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OPTIMIZATION OF URBAN TRANSPORT VEHICLE TASKS FOR LARGE, MIXED FLEET OF VEHICLES AND REAL-WORLD CONSTRAINTS

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Abstract:

Planning the operation of urban public transport vehicles is the first stage of operational planning and consists of combining timetable trips, which are input data, into blocks that constitute the daily tasks of vehicles. For a large mixed fleet of vehicles of various types, especially those with battery power that requires recharging, operating from many depots, with numerous requirements and rolling stock constraints, the problem is a major engineering challenge, even for an experienced team of planners. IT solutions based on realistic, mathematical decision-making models and fast optimization algorithms can be of a great assistance. For the problem formulated this way, a mathematical decision model with a multi-criteria objective function was built, taking into account technical, economic, and ecological criteria and binary decision variables. The model takes into account the real requirements and constraints, a mixed fleet of different types of vehicles, including electric buses, multiple depots, technical trips (deadheads), and battery charging. The considered problem is an NP-hard combinatorial optimization one. The use of classical, exact algorithms to solve this problem is not possible for timetables with many thousands of line trips and fleets of hundreds or thousands of vehicles. This research proposes an original, dedicated heuristic algorithm, enabling to obtain an acceptable, but still suboptimal solution, in a very short time. The tests of the proposed algorithm were carried out on real-life databases of public transport systems of the two selected medium and large Polish cities. In particular, multiple depots, a mixed fleet of different types of vehicles, and real-world constraints were taken into account. The results of the computer experiments carried out using the developed heuristic were compared with the results obtained manually by a team of experienced and expert planners. For the developed multi-criteria decision-making model results comparable to and better than those prepared manually by experts were obtained in a very short time using the proposed heuristic. It is the basis for the further development works on expanding the model and improving the optimization algorithm.

Keywords: *urban public transport, optimization of vehicle schedules, fast heuristics*

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1. Introduction

The fast-growing public transport sector has experienced rapid technological and organizational change in the recent years. The fight against air pollution in agglomerations has led to a growing interest in electric rolling stock, which is increasingly replacing traditional one with combustion engines. Following the current market trend, it can be assumed that electric rolling stock will reach a dominant position in urban public transport worldwide within the next few years. This raises technological and organizational problems and requires replacement and expansion of the infrastructure for its operation. The different traction characteristics, the lower mileage in 'full charge' compared to traditional vehicles, and the longer recharging time in relation to refueling time force major changes in the methods of planning and organizing operations on public transport lines.

The above-mentioned problems require an increasing effort from transport organizers and operators to optimally plan the use of new rolling stock. One of the most important public transport planning issues is the vehicle scheduling problem (VSP) that consists of assigning a set of scheduled (line) trips to a set of vehicles blocks or schedules. The VSP is the subject of research carried out by many research teams, and in the recent years there has been a significant increase in works on planning electric vehicle fleets. The VSP is an NP-hard combinatorial optimization problem, and in the case of electric buses rolling stock, the problem is even more complex. Moreover, in a real-life public transport, there is a large number of decision variables and various constraints. All of these makes manual scheduling of vehicle blocks very difficult and time-consuming. In this paper, we consider the VSP with numerous constraints specific for the electric buses, such as the maximum vehicle mileage on a charged battery, the battery charging rate, the minimum charging time, the minimum battery charging level, the location of the charging stations in the transport network, the number of charging stands at each charging station, and the preferred charging times for batteries due to different cost rates.

The objective function of the optimization problem should take into account factors that affect the cost and efficiency of rolling stock use, such as minimal number of vehicles (blocks) and technical trips (deadheads), and expected duration of breaks between consecutive line trips in vehicle blocks. Although there are a large number of scientific works devoted to the issue of VSP, a significant part of them present solutions that were tested on small transport networks with at most a few hundred line trips. Meanwhile, in practice, in large size cities, the number of public transport trips can reach several thousand per day. Practical issues also require taking into account factors such as more than one depot, the possibility of changing lines within a block considering the given predefined preferences, fleet heterogeneity, and limitations related to electric vehicle charging infrastructure. In the literature such a problem is known as Multi Depot- Multi Type Vehicle Scheduling Problem (MD-MTVSP). In addition, most authors consider only one or two optimization criteria in the objective function. The model of the MD-MTVSP proposed in this paper considers a multi-criteria objective function – eight criteria including transport operation cost, impact on environment, total length of deadhead trips, total time of standstills between trips and line change preferences on the vehicle block.

In this paper, we propose a dedicated heuristic algorithm to solve the MD-MTVSP. The proposed algorithm is an extension of the layered algorithm as proposed by Valouxis and Housos (2002) and Kisielewski (2019).This extended version takes into account most of the practical limitations of the MD-MTVSP. The developed heuristics makes it possible to obtain a solution in the form of a set of vehicle blocks in a very short time, even for very large test data instances. The developed algorithm was verified and validated using two real urban transport databases. The test results showed that the algorithm can be an effective tool for planning public transport even in large urban agglomerations.

