### **APPLICATION OF VIBRATION SIGNALS IN RAILWAY TRACK DIAGNOSTICS USING A MOBILE RAILWAY PLATFORM**

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#### Abstract:

The article presents a comprehensive method for using vibration signals to diagnose railway tracks. The primary objective is to gather detailed information on track conditions through a passive experiment. This involves using mobile diagnostic tools and techniques to assess railway infrastructure. The article elaborates on the range of diagnostic activities conducted in accordance with detailed railway regulations and highlights the benefits and capabilities of mobile diagnostics in railway transport. The research includes mobile field measurements across the general railway manager's network, employing vibration signals to detect and evaluate track conditions. The methodology section provides a thorough description of the mobile measurement rail platform, detailing the equipment used, the routes taken for measurements, and the processes of data acquisition and processing. The data obtained from these measurements is crucial for understanding the actual technical condition of the railway tracks. The method of obtaining and processing data is explained in relation to the real technical condition of the railway track. This involves using transducers with specific parameters and parametrically defined signal recording, along with dedicated analysis techniques in post-processing. Vibration signals serve as the primary carrier of information in this diagnostic method. The article details the step-by-step procedures for collecting and analyzing these signals to provide accurate assessments of track conditions. Based on the results from the mobile measurement rail platform, the article characterizes various areas of diagnostics where vibration signals are particularly effective for technical evaluation. These areas include identifying track defects, monitoring track surface and railway crossing and assessing the overall structural health of the railway infrastructure. The use of vibration signals offers a non-invasive and efficient means of track diagnostics, providing real-time data for maintenance and repair decisions. In conclusion, the article underscores the significance of mobile diagnostics in enhancing the safety and reliability of railway transport. By leveraging vibration signals and advanced data processing techniques, this method provides a framework for continuous monitoring and assessment of railway track conditions, ultimately contributing to improved maintenance strategies and operational efficiency.

Keywords: railway transport, on-board monitoring - OBM, rail defects

#### To cite this article:

Licow, R., Tomaszewski, F., (2024). Application of vibration signals in railway track diagnostics using a mobile railway platform. Archives of Transport, 71(3), 127-145. DOI: https://doi.org/10.61089/aot2024.gk7vs246



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#### 1. Introduction

Maintaining railway lines at appropriate quality standards, in terms of their geometry and the diagnostics of surface elements, is crucial for infrastructure managers to meet the required standards for safe operation and to ensure an adequate level of safety and comfort for passengers (Sadri et al., 2020). In Poland, according to data from the Office of Rail Transport in Poland from 2023, railway traffic is constantly increasing, contributing to the accelerated fatigue of infrastructure elements, leading to faster wear and potentially hazardous situations.

This article presents a novel approach to railway track diagnostics using vibration signals collected through a mobile railway platform. The novelty of this research lies in its innovative and modern approach to railway track diagnostics concerning the railway transport system currently in use in Poland. Globally, research is being conducted on effective track diagnostics using onboard devices (Mori et al., 2010; Naganuma et al., 2010). This article provides a broader perspective on the diagnostics of individual railway elements, taking into account the specifics of the Polish railway transport system.

Therefore, there is a need for continuous and unambiguous control of track quality based on the information received (Wang et al., 2021). A complicating factor affecting the clarity of information obtained about track conditions is the complexity of the railway system, which includes several combinations of track elements, significantly influencing the variability of results. These elements include varied rails, sleepers, fastenings, terrain conditions, and the vehicles in operation. Additionally, when researching the components of surface elements, factors such as their year of construction, degree of wear, subgrade (Nielsen et al., 2018) and its individual layers should be considered (Arcieri et al., 2024).

This study involves recording vibration signals using accelerometers mounted on a vehicle during normal operation. Based on the results obtained from real conditions of the railway transport system, further analyses were conducted. An algorithm for processing the measurement data was developed, along with a concept for the architecture of a rapid onboard railway diagnostic system.

