousands of line trips or even more than 10 000. In this paper, we propose a metaheuristic (evolutionary) algorithm to tackle a real-world instances of the CSP solved sequentially along with the EVSP, i.e., including its already obtained solution. The proposed approach allows for solving even very large instances of the problem leading, in a reasonable computational time, to solutions of acceptable quality and better than manual ones.

3. Problem Definition and Mathematical Formulation

In general, the considered problem is to determine the assignment of vehicle trips to driver duties satisfying numerous labor regulations. It comes directly from this definition that to solve such a problem it is necessary to combine it with the VSP. It can be done by an integration of these two problems, leading to the Vehicle-Crew Scheduling Problem (VCSP) or by sequencing them, i.e., solving the CSP as a second and incorporating an already existing solution of the VSP. Thus, vehicle trips, which constitute the solution of the VSP, are income data to the CSP and highly influence its solution. A sequence of trips to be covered by particular vehicles on a daily basis is forming the so called vehicle blocks. Similarly, a sequence of vehicle blocks as well as their parts to be covered by particular drivers on a daily basis constitute driver daily duties, the so called duties.
In particular, taking into account the above, the considered problem is to determine the assignment of vehicle blocks, if necessary, divided into parts according to a set of predefined rules, to driver duties. All that based on such characteristics of each block or its part as starting and end times, and also locations being possible relief-points (including stops, terminuses, charging stations of the electric vehicles – EVs, and multiple depots). A searched solution must ensure that each entire vehicle block is covered exactly once, taking into account its parts, as well, and that each driver performs a feasible sequence of blocks or its parts – forming driver daily duties (Perumal et al., 2022). Thus, it constitutes the Multi-Depot Crew Scheduling Problem (MD-CSP) where, usually, the goal is to minimize the total generalized cost of driver duties including idle times, e.g., breaks, as well.
The above defined CSP was formulated as a binary linear programming (BLP) mathematical model. The model is a combination of the connection-based network flow approach and the generalized assignment problem (GAP). Such a combination was used by Beasley and Cao (1996) to propose the generic formulation of the CSP. However, their formulation was limited to one practical constraint only (skipping mathematical ones), i.e., fixed start and finish times such that each crew does not exceed a limit on the total time it can spend working. Moreover, they associate a unit cost with a transition between two consecutive tasks only, omitting costs of particular tasks. Such an approach is valid if unit working costs of all crews (drivers) are exactly the same. If not, this assumption has to be relaxed, as done in the proposed formulation.
As for the sequential optimization of the CSP being secondary to the VSP we use the approach as in Simões et al (2021), i.e., from the integrated formulation of the MD-VCSP, e.g., as in (Steinzen et al, 2010), only a part of the objective function and constraints associated with a crew schedule are considered.
The assumptions:

- Vehicle blocks, if necessary, are divided into parts, within a pre-processing stage, according to a predefined rules of selecting appropriate driver relief-points.

– Start (earliest – e_i) and end (latest – l_i) times of particular tasks are given as an ordinal numbers representing time points (minutes) enumerated starting with some predefined beginning point, e.g., the earliest time of the first task during a day. While, the difference of these two ordinal numbers ($l_i - e_i$) for each one particular task, gives its duration in minutes, the difference ($e_i - l_{i-1}$) for each two consecutive tasks, gives the duration of a break between them in minutes, as well.
– Tasks are ordered and numbered in ascending according to their start time (e_i), and as a second level the duration from short to long ones, thus according to their end time (l_i).
– Drivers can start and end their work in different depots or relief-points.
– Relief-points may be depots, terminuses, bus stops and, if the EVs are operated, recharging stations, as well.
– Based on the above, the maximum number of available drivers can be represented by the predefined cardinality of a set of driver duties S.
– Having the VSP already solved the depots do not have to be included into the CSP, even though its multi-depot (MD-CSP) version is considered. This will be discussed in details later.

The nomenclature used to define the optimization model is presented in Table 1.