The remainder of the paper is organized as follows. In the next section, related works from the literature are briefly summarized and research gaps are highlighted. In Section 3, the research problem definition and model formulation are presented. Then, the proposal of the heuristic algorithm is presented in Section 4. The description of real-world instances used for computational experiments and the obtained results are provided in Section 5. Finally, in Section 6, conclusions are drawn and proposals of future works are given.

2. Literature review

The multi-depot vehicle scheduling problem for a large mixed fleet of vehicles and real-world constraints is a complex issue that requires careful consideration of various factors such as vehicle scheduling, charging scheduling, and operational constraints. Several studies have addressed different aspects of this problem, offering valuable insights and solutions. The topics of research papers on the Vehicle Scheduling Problem (VSP) using electric vehicles and a mixed fleet of vehicles are considered by many authors. These studies highlight the complexity of the problem and the need to use advanced mathematical models and algorithms to solve it. They also highlight the importance of taking into account various factors in the planning process, such as operational costs, environmental impact, and real-world constraints. The aim of the following review is not to provide a thorough analysis of the literature on the VSP, but to provide a synthetic overview of papers closely related to the mixed vehicle fleet scheduling problem we have undertaken, considering complex real-world conditions and constraints.

More than a dozen review papers on VSP issues can be found in the literature (see, e.g., Ceder (2007) and Hassold and Ceder (2007)). One of the more interesting papers on the review of vehicle scheduling problems considering the electric fleet is the article by Perumal et al. (2022). The authors reviewed 43 articles related to electric bus technologies and provided an overview of various strategic, tactical, and operational problems in the electric bus scheduling process. The paper by Bunte and Kliewer (2009) reviews solutions to this problem and its extensions. The paper discusses modelling approaches for different types of vehicle scheduling problem and provides a state-of-the-art and comprehensive overview based on a general definition of the problem. In the paper Valouxis and Housos (2002) propose a VSP solution in two stages. In the first stage, an initial solution is calculated using a fast heuristic procedure QS (Quick Search), based on three algorithms: optimal allocation, set partitioning and the shortest path in the graph. Then, the QS solution is improved using linear programming with columnar generation. The paper presents sample results obtained for a small number of up to 89 vehicles. Similar approach is presented in the monograph by Kisielewski (2019) where a fast heuristic algorithm based on line trip layers is followed by the evolutionary algorithm with various kind of crossover and mutation achieving good results for large instances of a few thousand of line trips.

Recent papers in the area of a vehicle scheduling problem address the use of mixed type vehicle fleet. Chung and Chiou (2023) examined the cost structure of the bus fleet with different compositions of electric and diesel vehicles. The authors showed the impact of introducing electric buses into the fleet of vehicles used for public transport. In contrast, Zhang et al. (2022) presented a partial mixed-route strategy that allows multiple transit lines to operate more cost-effectively. The problem was formulated as a mixed integer programming model and an Adaptive Large Neighborhood Search (ALNS) algorithm with new mechanisms specific to this problem was proposed for its efficient application. The authors obtained good results, but the size of the test instance was small – four bus lines and 620 timetable trips were considered.

In Wang et al. (2022), a bi-level, multi-objective linear programming model was developed for the problem of combined vehicle and driver scheduling on a bus route served by a mixed fleet. The improved particle swarm algorithm was used to solve the problem, but was verified for only an example line with a few dozen tips. A similar problem was presented in the work of Árgilán et al. (2012). The authors developed a sequential heuristic method to solve a combined vehicle and driver scheduling problem. The model presented was based on the modification of vehicle schedules to meet driver requirements.

In another paper, Sung et al. (2022) presented a simulation model and a heuristic algorithm to deal with a complex electric bus scheduling problem without significant simplifications. A total cost criterion including vehicle use, chargers, and energy prices was considered to evaluate solutions.

Rinaldi et al. (2020) presented a Mixed Integer Linear Program (MILP) to address the problem of optimal scheduling of a mixed fleet of electric and hybrid/non-electric buses. The authors formulated and solved Single Depot Electric Vehicle Scheduling Problem (SDEVSP). They proposed an extension to this problem that considered the charging and discharging dynamics of a mixed fleet of all-electric and hybrid-electric buses. In order to solve real-world problems effectively and efficiently, they developed an ad hoc problem decomposition scheme. Although the experimental study was carried out on real data, four lines with 252 trips and six lines with 288 trips were considered for the two case studies presented, respectively. An additional test set was also developed using the technique of artificially increasing the number of lines and the density of trips. In this case, the test instance mismatch comprised 21 lines, 1008 trips, and 210 buses. No optimal solution was obtained for this set (optimality tolerance GAP factor of about 7%) and the times to obtain acceptable solutions were about 1.5 h.

Gintner et al. (2005) considered a vehicle scheduling problem with multiple depots and multiple vehicles (MD-VSP). The authors proposed a two-phase algorithm that gives near-optimal solutions and allows these solutions to be determined for very large practical instances. Unfortunately, the proposed solution does not consider the electric fleet.

Mahadikar et al. (2015) addressed the issue of minimizing the length of deadhead trips. In the study, they focused on minimizing technical kilometers by optimizing the allocation of buses to depots according to the shortest distance between the depot and the start/end points of the line. In the model presented, they also took into account capacity and service life when allocating buses. Although the authors obtained an exact solution (using branch-and-cut procedure), the objective function was only concerned with minimizing the length of technical trips and also the electric vehicle fleet was not addressed.