The article is divided into three main parts. The first part is a detailed literature review on the research and implementation of solutions in railway line diagnostics. It also references solutions used in other

countries and systems effectively applied in research involving prototypes of specialized devices. Next, the methodology of the research is described, including details about the mobile measurement railway platform with different sets of instruments. The article outlines railway routes, travel speeds, and the configuration of the measuring vehicle, taking into consideration its movement dynamics. The pivotal section of the article is the case study and discussion, involving the analysis of research results aimed at validating and exploring the feasibility of using vibration signals to detect specific rail track damages based on field tests. The article also explores the capabilities of assessing railway crossings and surface conditions using the developed measurement rail platform. Lastly, the article synthesizes the findings and presents conclusions drawn from the research.

#### 2. Literature review

In Poland, railway surface diagnostics are guided by instruction No. 8 issued by the General Railway Manager of PKP Polish Railway Lines. Along with other instructions, it details the specific processes for conducting rail inspections. Defectoscopic tests, regulated by instruction Id-8 and further described in instruction Id-10, are central to these diagnostics. These tests include visual inspections, technical inspections measuring track geometric parameters, and the use of specialized equipment such as handcars and measuring wagons. Whether measurements are carried out by specialized vehicles or hand-held devices, the tested section of track must be closed during tests or conducted during transportation breaks (Licow, 2018).

Several researchers have made significant contributions to the field of railway diagnostics. (Kostrzewski et al., 2021) provides a comprehensive literature review and bibliometric analysis of railway transport system monitoring (Chudzikiewicz et al., 2013).

Recent advancements have seen the implementation of onboard monitoring technologies for real-time assessment of railway infrastructure (Hoelzl et al., 2023). (Hoelzl et al., 2022) highlighted the use of these systems on Federal Swiss Railways, which allow continuous and precise evaluation of track conditions, enhancing safety and efficiency. Similarly, (Tsunashima et al., 2023) described systems that monitor tracks during normal operations (Westeon et al., 2007), using innovative data analysis and signal processing algorithms to identify and locate defects accurately (La Paglia et al., 2022).

A lot of research focuses on detecting specific damages to railway tracks and monitoring the dynamic state of vehicles through acceleration measurements (Wang et al., 2006; Barbosa et al., 2023) and subsequent data analysis, including lightweight vehicles such as trams and metro systems (Firlik et al., 2012; Faccini et al., 2023). Research in regional railways also highlights these methods (ONO et al., 2023).

The authors of these studies emphasize the importance of mathematical criteria, such as the Nyquist criterion, which must be satisfied to obtain undisturbed signals for further analysis and to achieve unambiguous results (De Rosa et al., 2019). The findings of (Nielsen et al., 2020; Stoura et al., 2023) indicate the ability to identify wavelengths and irregularities and roughness on the rail running surface in accordance with European standard UNE-EN 13848-1. High speeds (La Paglia et al., 2023) result in relatively high levels of acceleration, which are crucial for improving the signal-to-noise ratios used in the estimation process. When investigating damage to the rail running surface, the condition of the rolling stock wheels should also be considered (Rodriguez et al., 2021). To achieve this, two inertial modules were sequentially employed on moving wheels, significantly enhancing the reliability of defect detection by approximately 10%. This approach is deemed justified if specific defect identification remains unclear.

An important element in signal processing is filtering methods. The authors (Hoelzl et al., 2023) presented an advanced method for assessing railway track stiffness, utilizing the Vold-Kalman filter to analyze accelerations of vehicle axle boxes. This work makes a significant contribution to methods and approaches for maintaining and diagnosing railway infrastructure, offering a new approach to track condition monitoring. Meanwhile, the work by Muñoz et al. (2021) focuses on the application of Kalman filtering techniques to estimate lateral track irregularities. Geometric defects can affect the stability and safety of railway transport systems, making their accurate detection and estimation crucial for maintenance and operational purposes. Advanced diagnostic systems utilize measurement units such as IMUs (Inertial Measurement Units), INS (Inertial Navigation Systems), satellite navigation (GNSS), railhead optical scanners, and distance meters (odometers) (Uhl et al., 2010). IMUs and related technologies are pivotal in modern diagnostics. (Rosano et al., 2024; Tsunashima et al., 2014) discussed the use of acceleration and contact force measurements to monitor and improve the technical condition of railway infrastructure. These technologies provide valuable data that aids in predicting and identifying defects early, thus facilitating proactive maintenance (Chrzan, 2022). For instance, (Carnevale et al., 2021) introduces an algorithm that significantly enhances the accuracy of location measurements collected by diagnostic systems installed on railway vehicles. This algorithm integrates data from GPS and INS systems, improving localization accuracy even in challenging environments such as tunnels.