Table 1. The optimization model nomenclature

Indices	
i, j	task, i.e., vehicle block or its part, $i = 1, \dots, \lvert T\rvert, \lvert T\rvert + 1$ and $i < j$, where $i = \lvert T\rvert + 1$ denotes a dummy task allowing to assign only one task i to a given driver duty ($X_{i\ j=\lvert T\rvert+1\ s} = 1$)
(i, j)	connection arc that joins pair of tasks
s	driver duty, $s = 1, \dots, \lvert S\rvert$
Sets	
A	set of arcs, all or only feasible, when considering the CSP solution, resulting from the VSP feasible solution, i.e., entire vehicle blocks or, if partitioned, their parts, including dummy tasks, as well, $(i, j) \epsilon A$
T	set of tasks, $i \epsilon T$
S	set of driver duties, $s \epsilon S$
Parameters	
b_i	vehicle block number task i belongs to (tasks being parts of blocks have the same numbers, whereas tasks $j = \lvert T\rvert + 1$, have the same block number as task i on arc (i, j)) [-]
b_{max}	maximum number of vehicle blocks assigned to a single driver duty [-]
bt_{min}	minimum break/waiting time between two consecutive tasks on driver duty [min.]
bt_{max}	maximum break/waiting time between two consecutive tasks on driver duty [min.]
$c_{t(s)}$	unit cost of work of a driver on duty s [monetary units/min.]
dt_i	duration time of task i [min.]
dt_{min}	minimum duration time of a single driver duty [min.]
dt_{max}	maximum duration time of a single driver duty [min.]
$DVIT$	total driver's daily vehicle inspection time [min].
e_i	earliest/start time of task i [-]
l_i	latest/end time of task i [-]
lt_i	number of line trips covered by task i [-]
lt_{min}	minimum number of line trips assigned to a single driver duty [-]
M	constant, very large positive number [-]
wt_{ij}	waiting time of task (vehicle block/line trip) change on arc (i, j) [min.]
Variables	
X_{ijs}	binary decision variable, $X_{ijs} = 1$, if driver duty s carries out task i directly preceding task j, 0 otherwise
Y_s	auxiliary binary variable, $Y_s = 1$, if driver duty s is used in a schedule, 0 otherwise

Eq. (1) presents the objective function, whereas Eqs. (2)-(10) the constraints.
Minimize:

$$\sum_{s\in S}\sum_{(i,j)\in A}\left(dt_i + wt_{ij}\right)c_{\mathrm{t}(s)}X_{ijs} + \sum_{s\in S} DVIT\, c_{\mathrm{t}(s)}Y_s \tag{1}$$

subject to:

$$\sum_{s\in S}\sum_{(i,j)\in A} X_{ijs} = 1, \qquad \forall i \in T \tag{2}$$

$$\sum_{(i,j)\in A} X_{ijs} - \sum_{(j,i)\in A} X_{jis} = 0, \qquad \forall i \in T, \forall s \in S \tag{3}$$

$$MY_s - \sum_{(i,j)\in A} X_{ijs} \geq 0, \qquad \forall s \in S \tag{4}$$

$$bt_{min} \leq \left(e_j - l_i\right)X_{ijs} \leq bt_{max}, \quad \forall i,j \in T, \forall s \in S \tag{5}$$

$$\sum_{(i,j)\in A} X_{ijs} \geq lt_{min} \;\text{ or }\; \sum_{(i,j)\in A}\left(dt_i + wt_{ij}\right)X_{ijs} \geq dt_{min}, \quad \forall s \in S \tag{6}$$

$$\sum_{(i,j)\in A}\left(dt_i + wt_{ij}\right)X_{ijs} \leq dt_{max}, \qquad \forall s \in S \tag{7}$$

$$\sum_{(i,j)\in A} b_i X_{ijs} \leq b_{max}, \qquad \forall s \in S \tag{8}$$

$$X_{ijs} \in \{0,1\}, \quad \forall (i,j) \in A, s \in S \tag{9}$$

$$Y_s \in \{0,1\}, \quad \forall s \in S \tag{10}$$

The objective function (1) is the minimized sum of crew cost when covering all the line trips present in a timetable and cost of waiting time between them plus cost of the total driver's daily vehicle inspection time. The crew cost is associated with drivers' working time, including breaks (waiting time).
The constraints (2)-(10) have the following meaning:

– constraint (2) ensures that each task is assigned to one and only one driver duty,
– constraint (3) conserves the flow,
– constraint (4) specifies the use of a driver duty in a schedule,
– constraint (5) ensures the time precedence of consecutive tasks carried out on a given driver duty, and that the break/waiting time between two consecutive tasks on that duty stays between its predefined minimum and maximum values,
– constraint (6) ensures that tasks assigned to each driver duty cover at least a predefined number of line trips or the duration of a duty is no shorter than a predefined time,
– constraint (7) ensures that the duration of a duty is no longer than a predefined time,
– constraint (8) ensures that tasks assigned to each driver duty cover no more than a predefined number of vehicle blocks,
– constraints (9) and (10) define the domains of the decision and auxiliary variables.

One can notice, that the multiple-depots are not included into the above model (no depot index), even though a problem is considered to be the MD-CSP one. It is due to the assumption (see above) that divers can start and end their duties in different depots or any other predefined relief-points. This assumption is crucial when minimizing the number of driver duties, thus, efficiently manage the number of necessary drivers. Depots in which drivers start and end their duties result from the assigned vehicle blocks, which, as mentioned above can be different. Which depots, it is defined in a solution of the VSP where they are already assigned to particular vehicle blocks. This information is included into the CSP as an input data.
The mentioned earlier model proposed by Beasley and Cao (1996) was solved to optimality for the instances up to 500 randomly generated driver tasks. In turn, in the authors' previous work (Duda et al., 2022), the Multi Depot- Multi Type Vehicle Scheduling Problem (MD-MTVSP) was solved to optimality using CPLEX Solver for the instances up to 1 000 line trips. However, for practical large problems (with the number of line trips > 1500 per day) the MIP solvers cannot find even a feasible solution. Therefore, a dedicated and efficient original Evolutionary Algorithm (EA) has been developed to solve

the above BLP model of the MD-CSP. The proposed algorithm is presented in the next section.