Summarizing the above analysis of previous work on the MD-MTVSP, it can be concluded that despite the large number of publications related to this problem, there is a lack of papers presenting detailed models and algorithms for real-world problems taking into account both the complexity of the constraints and the size of the instances (number of trips greater than 5 000). In this paper, we propose an algorithm and heuristics to take into account real-world constraints and show that it is possible to solve even very large instances of the problem very quickly and that the quality of the obtained solutions is acceptable and better than manual solutions.

3. Problem Definition and Formulation

The problem considered is to determine the assignment of trips to vehicle schedules based on the detailed timetable line trips with the departure and arrival times of the vehicles, the starting and end locations and the technical trip times between all pairs of end of line locations, including depots. The searched solution must ensure that each line trip is covered exactly once and that each vehicle performs a feasible sequence of trips – forming vehicle “blocks”. A vehicle block refers to a specific grouping of trips assigned to a single vehicle within a given day, including a pull-out from the depot, a sequence of trips from the timetable, i.e., line ones, necessary deadhead trips, and a pull-in back to the depot (Ceder, 2007; Perumal et al., 2022). Scheduling of vehicle blocks is a special case of the Multi-Depot Vehicle Scheduling Problem (MD-VSP) where, usually, the goal is to minimize the total cost of trips including deadhead ones.

Presented in this paper mathematical model uses a well-known and widely used multi-commodity with connection-based networks approach. Duda et al. (2022) presented a complex model that consists of an objective function that is a weighted combination of five components and constraints that precisely check whether there is enough state of charge (SoC) left so that the bus can perform a specific trip, and considering the necessary recharging of a battery, as well.

In this work, we focus on the real-world case study and minimization of the number of vehicle blocks used to cover all line trips. Contrary to the most of the models presented in the literature, the considered model takes into account also heterogeneous fleet of buses (different bus types, including electric ones). The nomenclature used to define the optimization model is presented in Table 1.

The objective function includes multiple criteria, such as:

– costs (costs of trips – KE_1, costs of charging – KE_2, variable costs of using vehicles – KE_3, fixed costs related to the number of vehicles used – KE_4),
– impact on the environment (CO_2 emissions – KS_1),
– technical issues (total length of deadhead trips – KT_1, total time of standstills between trips – KT_2),
– additional (preferences regarding changes of lines on a vehicle block – KD_1).

Formulas (1) and (2) present, respectively, the general and the detailed form of the objective function.

Minimize

$$\alpha_1 KE_1 + \alpha_2 KE_2 + \alpha_3 KE_3 + \alpha_4 KE_4 + \beta_1 KS_1 + \gamma_1 KT_1 + \gamma_2 KT_2 + \theta_1 KD_1 \quad (1)$$

Minimize

$$\alpha_1 \sum_{k\in K}\sum_{(i,j)\in A} d_{ij} c_{\Pi(k)} X_{ijk} + \alpha_2 \sum_{(i,j)\in A} g(z_j) W_{ij} + \alpha_3 \sum_{k\in K} u_{\Pi(k)} Z_k + \alpha_4 \sum_{k\in K} Z_k + \beta_1 \sum_{k\in K}\sum_{(i,j)\in A} d_{ij} f_{\Pi(k)} X_{ijk} + \gamma_1 \sum_{k\in K}\sum_{(i,j)\in A} d_{ij} X_{ijk} + \gamma_2 \sum_{k\in K}\sum_{(i,j)\in A} \big(z_j - (z_i + s_i)\big) X_{ijk} + \theta_1 \sum_{k\in K}\sum_{(i,j)\in A} o_{l_i,l_j} X_{ijk} \quad (2)$$

Subject to:

$$\sum_{k\in K}\sum_{(i,j)\in A} X_{ijk} = 1, \qquad \forall i \in T \quad (3)$$

$$\sum_{(i,j)\in A} X_{ijk} - \sum_{(j,i)\in A} X_{jik} = 0, \quad \forall i \in T, \forall k \in K \quad (4)$$

$$\sum_{\delta\in D\setminus\{\Delta(k)\}} \sum_{i\in V} X_{a^{\delta}ik} = 0, \ \forall k \in K \quad (5)$$

$$\sum_{\delta\in D\setminus\{\Delta(k)\}} \sum_{i\in V} X_{ib^{\delta}k} = 0, \qquad \forall k \in K \quad (6)$$

$$MZ_k - \sum_{(i,j)\in A} X_{ijk} \ \geq 0, \forall k \in K \quad (7)$$

$$\sum_{k\in K, \Delta(k)=\delta, \Pi(k)=\pi} Z_k \leq C_{\pi}^{\delta}, \quad \forall \delta \in D, \forall \pi \in P \quad (8)$$

$$Y_i \geq v_i^{\min}, \quad \forall i \in V, \quad (9)$$

$$Y_i - \sum_{k\in K} X_{ijk} e_i^{\Pi(k)} - v^{\text{charge}} - M\left(1 - E_{ij}\right) \leq 0, \quad \forall (i,j) \in A, \quad (10)$$

$$W_{ij} - \big(z_j - (z_i + s_i + t_{ij})\big) r_{\Pi(k)} \sum_{k\in K} X_{ijk} \leq 0, \quad \forall (i,j) \in A, \quad (11)$$

$$Y_i - \sum_{k\in K} X_{ijk}\left(e_i^{\Pi(k)} + e_{ij}^{\Pi(k)}\right) + W_{ij} + M\left(1 - \sum_{k\in K} X_{ijk}\right) \geq Y_j, \quad \forall (i,j) \in A, \quad (12)$$

$$X_{ijk} \in \{0,1\}, \quad \forall (i,j) \in A, k \in K \quad (13)$$

$$Z_k \in \{0,1\}, \quad \forall k \in K \quad (14)$$

The constraints (3)-(14) have the following meaning:

- constraint (3) ensures that each trip is operated by only one vehicle,
- constraint (4) conserves the flow,
- constraint (5) states that each vehicle starts at a designated depot,
- constraint (6) states that each vehicle ends at a designated depot,
- constraint (7) specifies the use of a block/vehicle,
- constraint (8) limits the number of vehicles of type π available at the depot δ,
- constraint (9) ensures that the lower battery charge level (SoC). at which the vehicle ends a trip, does not fall below $v_i^{\min}$, i.e., a sufficient energy to complete the route i,
- constraint (10) limits the upper battery charge level v^{charge} above which a battery cannot be recharged (e.g., charging up to 50% of the SoC),
- constraint (11) limits the amount of energy used for charging due to the available charging time,
- constraint (12) limits the energy balance for two consecutive trips,
- constraints (13) and (14) define the domains of the variables.

The above described model was solved in (Duda et al., 2022) for the problems of a smaller size (up to 1 000 line trips) using CPLEX Solver. However, for practical problems (with the number of line trips > 1 500) MIP solvers cannot find a feasible solution. Therefore a dedicated, efficient and proprietary heuristic algorithm for solving the vehicle scheduling problem has been developed. The proposed heuristic is presented in the next section.issue for the company's image on the manufacturer market.

Table 1. Nomenclature used to define optimization model

Indices	
i	node representing the line trip or the starting point of a deadhead trip
(i,j)	trip connection arc that connects two consecutive line trips or a line and deadhead ones
k	vehicle block
δ	depot
π	vehicle (bus) type
Sets	
V	set of all nodes in the trip graph (depots, trip endpoints, charging places, etc.)
T	set of trip nodes
A	set of all edges of the transport graph; there are four types of edges: (i) A_1 – connecting depots with line trips (pull-out trips), (ii) A_2 – connecting two line trips, (iii) A_3 – connecting line trips with endpoint depots (pull-in trips), (iv) A_4 – connecting line trips with charging points
D	set of depots
P	set of vehicle types
L	set of lines
K	set of vehicle blocks
Parameters	
e_i^π	energy used by vehicle of type π during trip i in relation to a battery capacity [%]
e_{ij}^π	energy used by vehicle of type π during the deadhead trip on link (i,j) in relation to a battery capacity [%]
$g(z_j)$	average cost of charging 1kWh with the tariff for the scheduled trip j end time [PLN/1kWh]
r_π	increase rate in the battery state of charge (SoC) for a vehicle of type π [%/min.]
s_i	duration of trip i [min.]
d_{ij}	length of deadhead trip on arc (i,j) [km]
t_{ij}	duration of deadhead trip on arc (i,j) [min.]
v_i^{min}	minimal battery SoC required for trip i [%]
v^{charge}	maximal battery SoC at which it can be recharged [%]
f_π	emission (carbon footprint) of vehicle of type π on a 1 km ride [kg CO_2]
u_π	cost of use vehicle of type π [PLN]
c_π	cost of 1 km ride using vehicle of type π [PLN/km]
z_i	scheduled time of departure for trip i [min.]
C_π^δ	number of available vehicles of type π in depot δ [-]
o_{l_1,l_2}	line change preference (from line l_1 to line l_2) [-]
h	minimal working time of vehicle block [min.]
M	constant, very large positive number [-]
$\Pi(k)$	vehicle type required for vehicle block k [-]
$\Delta(k)$	home depot for vehicle block k [-]
a^δ	begin node (depot) [-]
b^δ	end node (depot) [-]
Variables	
X_{ijk}	binary decision variable, X_{ijk} = 1, if vehicle block k carries out the deadhead trip on arc $(i,j) \in A$ or carries out line trip j after trip i
E_{ij}	binary decision variable; E_{ij}= 1, if a vehicle's battery is charged on arc (i,j)
Y_i	auxiliary variable, SoC of the battery before starting trip i in relation to a battery capacity [%]
W_{ij}	continuous auxiliary variable – amount of energy used for charging a battery after trip i and before trip j [kWh]
Z_k	auxiliary binary variable, Z_k = 1, if vehicle block k is used in the schedule

4. Proposed Heuristic Algorithm

The proposed approach is based on the layered algorithm, which is one of the basic procedures used when combining vehicle trips into blocks. All line trips are assigned to layers using the following rules. First, all line trips are arranged in ascending order according to their end times. The end time t_1 of the first trip is used to create the first layer, i.e., all trips that start before t_1 form the first layer. The remaining trips are put into subsequent layers in a similar manner. In this way, layers are created until the set of line trips is empty as presented in Figure 1. Each layer is associated with a specific time of the day.