The integration of AI (Avici et al., 2021), particularly deep learning and convolutional neural networks (CNNs), has revolutionized railway diagnostics. (Di Summa et al., 2023) reviewed the use of deep learning techniques for monitoring infrastructure, comparing them with traditional methods and highlighting their advantages in accuracy and efficiency. (Faghih-Roohi et al., 2016) demonstrated the effectiveness of CNNs in detecting surface defects such as cracks and corrosion, showing promising results in real-world applications. (Arcieri et al., 2024) explored the use of Partially Observable Markov Decision Processes (POMDP) and reinforcement learning to optimize maintenance decisions under uncertainty. This approach allows for better decision-making strategies by incorporating incomplete or partially observable information, typical in railway infrastructure management.

#### 3. Research methodology

#### 3.1. Mobile measurement platform

The measurements were planned using two mobile measurement sets for selected routes, as discussed in section 3.2. The first research set (A) consisted of a platform of type Xk 03–0517 and a rail vehicle DH–350.11 (Fig. 1). The second measurement set (B) included the platform of type Xk 03–0517, a separating wagon, and a rail vehicle DH–350.11 (Fig. 2).





The total length of the rail platform Xk 03–0517 with buffers is 12.240 m. It is a platform built on two two-axle bogies with flanged wheels, without its own drive. The mass of the platform is 17,850 kg. The second component of the mobile measuring set is the DH-350.11 hydraulic rail vehicle. The length of the rail vehicle is 12 m, and the power of the drive engine is 350 kW. The maximum speed of the rail vehicle is 80 km/h. The four-axle railway platform, serving as a separation element in campaign B, mitigated the adverse effects resulting from the operation of the DH-350 rail vehicle. In both configurations of the measuring sets, the DH-350.11 acted as the driving element (pulling and pushing), while the Xk 03-0517 platform served as the measuring element with installed equipment.

In measurement set A, two 4504A triaxial transducers and a 3050-A-060 B&K type signal recording cassette were used to measure and record vibration signals. The recording cassette recorded at a sampling frequency of 51.2 kHz. Three-axis vibration transducers were mounted on the bearing housing (axle box) of the second wheel of the first bogie (in the direction of travel of the platform) on both sides of the rail platform (Fig. 1). Vibration signals were recorded in three directions: x - aligned with the movement of the rail vehicle, y - horizontal to the movement of the rail vehicle, and z - vertical to the movement of the rail vehicle (perpendicular to the axis of the wheel set and the railway track), separately for the right and left wheels. The frequency range for each axis of the 4504A vibration transducer was as follows – for the x-axis: 1 to 11000 Hz; for the y-axis: 1 to 9000 Hz; and for the z-axis: 1 to 18000 Hz. The signal recording cassette was placed on the measurement platform and controlled via the Wi-Fi network from a mobile phone (Fig. 3a).

Measurement set B consisted of two single-axis transducers of type 4508 and a signal recording cassette of type 7533 B&K. The single-axis transducers were mounted under the upper shelf of the side part of the axle box on the second wheel of the first bogie (in the direction of travel of the railway platform) on both sides of the railway platform (Fig. 2). Vibration signals were recorded in the z direction - vertical to the movement of the rail vehicle, separately for the right and left wheels. The signal recording cassette was placed on the measurement platform and controlled from a computer via LAN (Fig. 3b).



Fig. 2. Mobile measurement set in variant B

The diagram presented in Fig. 3 outlines the stages of data acquisition, the method of obtaining them, and the subsequent processing stages, including the analysis of the technical condition of the railway track during post-processing.