4. Proposed Evolutionary-based Algorithm

A traditional formulation of the CSP, used in the most of commercial optimization tools, has the drawback such that the crew duties are based on a fixed underlying vehicle schedule. Whereas several optimal vehicle schedules often exist, but the traditional formulation considers only one of them. Yet a VSP solution that is not considered may in fact lead to a better crew schedule. To avoid this drawback a solution of the VSP (in particular the MD-VSP) being a base for a crew duties planning is initially modified to improve a solution the CSP (in particular the MD-CSP).

The entire approach to solve the MD-CSP as a secondary to the MD-VSP (the sequential optimization) is composed of the four following stages:

– Solving the MD-VSP – deterministic but fast layer-based heuristic algorithm along with the Hungarian one is used to generate vehicle blocks.
– Preprocessing-stage – simple but fast heuristics is used to modify already existing vehicle blocks by putting into them potential relief-points allowing for much more flexible, thus, real-world constraints oriented solution of the CSP.
– Main-stage – the Evolutionary Algorithm (EA) is used to transform (divide) vehicle blocks based on the relief-points into feasible (meeting the real-world constraints) and ready to use (being good enough for direct implementation) pieces of work to be next assigned to daily driver tasks (duties).
– Post-processing-stage – deterministic but fast layer-based heuristic algorithm along with the Hungarian one is used to assign pieces of work to driver duties.

The main- and post-processing stages are interwoven, i.e., the latter is executed within the former, in between its first and second step – see below.

The first and last stages are based on the two well-known algorithms. The first is the layer-based algorithm, that is one of the basic procedures used when combining line trips, i.e., timetabled ones into vehicle blocks (Valouxis & Housos, 2002). In short, all trips are assigned to the so called layers. First, all trips are arranged in ascending order according to their end times (l_i). The end time l_1 of the first trip is used to create the first layer, i.e., all trips that start before l_1 ($e_i < l_1$) form the first layer. The remaining trips are put into subsequent layers in a similar manner. In this way, layers are created until the set of trips is empty. For further reading see (Kisielewski, 2019). However, in the post-processing-stage the layer-based algorithm was adapted to combine pieces of work (instead of line trips) into driver duties (instead of vehicle blocks). The second one is the Hungarian algorithm, that is one of the best algorithms for solving the generalized assignment problem (GAP), is used twice. Once, in the first stage to assign all line trips from consecutive layers to vehicle blocks adding technical trips (deadheads) and taking into account the real-world constraints. Alternatively, a greedy algorithm can be used here. The assignment procedure continues until all layers are checked. Thus, a solution of the MD-VSP is reached. And the second time, in the post-processing stage to assign pieces of work to driver duty. Thus, a solution of the MD-CSP is reached.

In between, there are the two stages being crucial for the entire approach.

In the preprocessing-stage already existing vehicle blocks are modified by putting into them relief-points according to some predefined rules, which are real-world constraints oriented. The rules are as follows:

– at the very beginning all entire vehicle blocks constitute an indivisible pieces of work to be scheduled, as in the classical CSP;
– some combinations of trips cannot be separated by relief-points, e.g., deadhead once and the first or last line trips in a vehicle block;
– relief-points can be placed in all depots, unless the previous rule is met or breaks are not too short;
– all perspective pieces of work between the two consecutive relief-points are accepted, unless their duration is neither too short nor to long;
– not accepted pieces of work (including entire vehicle blocks) having duration time longer at least twice as much as the maximum one are further divided into two parts, if possible having the same or similar duration;
– pieces of work (including entire vehicle blocks) can be divided into two parts at first at pre-selected stops allowing for the exchange of

drivers, and at second at terminuses or charging stations of the EVs;

- if the two parts meet all organizational constraints, e.g., are not too short, both are accepted;
- not accepted pieces of work (including entire vehicle blocks) having duration time of an inter-trip breaks longer than the maximum one are further divided into parts if, at the locations where the breaks occur, the exchange of drivers is possible, and accepted;
- pieces of work (including entire vehicle blocks) still not accepted are divide more (e.g., +20%) or less (e.g., - 20%) in the middle (conserving their duration) into two parts at stops or other points that allow for the exchange of drivers, and accepted;
- remaining not divided vehicle blocks or its parts are finally accepted as they are.

The above-presented rules of placing relief-points into vehicle blocks are used to mark potential points of their division that is going to be done in the next stage.

In the main-stage, vehicle blocks are divided into parts, i.e., pieces of work. The division can be done only at the relief-points. The aim of the division is to transform the entire vehicle blocks into pieces of work that make the MD-CSP solutions better, i.e., much more real-world constraints oriented.

It is the EA that is used to transform vehicle blocks. The input data feeding the EA are the vehicle blocks with relief-points placed into them.