Next, line trips from the subsequent layers are combined into vehicle blocks. The process goes through all of the layers, one by one. Line trips from a particular layer cannot be added to the same vehicle block.

When screening candidates for the next line trips in a given vehicle block, different aspects may be considered. The most popular ones, as formulated by Kisielewski (2019), are:

- appropriate vehicle type,
- chronology,
- length of the break between trips,
- preferences of a line change,
- multi-linearity,
- locations of the line trip ends,
- connection possibility between the line trip ends.

The developed heuristics originally used the cost function (1) as in the model presented in the previous chapter. However, when instances with more trips and lines where solved, such an approach proved to be impractical, as, for example, there were too many line changes in a single block. The objective function was therefore replaced by the block-trips assignment function (that calculates the elements of the cost matrix used by the Hungarian algorithm). It focused primarily on the three most practical aspects of vehicle blocks planning, i.e., on the components: KT_1, KT_2 and KD_1. Additionally, a component responsible for selecting the preferred type of vehicle was added. The role of this component was, among others, to limit the use of electric buses to lines (trips) where they were primarily required. Cost function (1) was then used in a simulation stage to select best solution produced by the proposed heuristic algorithm.

In the analyzed case, the assignment of a certain trip j to the block k is based on the value of the following formula (15):

$$F_{kj} = \omega_1 \frac{f(w_{ij})w_{ij}}{w_{\max}} + \omega_2\left(\frac{10-o_{ij}}{10}\right) + \\ + \omega_3\left(\frac{\lambda_k \chi}{\lambda_{\max}}\right)^2 + \omega_4 \frac{t_{ij}}{t_{\max}} + \omega_5 \frac{d_{ij}}{d_{\max}} + \\ + \omega_6 \frac{10 - l_{j\Pi(k)}}{10} + \omega_7 \frac{n_k}{n_{k\max}} \tag{15}$$

where:

- ω_1 – weight coefficient for the inter-trips break time; $\omega_1 \in [0, 100]$,
- f – impact function of the inter-trips break time,
- w_{ij} – inter-trips break time,
- $w_{\max}$ – maximum time of the inter-trips break,
- ω_2 – weight coefficient for the line change; $\omega_2 \in [0, 100]$,
- o_{ij} – preference indicator for the line change on arc (i,j); $o_{ij} \in [0, 10]$,
- ω_3 – weight coefficient for the line changes number within a vehicle block; $\omega_3 \in [0, 100]$,
- χ – 0, if the trip j doesn't insert a new line into the vehicle block k or 1 otherwise,
- λ_k – number of line changes within the vehicle block k after inserting the trip j,
- $\lambda_{\max}$ – maximum number of line changes in a vehicle block – average number of line changes should not exceed 3, therefore it is assumed that $\lambda_{\max} \in [5, 10]$,
- ω_4 – weight coefficient for the technical trips time; $\omega_4 \in [0, 100]$,
- t_{ij} – duration of the technical trip on arc (i, j),
- $t_{\max}$ – maximum time of a technical trip,
- ω_5 – weight coefficient for the technical trips length; $\omega_5 \in [0, 100]$,
- d_{ij} – length of the technical trip on arc (i, j),
- $d_{\max}$ – maximum length of a technical trip,
- ω_6 – weight coefficient for fitness of the fleet type matching; $\omega_6 \in [0, 100]$,
- $l_{j\,\Pi(k)}$ – preference indicator for matching vehicle type to the trip j; $l_{j\Pi(k)} \in [0, 10]$, where max values mean arbitrary pre-assigned vehicle types,
- ω_7 – weight coefficient for short blocks; $\omega_7 \in [0, 100]$,
- n_k – number of trips in the vehicle block k,
- $n_{\max}$ – maximum number of trips in a vehicle block.

At least one of the ω_4 or ω_5 weights (that is, either time or distance) has to be equal 0.
All trips from a layer are assigned to vehicle blocks using the well-known Hungarian algorithm which is one of the best algorithms for solving the assignment problem. Alternatively, a greedy algorithm can be used. The assignment procedure continues until all layers are checked. The constraints (3)–(14) are taken into account on every assignment of the trip from the layer. If they are violated, F_{kj} value is set to a predefined large positive constant *M*, which is meant to prevent the trip *j* to be assigned to the vehicle block *k*.
The break time impact function *f(w)* concerns the duration of a break between two consecutive line trips. The function is presented in Figure 2, where w_{min} is the minimum break time, w_{opt} – optimal, expected break time, and w_{max} – maximum break time. The f_{min} value is a parameter that can be set by the user.

5. Computational Experiments and Results

5.1. Real-World Problem Instances – description

The proposed heuristic algorithm that minimizes, as a primary goal, the number of vehicle blocks in urban public transport was tested using real-life data from two cities in Poland. The cities are in the top ten Polish cities with the largest number of inhabitants.