Measurement set B consisted of two single-axis transducers, type 4508, and a signal recording cassette, type 7533 B&K. The single-axis transducers were mounted under the upper shelf of the side part of the axle box on the second wheel of the first bogie (in the direction of the railway platform's movement) on both sides of the railway platform (Fig. 2).

Vibration signals were recorded in the z direction – vertical to the movement of the rail vehicle – separately for the right and left wheels. The cassette recording the signals was placed on the measurement platform and controlled from a computer via LAN (Fig. 3b).

The presented diagram (Fig. 3) specifies the stages of data acquisition, the method of obtaining the data, and the subsequent processing stages, including the researched technical conditions of the railway track in post-processing analyses.



Fig. 3. Scheme of the architecture system for collecting and processing measurement data

All the signals are then sent to the created database of source files. In this database, the recorded signals are organized according to their timestamp and assigned to a specific railway line, with information about the track being tested and the starting and ending stations (stops) of the measurement. The database of source files was designed to allow retrieval of the original signal at any time.

The next stage of data acquisition involves using mass memory to download specific signals for further analysis. The preliminary analysis of the recorded signals consists of selecting specific signal time windows (regions) relevant to a particular analysis. For example, signal regions from km 38.000–38.500 may be selected due to the occurrence of squat defect No. 227. Information about defects occurring in specific locations is based on knowledge from the railway manager. Determining where to cut these regions depends on the timestamp or listening to the recorded audio signal with mileage indications.

In the next step, the signals prepared in this manner are parameterized for each measured vibration direction from the two wheels of the vehicle. Signal parameterization, considering vibration directions, involves defining specific characteristics relevant to particular surface conditions. Variant (a), utilizing six signals, allows for a broader spectrum of directional vibration analyses compared to variant (b). The results of the analysis are stored in a backup database, where they are categorized into specific entities describing the type of each defect.

#### 3.2. Researched railway lines

The measurements were conducted as part of two measurement campaigns (Fig. 4). The first measurement campaign (Route A) was conducted in 2019. It involved continuous measurement of vibration signals on railway line No. 203, Tczew – Chojnice section, and railway line No. 211, Chojnice – Brusy section.



Fig. 4. Routes of measurement campaigns [own work based on the Interactive map of PKP PLK S.A.]

The second measurement campaign (Route B) was conducted in 2021. This campaign involved continuous measurement of vibration signals on the following lines: Line No. 9, Gdańsk Południe – Gdańsk Główny section; Line No. 202, Gdańsk Główny – Gdańsk Wrzeszcz section; Line No. 248, Gdańsk Wrzeszcz – Gdańsk Osowa section; and Line No. 201, Gdańsk Osowa – Gdynia Port section.

Table 1 presents details related to the measurement of vibration signals on individual railway lines within Routes A and B. The tests were conducted on single- and double-track lines with various railway surface elements. The vehicle moved at variable speeds in the range of 10 to 80 km/h. The table also includes information about the condition of the

Table 1 Description of the researched railway lines

measurement rail platform, whether it was pulled by a DH-350 or pushed by it on the return journey.

#### 3.3. Data processing algorithms

The article undertakes the analysis of three cases: a) identification of the squat defect, b) assessment of the condition of a railway crossing based on the type of surface used at the crossing, and c) evaluation of the track surface condition considering the type of fastening and railway sleeper. The analysis results are presented in sections 4.1 - 4.3. The data processing models are shown in Figure 6. For each listed case, an individual model was applied, which clearly indicated the feasibility or infeasibility of diagnostic identification using vibration signals.

Railway line number	Kilometer	Track number	Type of line (on the research section)	Speed [km/h]	Condition of the measuring wagon
Route A					
203	5.423 - 96.227	1	double track	40 - 80	pulled
211	0.672 - 26.598	-	single track	20 - 30	pulled
211	26.598 - 0.672	-	single track	20 - 30	pushed
203	6.227 - 5.423	2	double track	40 - 80	pushed
Route B					
9	325.848 - 327.741	1	double track	80	pulled
202	-0,381 - 4.180	1	double track	40 - 80	pulled
248	-0,266 - 18.246	1	double track	40 - 80	pulled
201	188.529 - 210.763 210.763 - 188.529	$\frac{1}{2}$	double track	10 - 20	pulled / pushed
248	18.246 - 1.103	2	double track	40 - 80	pushed
202	4.180 - 1.000	2	double track	40 - 80	pushed



Fig. 5. Algorithm of data processing

Algorithms for data processing describe the equations used in the model and transformations necessary to ultimately obtain information on the signal's usability with the data processing model for railway track diagnostic identification. The data were tested under various terrain conditions, as each railway line traversed by the vehicle exhibits different types of track infrastructure, degrees of degradation, and permissible speeds applicable to the routes.