The crucial element of the EA is a chromosome representing the problem to be solved. The problem is at the beginning coded as the chromosome, and eventually, when the optimization procedure is finished, encoded back to the problem, i.e., its solution. Genes in the chromosome we propose represent particular relief-points in the all consecutive vehicle blocks. The length of the chromosome equals to the number of all relief-points resulting from the preprocessing-stage. Values of the particular genes are binary (that makes the algorithm a Genetic one, actually) and can be set to 1 (true) or 0 (false) meaning that a given relief-point divides particular vehicle block into parts or not, accordingly – see Figure 1 (down).

The following standard steps are performed by the EA:

- Step 1 – Initialization: the initial population (first generation) of N individuals is generated assuming that at least one individual or some predefined percent (e.g., 10%, 20%, 30%, …) of them contains all the genes set to 1 (true), and the rest of individuals are generated randomly according to the uniform distribution. Here the post-processing-stage is executed.
- Step 2 – Evaluation: value of the fitness function is computed for each individual in the population. The fitness function presents Eq. (10).
- Step 3 – Selection: for the first, the best one individual is selected taking into account value of the fitness function, for the second, the rest n – 1 individuals that go to the set of parents are selected using the roulette method. As a result the set of parents contains n individuals.
- Step 4 – Creation of offsprings: it is done using a crossover operator. The one-point crossover is done randomly, both, concerning the selection of two parents that will produce two offsprings, and the crossover point. The number of n offsprings is produced.
- Step 5 – Creation of a new population (next generation): the new population is composed of N – n best, concerning the fitness function, individuals from the previous population plus n offsprings.
- Step 6 – Mutation: for each individual in the new population and each gene in the chromosome representing this individual a random value is drown according to the uniform distribution. This value compared to some predefined threshold decides if value (0 or 1) of this gene is flipped to the opposite one (1 or 0).

The post-processing-stage along with steps 2 to 6 are repeated until a stop criterion is met. It is the predefined computational time or a number of iterations without improvement of the value of fitness function, depending which occurs first.

The logic of the transition between the pre- and post-processing-stage, including the main one in between, i.e., from transforming (putting the relief-points and dividing) vehicle blocks into pieces of work up to assigning them to driver duties presents Figures 1, 2, and 3.

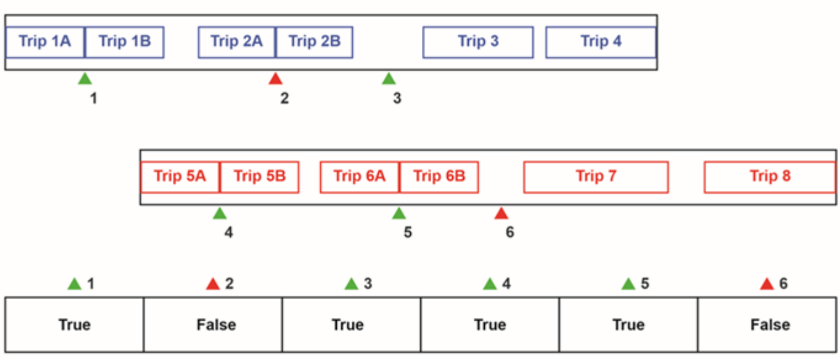

Fig. 1. Vehicle blocks divided into pieces of work (up) based on relief-points defined in the chromosome with six genes (down)

Fig. 2. Pieces of work divided into layers

Fig. 3. Driver duties with pieces of works assigned to them

As Figure 1 (up) shows at first (the preprocessing-stage) the six potential relief-points are put into two vehicle blocks at the all possible places staying in conformity with the rules described earlier. Each single gene in the chromosome denotes one relief-point – Figure 1 (down). Next (the main-stage) vehicle blocks are divided into six pieces of work at the selected relief-points – Figure 2, by setting values of particular genes to true – Figure 1 (down). Eventually (the post-processing-stage) the pieces of work are combined into three driver duties – Figure 3. The first (upper one) driver duty and the last (lower one) are not combined together into one duty due to their total duration time that exceeds the expected maximum one.

The last but not least element is the fitness function used in the EA. It is represented by a weighted sum of particular generalized cost components as in Eq. 11. Value of the fitness function is dimensionless and minimized for it is composed of generalized costs. It has to be kept in mind, that the execution of the EA is intermitted by the post-processing-stage. As a result, when the fitness function is computed the driver duties are already generated, thus, the number of them is known.

$$Min\ F_S = \omega_1 \frac{S_T}{S_{\max}} + \omega_2 \frac{\sum_{s=1}^{S_T} \sum_{i=2}^{W_s} f(w_{i-1i}) w_{i-1i}}{t_T} + \omega_3 \left(\frac{R - \sum_{s=1}^{S_T} r_s}{R} \right) + \omega_4 \frac{S_D}{S_T} + \omega_5 \frac{\sum_{s=1}^{S_T} t_{dev.}}{S_T} + \omega_6 \frac{\sum_{s=1}^{S_T} \lambda_s}{\lambda_{\max}} + \omega_7 \frac{S_N}{S_T} \quad (11)$$

where:

$\omega_{1...7}$ – weight coefficients; $\omega_{1...7} \in [0, 1]$,

S_T – total number of all driver duties,

$S_{\max}$ – maximum number of driver duties (number of available drivers),

W_s – number of pieces of work within driver duty s,

f – impact function of the inter-piece of works break time,

w_{i-1i} – inter-piece of works $i-1$ and i break time,

t_T – total duration time of all driver duties,

R – total number of all line trips within all vehicle blocks (trips scheduled when solving the VSP),

r_s – number of line trips within driver duty s,

S_D – number of degenerated driver duties, e.g., lasting for less than 2 hours or composed of less than 3 line trips,

$t_{\text{dev.}}$ – deviation of the actual duration time of driver duties in relation to its expected value, e.g., 8 hours,

λ_s – number of vehicle block changes within driver duty s,

$\lambda_{\max}$ – maximum number of vehicle block changes in a single driver duty – average number of block changes should not exceed 3, therefore it is assumed that $\lambda_{\max} \in [5, 10]$,

S_N – number of driver duties without required break (rest) time in social points.

The break time impact function $f(w_{i-1i})$ concerns the duration of a break between two consecutive line trips i. The function is presented in Figure 4, where $w_{\min}$ is the minimum break time, w_{opt} – optimal, expected break time, and $w_{\max}$ – maximum one. The $f_{\min}$ is a fourth parameter that can be adjusted in the range of [0, 1].

The four parameters ($w_{\min}$, w_{opt}, $w_{\max}$ and $f_{\min}$) of the break time impact function $f(w_{i-1i})$ allow to implement the obligatory driver's rest time requirements. In this research the EU's (thus, Polish as well) rest time regulations are considered, i.e., for driver duty duration:

- below 6 hours: no breaks required,
- between 6 and 8 hours: one 30-minute break or two 15-minute breaks,
- over 8 hours: one 45-minute break or three 15-minute breaks.

The demand on the between-trip breaks implemented with the $f(w_{i-1i})$ function forces adequate rest time breaks during scheduled duties. Moreover, even if the expected breaks are not met, the solution (an individual) is not excluded from the population but gets the very big cost, which with high probability excludes it from a selection for the next generation.

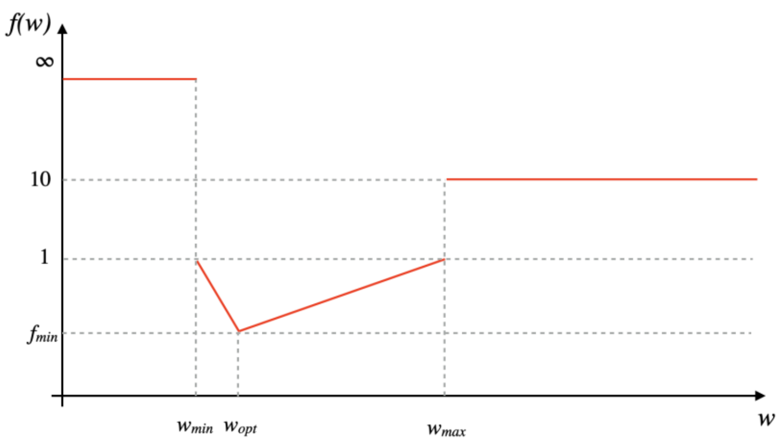

Fig. 4. Piecewise-linear impact function for evaluation of inter-trips break times

5. Computational Experiments and Results

5.1. Real-World Problem Instances – description

The proposed EA that minimizes as a primary goal the total crew working costs when inspecting and driving vehicles, and also waiting for the next task to be done was tested using a real-world data sets from three cities in Poland. The cities are in the top fifteen of the Polish cities with the largest number of inhabitants.

The first of the considered cities, denoted by C1, is the largest one with a population of more than 800 thousand inhabitants. In the analysis for the comparison to two other cities only bus transport was considered in the city C1. There are 4 bus depots in the city used by 2 public transport operators. The mixed fleet of over 600 buses altogether in both operators, over 500 buses in bigger operator, includes standard (12 m long) and articulated (18 m long) buses. Part of them is powered by traditional internal combustion (ON, CNG) and the other by electric engines. There are 172 bus lines in the city, which cover almost 2 300 km in total. During the working day there are about 11 200 line trips to be scheduled when solving the VSP. Regarding electric vehicles, there are 9 charging stations in the city transport network equipped with fast chargers. They are dedicated for recharging bus batteries on-route (using the plug-in or pantograph connection). There are also slow chargers in the depots for overnight charging (plug-in connection only). As for the CSP, in bigger operator, there are about 500 vehicle blocks during a working day to be divided into parts if necessary in aim to create pieces of driver work, i.e., duties for drivers. Each single vehicle block contains almost 19 line trips and its duration accounts for more than 16 hours, on average. Taking into account the above, the daily number of driver duties in bigger operator to be scheduled range up to 992 depending on the rules applied when dividing vehicle blocks into driver pieces of work (tasks).