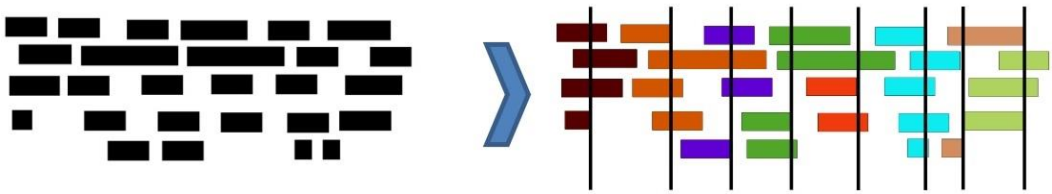

Fig. 1. Scheme of assigning line trips to layers

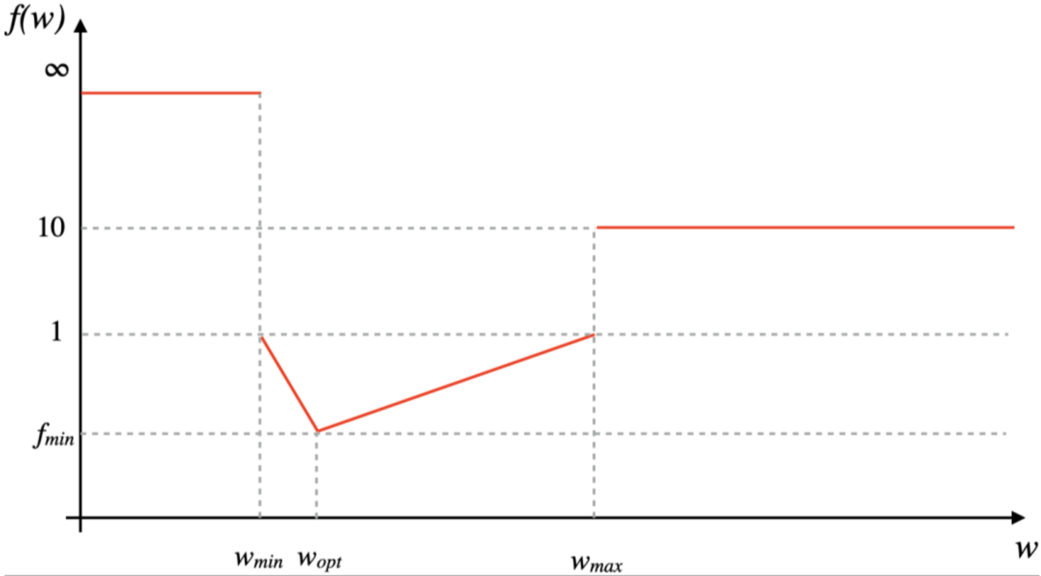

Fig. 2. Piecewise-linear impact function for evaluation of inter-trips break times

The first of the considered cities is a medium size one with the population ranging from 300 to 400 thousand inhabitants. There are 3 depots in the city, used by a public transport operator. The mixed fleet of about 350 buses includes standard (approximately 9–12 m long) and articulated (approximately 18 m long) ones, part of them is powered by traditional, internal combustion (ON, CNG) and the other part by electric engines. There are 74 bus lines in the city, which cover almost 1 100 km in total. During the working day, there are about 3 800 line trips to be scheduled. Regarding electric vehicles, there are 16 charging stations in the city transport network equipped with fast chargers. They are dedicated for charging bus batteries on-route. There are also slow chargers in the depots for overnight charging.
The second of the considered cities is a large one with a population of more than 800 thousand inhabitants. There are 4 depots in the city. The public transport operator uses also a mixed fleet of buses. The fleet with about 605 buses includes the same types of vehicles as the fleet in the first of the considered cities. There are 172 bus lines in the city, which cover 2 300 km in total and there are about 11 200 line trips to be scheduled during a working day. In the city transport network, there are 9 charging stations equipped with fast chargers, there are also slow chargers located in the depots.
In both cases, the following real-world expectations for parameters with subscript max in Eq. (15), were taken into account: i) maximum time of the inter-trips break (e.g., w_{max} = 2 hours); ii) maximum number of trips in the block b (e.g., n_{kmax} = 50); iii) maximum acceptable number of line changes in the block (e.g., λ_{max} = 5); iv) maximum time of the technical trip (e.g., t_{max} = 20 minutes); and v) maximum length of technical trips (e.g., d_{max} = 20 km). The above listed expectations may be considered as soft constraints included into the function given by Eq. (15) as a penalty cost of the assignment. However, as pointed out in Section 4, when the values of parameters associated with these expectations are exceeded, then value of the F_{kj} is set to a predefined large positive value M, to make the assessed assignment unlikely to be accepted. From this point of view, they also play a role of a kind of hard constraints.