Identification of rail surface defects (Fig. 5a), in the context of this article, pertained to identifying the squat No. 227 defect from the catalog of Polish Rail-way Lines. The recorded signal had a sampling frequency of 51.2 kHz. The frequency ranges for each axis of the 4504A vibration transducer were as follows: for the x-axis: 1 to 11000 Hz; for the y-axis: 1 to 9000 Hz; and for the z-axis: 1 to 18000 Hz. Thus, for each axis dedicated to processing the signal using Fast Fourier Transform, the Nyquist criterion was satisfied.

In the case of identifying the squat defect, time-domain analysis proved challenging due to numerous other occurrences such as rail joints (welds), cracks, or conventional joints, which contribute to informational noise and lack of clarity in assessing the squat defect. Vertical displacement analysis based on recorded vibration signals over time was proposed. The computational procedure was conducted in the dedicated BK Connect computing environment and involved deriving vertical displacements from vibration signals during the time-domain analysis.

The first and second derivatives with respect to time of the quantity x(t) – displacement, are called velocity (1) and acceleration (2), respectively.

$$v(t) = x'(t) \tag{1}$$

$$a(t) = x''(t) \tag{2}$$

where:

x(t) - displacement as a function of time,

v(t) - velocity as a function of time, the first derivative of displacement x(t) [m/s],

a(t) - acceleration as a function of time, the second derivative of displacement x(t) [m/s<sup>2</sup>].

After performing the double integration of the vibration signal and obtaining vertical displacements, the next step was to perform filtering to eliminate undesired frequencies. Since filtering is conducted in the frequency domain, it requires converting the data from the time domain to the frequency domain. This domain transformation was performed using the Fast Fourier Transform which is expressed as:

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$$
(3)

where:

X(k) - value of the Fast Fourier Transform at frequency k (k=0,1,...,N-1),

 $\mathbf{x}(\mathbf{n})$  - sample of displacement signal in the time domain,

N - number of samples of the vibration signal,

j - imaginary unit ( $j^2 = -1$ ).

After computing the signal in the frequency domain, a filtering function was implemented. A high-pass filter is used to enhance the waveform amplitude. An Infinite Impulse Response (IIR) filter is employed with an accuracy of 0,1 dB. The high-pass filter is designed to remove the DC component and low-frequency drift. However, its application reduces accuracy below fs/3000 Hz. In the case of squat defect observation, low frequencies up to 100 Hz are critical, thus acknowledging the accuracy reduction in the range of 0 to 6,7 Hz. The transformed results were analyzed in the time-amplitude-frequency domain to localize the defect under investigation. The results obtained from the three measurement axes (x, y, and z) revealed a significant identification potential in the z-direction. The final step in squat defect analysis involved third-octave analysis using the Constant Percentage Bandwidth module in BK Connect software. Third-octave analysis was conducted exclusively for the signal recording direction z, vertical to the railway vehicle movement.

The next group of analyses focused on evaluating the condition of a railway crossing based on the type of surface used at the crossing (Fig. 5b). In this analysis, the original vibration acceleration signal was initially analyzed in the time domain. Vibrations provided clear information only when considering two directions of their registration simultaneously. Therefore, the analysis focused on the y-direction horizontal to the movement of the railway vehicle and the z-direction - vertical to the movement of the railway vehicle.