The second of the considered cities, denoted by C2, is a medium size one with the population ranging from 300 to 400 thousand inhabitants. There are 3 operators whose buses are located in their own depots in the city. The public transport operators use a mixed fleet of buses as well. One operator also operates trolleybuses. The fleet with about 300 buses includes the same types of vehicles as in the case of the C1. There are 74 bus lines in the city, which cover 1 100 km in total, and about 3 800 line trips to be scheduled during a working day. There are 16 charging stations equipped with fast chargers, and slow chargers located in the depots as well. There are almost 300 vehicle blocks daily. Each single block contains about 13 line trips, and its duration accounts for about 15 hours, on average. The number of duties to be scheduled range up to 454.

And finally, the third of the considered cities, denoted by C3, is the smaller one with the population about 200 thousand inhabitants. One public operator operates buses from 1 depot in the city. The public transport operator uses a mixed fleet of buses including electric ones. The fleet of about 200 buses includes the similar types of vehicles as in the case of the two previous cities (C1 and C2). There are 54 bus lines in the city, which cover 863 km in total and, and about 2676 line trips to be scheduled during a working day. There are 2 charging stations equipped with fast chargers, and also slow chargers in the depots too. There are about 200 vehicle blocks daily. Each single block contains about 13 line trips, and its duration accounts for about 13 hours, on average. The number of driver duties to be scheduled range up to 304.

In all the above described cases the following real-world expectations in relation to the model parameters defined in Table 1 and Eqs. (5-8) with subscripts min or max were taken into account: i) minimum number of line trips assigned to a single driver duty $lt_{min} = 4$, otherwise a duty is considered to be degenerated; ii) maximum number of vehicle blocks assigned to a single driver duty, thus, changes of vehicle on a duty $b_{max} = 5 \; to \; 10$; iii) minimum break/waiting time between two consecutive trips on driver duty $bt_{min} = 2 \; to \; 5 \; [min.]$; iv) maximum break/waiting time between two consecutive tasks on driver duty $bt_{max} = 60 \; to \; 120 \; [min.]$; v) minimum duration time of a single driver duty $dt_{min} = 120 \; [min.]$, otherwise a duty is considered as degenerated; and vi) maximum duration time of a single driver duty $dt_{max} = 600 \; to \; 630 \; [min.]$.

5.2. Results Discussion

The detailed results of the simulation experiments carried out for the large- (C1), medium- (C2) and small-medium size (C3) cities are presented in Tables 2, 3 and 4, respectively. The results cover a selected but crucial KPIs, all minimized, characterizing particular driver duty schedules. There are two new driver duty schedules (S2 and S3) proposed based on the optimization results, and the one (S1) already used by each city. The schedules were generated according to the three different scenarios, i.e.:

- Scenario 1 (S1) – the base-line driver duties schedule being a real-world one used in a given city (large, medium or small one) and prepared manually by expert planners of public transport operator (PTO) based on a real-world VSP solution prepared manually as well;
- Scenario 2 (S2) – the driver duties schedule prepared using proposed approach to solve the MD-CSP (the evolutionary-based algorithm – stages 2, 3 and 4) as a secondary to the MD-VSP (the sequential optimization) where the VSP solution is a real-world one used in a given city and prepared manually by expert planners of PTO;
- Scenario 3 (S3) – the driver duties schedule prepared using the entire proposed approach to solve the MD-CSP (stages 1 to 4) as a secondary to the MD-VSP where the VSP solution is obtained within the first stage using the deterministic fast layer-based heuristic algorithm along with the Hungarian one.

Solutions in scenarios S2 and S3 were obtained using the proposed EA and appropriately adjusted values of weights in the fitness function, its 7 cost components (Eq. (11)). Thus, more than one KPI can be minimized at the same time.

As can be seen in Tables 2, 3 and 4 driver duty schedules prepared manually (S1) are mostly focused on avoiding vehicle (block) changes by drivers during their duties (no changes at all). This is a well-known problem in practice. Drivers are usually very reluctant to change a vehicle during a shit. Not to mention to change it more than once. They find it very inconvenient. Such observations are confirmed by Wang et al. (2022). Andrade-Michel et al. (2021) state that to improve the VCSP (they consider drivers' reliability too) the number of vehicle swaps per driver should be minimized. However, the schedules obtained within scenarios S2 and S3 introduce on average 40 vehicle block changes daily, which result in 0,07 changes per one duty, only. In the other words 7% of duties which contain one change of a vehicle. But, this is enough to reduce at the same time the number of duties, thus, drivers by almost 6% on average. The vehicle block changes allow in turn for the reduction of the regular breaks time by almost 11%, on average. These are the brakes between consecutive pieces of work (driver tasks) scheduled for a given duty. But, taking into account the way the proposed EA transforms (divide) vehicle blocks into pieces of work based on the relief-points the breaks are an inter-line trip ones, as well. Therefore, it is the reduction of regular breaks time due to an introduction of the vehicle block changes that influences (reduces) the number of duties. To reduce the number of duties is especially important when facing the severe shortage of drivers which is a dominating trend in European Union (EU). And, this is what the current situation at the transport market looks like. The problem is not the availability of vehicles at all, but drivers.