5.2. Simulation Experiments

The experiments for the two real MD-VSP instances were carried out based on the previously presented model solved using a proposed heuristic algorithm created in C# compiled for the .NET 6 framework. The calculations were performed on a computer with an AMD Ryzen 4800H 2.9 GHz processor (8 cores, 16 threads) and 32GB RAM.
In addition, a dedicated software was developed to conduct a simulation. The software was also created in C# and compiled for the .NET 6 framework. Its role was to automatically change the values of particular weights of the assignment function (see Eq. (15)) within the range between 0 and 100 with a step equal to 20. There were 7 weights and 6 steps but, as pointed out in Section 4, $\omega_4 \cdot \omega_5 = 0$, thus, the total number of all simulations carried out was equal to 2 multiplied by 6 to the power of 6, giving over 93 thousand of results (different vehicle blocks schedules accompanied by resulting values of KPIs – Key Performance Indicators, see Tables 2 and 3) for each case (city) separately.
The simulation process carried out using the software dedicated to it allows one to tune the values of weights selecting those which assure expected levels of KPIs, thus, quality of solutions. The values of weights are tuned using the well-known grid search method according to which, as mentioned above, all the combinations of weights are checked and the best one can be selected automatically.
An average computational time to obtain a single solution (a vehicle blocks schedule) depends on the number of line trips to be planned, thus, in general, the size of a city. Such an average time equaled to 1.6 seconds for the medium size city and 11.2 seconds for the big one. Taking into account the number of generated solutions and the ability of the created application to handle a multithreaded process of calculations, the total simulation time for the medium-sized city accounted for about 5 hours, and for the big one 36 hours (using computer with the 8-core processor). For every solution, values of the selected KPIs were calculated, as well.

5.3. Results Discussion

The main results of the simulation experiments carried out are presented in Tables 2 and 3 for the medium and large city, respectively. The results cover selected, but crucial KPIs characterizing particular blocks schedules. There are four blocks schedules proposed for each city. The schedules were generated according to the four different scenarios, i.e.:

- S1 – the base solution being a real-life one used in a given city (medium or large size) and prepared manually by its public transport operator;
- S2 – the most economical solution that minimizes the number of vehicle blocks, obtained using the proposed model solved by the proposed heuristic algorithm;
- S3 – the high quality solution that minimizes the number of short, also called degenerated vehicle blocks (with less than 4 line trips), obtained using the proposed model solved by the proposed heuristic algorithm;
- S4 – the most functional solution that minimizes the total distance of technical trips (including deadhead ones), obtained using the proposed model solved by the proposed heuristic algorithm.

Solutions in scenarios S2–S4 were obtained by the appropriately adjusted values of weights of the assessment function (Eq. (15)) used in the proposed heuristic algorithm.

Table 2. Results for a medium-sized city

Scenario:	S1	S2	S3	S4
		Minimized		
KPI	**Manual**	**number of blocks**	**number of short blocks**	**technical trips distance**
Blocks [-]	282	**271**	272	272
Short blocks (< 4 line trips) [-]	3	0	**0**	1
Line changes [-]	123	747	224	191
Long breaks before/after 1 or 2 line trips [-]	0	0	0	3
Line trips distance [km]	55 201	55 201	55 201	55 201
Technical trips distance [km]	5 197	6 408	5 572	**5 492**
Total distance [km]	60 399	61 609	60 773	60 693
Line trips time [h]	2 842	2 842	2 842	2 842
Technical trips time [h]	207	416	361	355
Breaks time [h]	1 210	1 046	1 118	1 136
Total time [h]	4 259	4 304	4 320	4 333

Table 3. Results for a large-sized city

Scenario:	S1	S2	S3	S4
		Minimized		
KPI	**Manual**	**number of blocks**	**number of short blocks**	**technical trips distance**
Blocks [-]	605	**520**	521	537
Short blocks (< 4 line trips) [-]	0	3	**1**	4
Line changes [-]	703	2 690	2 874	1 484
Long breaks before/after 1 or 2 line trips [-]	2	14	16	16
Line trips distance [km]	141 437	141 437	141 437	141 437
Technical trips distance [km]	11 358	11 561	11 580	**10 337**
Total distance [km]	152 795	152 998	153 017	151 774
Line trips time [h]	5 950	5 950	5 950	5 950
Technical trips time [h]	385	436	437	391
Breaks time [h]	3 460	2 452	2 436	2 786
Total time [h]	9 800	8 838	8 822	9 128

As can be seen in Tables 2 and 3 vehicle blocks schedules prepared manually (the scenario S1) are mostly focused on the minimization of line changes within vehicle blocks (more than 3 times less of line changes than in the scenarios S2–S4). At the same time, it reduces the number of necessary technical trips, thus, their distance and time, as well. In the case of the large city, the number of long breaks before and/or after 1 or 2 line trips at the beginning and/or the end of vehicle blocks is also significantly smaller in the manual solution (S1). But the all these advantages of manual solutions are obtained mostly at the expense of the increased number of vehicle blocks, which is higher by 4% in the case of the medium city, and even 15% for the large one. The crucial is that, the increased number of vehicle blocks is much more costly (uneconomical) than the increased number of line changes, which in fact is both inconvenient for drivers and difficult for vehicle blocks schedules planners, but not so costly (much more economical). The two of the calculated KPIs (see Tables 2 and 3), i.e., the distance and time of line trips, are constant over the particular scenarios, since these trips come directly from the timetables that do not undergo any changes when planning vehicle blocks schedules. However, it can be expected that if such changes to the timetables are allowed and included into the vehicle blocks planning process, the results obtained will be better.