The next step involved frequency domain analysis. For this purpose, a Fast Fourier Transform was performed, and similarly to the squat defect analysis, a high-pass integrating IIR filter with 0,1 dB accuracy was applied in the frequency analysis. For the railway crossing diagnostic case, the frequency analysis window ranged from 7 Hz to 10000 Hz for z axis. In addition, a Power Spectral Density (PSD) Analysis was applied to determine the distribution of signal power in the frequency domain. This analysis will be expanded in future articles to address specific aspects related to the construction of railway crossings, including the use of fine aggregate fill at the approach and departure ends of the crossing plate.

The last group of analyses focused on assessing the condition of the railway surface by considering changes in the type of railway substructure and the type of fastening. In this analysis, particular attention was paid to time-domain analysis, examining vibration acceleration amplitudes from raw files in the directions of vibration registration - y (horizontal to the movement of the railway vehicle) and z (vertical to the movement of the railway vehicle).

#### 4. Analysis of research results

# 4.1. Implementation of vibration signals in the identification of squat defects on the running surface – case study

An example of squat defect No. 227 is presented in part of Figure 6a. Squats occur on the running surface of the rail and are initially characterized by peeling and cracking of the material, creating a characteristic "dimple" of a semicircular shape. The development of the defect fundamentally depends on the specific operating conditions of the line. A squat defect is a type of point defect. Squat rail defect No. 227 can have serious consequences for vehicle safety and dynamics. It affects the structural integrity of the vehicle, reliability, and ride comfort. However, on the railway network, there are also sectional rail defects, such as wheel burns, which can reach lengths of up to 1,5 meters.

The analysis of signal samples recorded on routes (Table 1) was conducted according to the data processing algorithm described in Section 3.3. In the conducted analysis, squat defect No. 227 (Fig. 6a) was identified using processed vibration signals. The signal, according to the proposed algorithm, was classified as a vertical irregularity. A double integration of the vibration signal was performed in

the vertical direction relative to the movement of the rail vehicle to obtain vertical displacements.

Subsequently, it was necessary to filter the integrated accelerations to eliminate undesired frequencies and random events. The data was analyzed in the time domain. However, for clear visualization purposes, the x-axis is dimensionless with assigned indices.

Figure 6 presents one of the example results obtained during rail vehicle passage over a squat defect. The calculated displacements during the passage over squat defects No. 1 and No. 2 are shown. The example uses results from defects occurring on the right rail (in the case of squat No. 1) and the left rail (in the case of squat No. 2). The authors chose to present this example due to the occurrence of two squats at a similar kilometer mark but on different rails. The analysis revealed that the results recorded by the transducer as vibrations on the axle box, which directly absorbs the impact from a specific rail, directly influence and are recorded by the transducer mounted on the adjacent axle box.

Additionally, the time domain analysis showed very similar results for both defects, even though they occurred on different rails. This applies to the entire set of squat defects recorded during passages on routes A and B. Based on this, it can be concluded that this analysis could be one of the components of the assessment of squat defects.

The results of the spectral analysis (Fig. 6c and d) indicate low frequencies, up to 100 Hz, at which the defect can be observed. Additionally, a characteristic feature observed during spatial visualization in the frequency domain is the shape of the defect spectrum. It takes on a rectangular flat shape with a specific amplitude. Figure 6d presents the results of the left wheel passage, initially recording the dynamic impact caused by the presence of a squat defect on the right rail, followed by the visualization of the squat defect on the left rail.

The images obtained from the conducted experiment will be verified in further analyses in the context of other running surface defects such as spalling, transverse cracking, or head checking. In future work, the authors intend to conduct analyses aimed at distinguishing between types of defects and assessing their degree of degradation.



Fig. 6. Results of the analysis of the signal recorded at the squat defect: a – squat, b – CPB analysis, c,d - spectrum analysis for the vibration signal vertical direction for the right Rz and left Lz wheels

The presented case study of the most common squat defect and the analysis of research results demonstrated that the use of vibrations allows for the diagnostics of the rail running surface. The signals collected by vibration sensors are analyzed using signal processing algorithms. These algorithms can detect characteristic changes in the vibration signals that may indicate the presence of squat defects.