Table 2. CSP (daily) solutions for a large-sized city (C1)

Scenario: **KPI** (minimized):	**S1**	**S2**	**S3**
Vehicle blocks [-]	502	502	**496**
Duties (= drivers) [-]	992	936	**912**
Long duties (> 10h) [-]	69	**44**	151
Working time [h]	8 081	7 767	**7 561**
Regular breaks time [h]	2 465	2 392	**1 862**
Long breaks time [h]	291	**189**	443
Total time [h]	10 837	10 348	**9 866**
Vehicle block changes [-]	**0**	7	149

Table 3. CSP (daily) solutions for a medium-sized city (C2)

Scenario: **KPI** (minimized):	**S1**	**S2**	**S3**
Vehicle blocks [-]	282	282	**272**
Duties (= drivers) [-]	454	**434**	443
Long duties (> 10h) [-]	97	**14**	42
Working time [h]	4 324	4 010	**4 000**
Regular breaks time [h]	875	877	**805**
Long breaks time [h]	316	**326**	439
Total time [h]	5 515	**5 213**	5 244
Vehicle block changes [-]	**0**	3	32

Table 4. CSP (daily) solutions for a small-sized city (C3)

Scenario: **KPI** (minimized):	**S1**	**S2**	**S3**
Vehicle blocks [-]	199	199	**172**
Duties (= drivers) [-]	304	294	**272**
Long duties (> 10h) [-]	0	0	0
Working time [h]	2 558	2 566	**2 447**
Regular breaks time [h]	713	721	**586**
Long breaks time [h]	302	311	**222**
Total time [h]	1 015	1 032	**808**
Vehicle block changes [-]	**0**	14	32

The introduction of vehicle block changes is the cost to be paid for the reduction of the number of necessary drivers, which seems to be not too high. Considering the C1 and scenarios S1 and S2 only, it can be concluded that in this case a 6% reduction of the number of duties can be obtained while reducing by one-third the number of long duties (> 10h) or long breaks time.

As for the results obtained for the particular cities (C1-C3) it can be observed that values of the all KPIs presented in Tables 2, 3 and 4 can be improved while minimizing the number of driver duties. There is only one exception, i.e., the number of vehicle block changes that increased in all analyzed cases (C1-C3). Moreover, no significant differences and regularities have been observed when solving the MD-CSP sequentially along with the MD-VSP for large, medium and small-size cities using as a starting point the VSP solution prepared manually or with a help of the proposed approach (stage one). Obtained solutions of the MD-CSP, their characteristics and quality do not depend both on the size of the analyzed city (C1-C3), and the starting point, i.e., the way a solution of the MD-VSP is obtained (S2-S3). When solving the MD-CSP good results can be obtained starting from both the manual and algorithm based VSP solutions.

To compare the solutions obtained using the proposed EA (S2 and S3) to the base-line, manual one (S1) relations of the KPIs are presented in Figure 5. The values of the KPIs for the scenarios S2 and S3 were divided by appropriate values obtained under the S1 scenario. Thus, in Figure 5 the increases (+%) and decreases (–%) of these relational values can be observed. Knowing that the lower values are the better for all the KPIs, advantages (–%) of the solutions obtained using the proposed EA can be observed, and at the cost of which disadvantages (+%) they were gained. The vehicle block changes KPI has been excluded from this analysis for its values obtained under the S1 scenario equal to zero. However, the number of vehicle block changes increased in scenarios S2 and S3 in comparison to the S1. And, the increase of the number of vehicle block changes is significantly higher (almost 9 times) in S3 than in S2. This implies that solving the MD-CSP based on the MD-VSP solutions obtained using the proposed EA results in higher numbers of vehicle block changes than for the solutions of the MD-VSP obtained manually. This shows that the deterministic, layer-based heuristic algorithm along with the Hungarian one which is used to generate vehicle blocks is fast but not good enough. In general, it can be stated that to reduce the number of duties, and thus the number of necessary drivers it is necessary to accept the introduction of the increased number of vehicle changes within the duties. But at the same time, both the number of long duties (by -15% on average over analyzed cases) and total time (-7%) can be reduced significantly. And finally, based on the entire set of solutions obtained for the all cities and scenarios, a correlation analysis was conducted for particular KPIs – Table 5.