To compare the results obtained using the proposed model solved by the proposed heuristic algorithm (S2–S4) to the base, manual solution (S1) relations the selected KPIs are presented in Figure 3. The values of the KPIs for the scenarios S2–S4 were divided by appropriate values obtained under the first, manual scenario (S1). Thus, in Figure 3 increases (+%) and decreases (–%) of these relational values can be observed. Taking into account that lower values are the better for all the KPIs, advantages (–%) of the solutions obtained using the proposed model solved with the proposed heuristic algorithm can be observed, and at the cost of which disadvantages (+%) they were gained. In general, it can be stated that to reduce the number of vehicle blocks and thus the distances covered and time spent for technical trips, it is necessary to accept the increased number of line changes and longer brakes within blocks.

And finally, based on the entire set of solutions obtained within the simulation (see Subsection 5.2), a correlation analysis was performed for particular KPIs – Table 4.

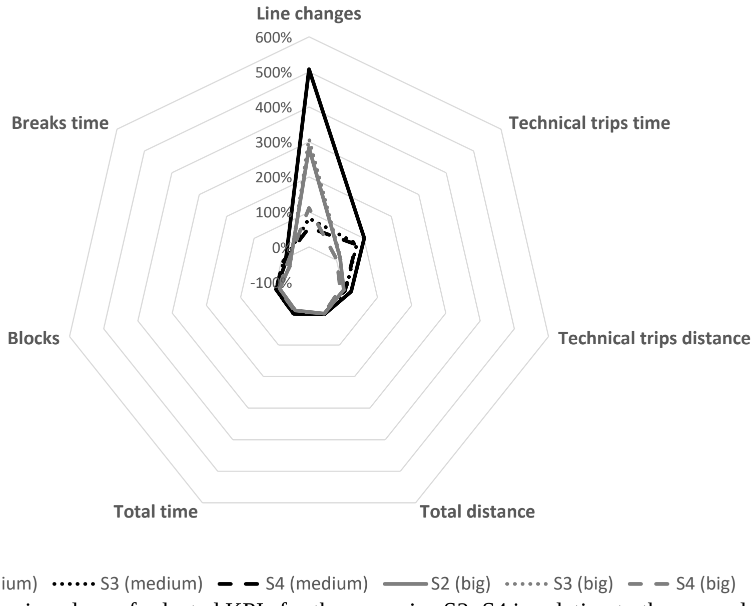

Fig. 3. Relative changes in values of selected KPIs for the scenarios S2–S4 in relation to the manual scenario S1

Table 4. Pearson correlation coefficient (r) for pairwise combinations of selected KPIs

KPI	Blocks	Short blocks	Line changes	Long breaks	Technical trips distance	Total distance	Technical trips time	Breaks time
Short blocks	0.01							
Line changes	**-0.47**	0.24						
Long breaks	**0.86**	-0.13	-0.49					
Technical trips distance	-0.30	0.51	**0.82**	-0.43				
Total distance	-0.30	0.51	0.82	-0.43	1.00			
Technical trips time	-0.33	0.49	**0.84**	-0.44	1.00	1.00		
Breaks time	0.89	-0.18	**-0.74**	0.93	-0.60	-0.60	-0.63	
Total time	0.92	-0.13	-0.68	0.95	-0.52	-0.52	-0.55	1.00

There are positive and negative relations between the KPIs characterizing the vehicle blocks schedules obtained with the different values of weights in the assessment function used by the proposed heuristic algorithm. The highest positive, that is, the full correlation (r = 1.0) comes from the two aspects of the simulation carried out. These are the fixed timetables and average vehicle speed used to convert the distance into time. Thus, these cases are not significant here. The most interesting and important relations (marked in bold in Table 4) occur between blocks, line changes, technical trips, and breaks. In general, it proves once again that, to reduce the number of vehicle blocks, thus the number of required buses and partially drivers, it is necessary to increase the number of line changes (negative moderate correlation). This, at the same time, increases distance and time of technical trips (positive strong correlations), but allows for the reduction of brakes between particular line trips in vehicle blocks (negative strong correlation). And finally, long brakes before and/or after 1 or 2 line trips at the beginning and/or the end of vehicle blocks, e.g., resulting from the daily pattern (peaks) of timetabled trips, significantly affect the number of vehicle blocks (positive strong correlation).

6. Conclusions and Further Work

The article presents a practical, mathematical MILP model of the multi-criteria optimization of an assignment of trips to mixed fleet of urban transport vehicles operating from multiple depots, and taking into account real-world constraints (MD-MTVSP). For the such model an appropriate original heuristic algorithm was proposed. The algorithm using dedicated assignment function to assess how particular trips fit vehicle blocks in particular stages of their construction (when assigning line trips from particular layers).

Carried out computational experiments (simulations) for the two real-life problem instances (i.e., medium and large cities characterized by thousands of line/timetabled trips a day), showed that the proposed model enables control over the tradeoff between various components (criteria) of the assignment function, depending on such business goals as economical, quality, and functional ones (particular scenarios).

The general observation is that there is a direct relationship between the minimized number of vehicle blocks in the schedules (that is, the primary goal both in theory and practice) and the number of line changes within particular blocks. Thus, the main way to reduce the number of blocks is to increase the number of line changes. Moreover, we observed that there is another promising way to reduce the number of blocks. It is to make some small changes to the timetables, i.e., the start times of line trips. And this aspect will be the direction of our further work in the presented research area.

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