### 4.2. Analysis of the condition of the surface at railway crossings - case study

The second broadly defined case study in the diagnostic assessment of the railway transport system is the evaluation of railway crossings. Railway crossings are a very complex system to assess because there are at least a dozen possible combinations of their construction. These surfaces can include small sized slabs, large sized slabs, rubber slabs, or combinations of slabs with asphalt on the outer sides of the track, with or without fine grained aggregate fill, which is placed at the edge of the entry slab from the track side. Each of these configurations requires effective diagnostics to mitigate undesirable vibration phenomena during passage.

In this case study, vibration signals were analyzed in both the time and frequency domains to explore the possibility of identifying railway crossings based on their surface type. In future work, the authors will thoroughly consider all types of surface configurations, the presence of fill, and the consideration of degradation states and the possibility of monitoring them using vibration signals.

The plots shown in Figure 7 pertain to the recorded vibration signals during the passage over a railway crossing with large-sized CBP slabs (a) and small-sized slabs (b).

A characteristic feature of the signal recorded at a railway crossing is the occurrence of symmetrical peaks in both the vertical and horizontal directions relative to the movement of rail vehicles, appearing at specific intervals corresponding to the time of passage. Therefore, it is reasonable to analyze both directions of vibrations - vertical and horizontal to the vehicle's movement - in the diagnostic assessment of railway crossings. In the analysis of the vibration signal in the time domain, impulse averaging was used, allowing for a detailed examination of shortterm signal changes that occur during the passage over an 'obstacle,' such as a railway crossing. The recorded vibration acceleration amplitudes were higher in the case of small sized slabs, reaching 40 m/s<sup>2</sup>, compared to the amplitudes obtained after passing over CBP slabs, which reached up to 16  $m/s^2$ .



Fig. 7. Vibration signal plot during the passage over a railway crossing with CBP slabs (a) and small-sized slabs (b)

In the next step, a spectrum analysis was conducted, aiming to identify the entry and exit points on the railway crossing in the area plot. The analysis was performed for all crossings on the route, as listed in Table 1, without distinguishing the type of surface at the crossing at this stage. The results of fifteen randomly selected samples for the Rz direction (vertical, right wheel of the vehicle) are presented in Figure 8.

The area plots on the left side of the chart were subsequently filtered to retain high amplitudes over time and remove random or out-of-range frequencies of the transducer, obtaining the results shown on the right side of Fig. 8. In this section, traces of signals generated at specific frequencies during the train's entry and exit from the railway crossing can be observed.

This trace allowed for the extraction of signal samples related to selected railway crossings, taking into account the surface construction of the railway crossing. Fig. 9 presents example samples of vibration signals in the  $L_z$  direction (vertical direction of

vibration recording on the left wheel of the vehicle) during passage over CBP slabs. Similarly, Fig. 10 shows the  $L_z$  signal recorded during passage over small sized slabs.

In most samples obtained from a single wheel, the frequency plots convey information regarding the train's entry and exit from the crossing slabs. In case of uncertainties, it is reasonable to retain information from the second wheel of the rail vehicle. Combining synthesized information from both wheels of the rail vehicle provides reliable and unambiguous information regarding the passage over the crossing slabs.

The proposed analysis aimed at identifying the type of railway crossing is Power Spectral Density (PSD) Analysis. The bottom row of windows in the figures presents the results of the PSD analysis, which was conducted based on frequency analysis involving the identification of frequency windows related to the train's passage over the crossing slabs.



Fig. 8. Identification of the frequency trace resulting from the train's entry and exit from the railway crossing



Fig. 9. Frequency spectrum and power spectral density for CBP slabs in the L<sub>z</sub> vibration signal recording direction



Fig. 10. Frequency spectrum and power spectral density for small sized slabs in the L<sub>z</sub> vibration signal recording direction

Using Power Spectral Density (PSD) analysis, it was possible to identify the signal power in the frequency domain considering the passage time. The results of the PSD analysis showed that the frequency bands during the entry and exit from the railway crossing had the highest power levels throughout the entire time window of the passage. The highest power level was recorded at a frequency of approximately 200 Hz regardless of the surface type. Subsequently, high power levels were recorded in a narrow band of 800 Hz for CBP slabs and in the range of 700-900 Hz for small sized slabs.