Table 5. Pearson correlation coefficient (r) for pairwise combinations of KPIs

KPI	**Vehicle blocks**	**Duties**	**Long duties**	**Working time**	**Regular breaks time**	**Long breaks time**	**Total time**
Duties	**1,00**						
Long duties	0,69	0,67					
Working time	**1,00**	**1,00**	0,69				
Regular breaks time	**0,96**	**0,98**	0,52	0,97			
Long breaks time	0,05	0,01	0,50	0,03	-0,16		
Total time	0,98	0,97	0,71	0,98	0,92	0,11	
Vehicle block changes	0,35	0,33	**0,67**	0,33	0,19	**0,59**	0,29

In Table 5 there are positive and negative relations between the KPIs characterizing the crew duties schedules obtained using the proposed evolutionary based algorithm. The most interesting and important relations are marked in bold. At first the full correlation (r = 1.00) between the number of vehicle blocks, duties and working time can be observed. The regular breaks time is also almost fully correlated with the number of vehicle blocks and duties. This can be observed in Tables 2, 3 and 4, as well. Thus, the higher number of vehicle blocks the higher number of duties and longer working and regular brakes times. As for the vehicle block changes, the increase in their number implies the increase in the number of long duties and requires longer breaks within them. In the other words to change a vehicle within a duty driver has to wait. Long breaks are usually connected with the inter-peak period.

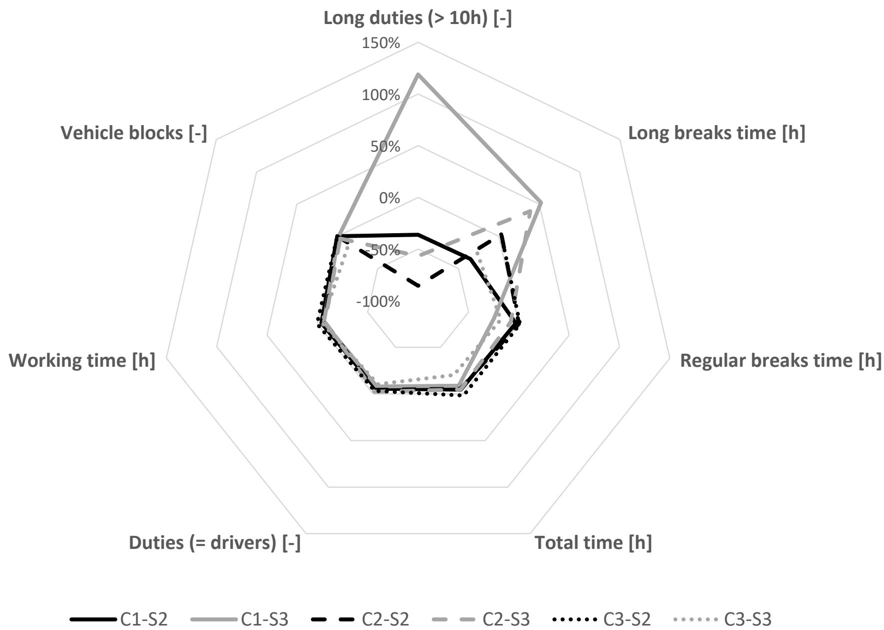

Fig. 5. Relative changes in values of selected KPIs for the scenarios S2 and S3 in relation to the manual scenario S1

6. Conclusions and Further Work

Carried out computational experiments for the three real-world problem instances (i.e., large, medium and small-size cities characterized by thousands of line/timetabled trips and hundreds of vehicle blocks a day), showed that depending on the individual conditions of the urban public transport system, there are real possibilities of reducing the number of vehicle blocks, thus necessary vehicles by approx. 1% up to 14%. Taking into account the individual expectations of public transport operators, e.g., taking into account the individual configuration of breaks for the so-called "congested lines", the matrix of preferences for line changes, queuing electric buses for charging, in practice this reduction stands for approx. 2-3%. Importantly, the reduced number of vehicles results in an increase in the number of line changes on vehicle blocks and usually an increased distance of technical trips, also break times between trips on vehicle blocks are reduced. But vehicle schedules created to reduce their number are not constructed for driver duties, which will result in more changes of vehicles on driver duties.

On the other hand, the effect of optimizing driver duties, as a secondary to the VSP, is the reduction in their number by approx. 6%. And this result depends significantly on the vehicle schedules, which was confirmed by the analyses described in the article, but also numerous analyses independent to the presented.

In the light of the worldwide trend of the increasing shortage of drivers and practical vehicle availability, operators can focus on minimizing the number of duties, while maintaining the vehicle schedules. The presented research has shown that crew scheduling optimization can significantly reduce carriers' operating costs, especially personnel ones by 2-3%.

The general observation is that there is a direct relationship between the minimized number of duties in the schedules (that is the primary goal, both in theory and practice), and the number of vehicle block changes within particular duties. Thus, the main way to reduce the number of duties is to allow vehicle block changes in the crew schedules. Moreover, we observed that there is another promising way to reduce the number of both vehicle blocks and driver duties. It is to make some minor changes to the timetables, i.e., the start times of line trips, which are practically imperceptible, and thus unimportant to passengers but at the same time allowing the reduction of the number of vehicles and drivers (Amberg & Amberg, 2023). And this aspect incorporated into a sequential, but also fully integrated multi-depot vehicle and crew scheduling problem (MD-VCSP) will be the direction of authors' further work combining the presented and previous research (Duda et al., 2022).

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