Additionally, the results of the frequency analysis indicated that the signal spectrum for the vertical (z) and horizontal (y) directions shows characteristic vibration acceleration amplitudes of 10 m/s<sup>2</sup> and 2.5  $m/s^2$ , respectively, concentrated around a frequency of approximately 200 Hz. In the horizontal direction, additional amplitudes of 1 m/s<sup>2</sup> were observed around 2000 Hz. Further analysis of the recorded vibration signals from the railway crossing with small sized slabs inside the tracks and asphalt outside showed similar occurrences of symmetrical peaks as observed for CBP slabs. The spectral analysis in the direction vertical to the vehicle's movement indicated the highest amplitudes around 200 Hz, followed by decreasing amplitudes at 500 Hz and smaller amplitudes at 2000 Hz. In the horizontal direction relative to the vehicle's movement, higher amplitude values of around 2 m/s<sup>2</sup> were observed in the frequency range from 1000 to 10000 Hz, compared to approximately 1 m/s<sup>2</sup> observed for CBP slabs.

In future studies on railway crossings, the authors will focus on determining the presence or absence of sand backfill at the crossing. The presence of backfill significantly impacts the displacement of internal slabs during low temperatures, thereby affecting traffic safety. Additionally, the analyses will include assessments of the type of crossing development, its technical condition, and other components of the railway surface.

## 4.3. Analysis of changes in the type of railway surface - case study

The analysis of changes in surface types can be utilized for diagnostic assessments conducted during railway line operations and for qualifying sections for element replacement. This analysis evaluates vibration signals recorded while traversing two different combinations of surface types.

The first case study analysis regarding the diagnostics of surface changes involves assessing the transition from wooden sleepers to concrete sleepers. This change is quite common, particularly at railway line junctions with turnouts. Wooden and concrete sleepers are also frequently combined on routes due to maintenance decisions to replace sections of degraded wooden sleepers with new concrete ones. In this case, the analysis examined the combination of surfaces built on wooden and concrete sleepers. In both cases of railway surface construction, K-type fastenings and S49 rails were used. Figure 11 illustrates the analysis results in both the time and frequency domains.

Figure 11 depicts the results of a case study for signals analyzed during changes in railway track surfaces. Figure 11a shows a randomly selected analysis over time. From the set of recorded signals during the track surface change scenario, it was observed that transitioning from wooden sleepers to concrete sleepers results in vibration accelerations ranging from 20 to 30 m/s<sup>2</sup> when recording in the vertical direction relative to the motion of the railway vehicle.

In the analysis of railway track surface changes, the predominant component of diagnostic assessment is the vertical vibrations (perpendicular to the direction of railway vehicle motion). However, these analyses should also consider vibrations transverse to the direction of railway vehicle motion. Similar values of transverse vibration amplitudes provide additional informational elements regarding the change in railway track surface.

For the analysis in terms of spectral content (Figure 11b), two cases of railway track surface changes were selected and compared solely in the direction of recording vertical to the motion of the railway vehicle, separately for the left (Lz) and right (Rz) wheels of the railway vehicle. Sample signals numbered 1 and 2 were used for comparison. Both signals exhibit very similar frequency characteristics and comparable amplitude levels. Signal number 1, on the right track (Rz\_1), shows higher amplitude levels around 800 Hz.

Analyzing changes in vibration signals in the context of changes in track surface types over longer periods allows for a better understanding of amplitude levels compared to momentary recordings. Therefore, continuous measurement during railway vehicle operation provides more insight than triggering and recording in narrow time windows (Figure 12). The second case to be analyzed in the context of railway track surface changes involves the modification of fastenings. In this analysis, wider time windows of vibration signals were utilized to visualize amplitudes under different surface conditions. Figure 12a) illustrates the transition from K-type fastenings to SB-type fastenings, while Figure 12b) depicts the reverse transition from SB-type fastenings back to Ktype fastenings.



Fig. 11. Spectrum signal when changing the surface with wooden sleepers to concrete sleepers, where Lz\_1 - vibration spectrum of signal No. 1 for the left wheel in the direction vertical to the vehicle movement; Lz\_2 - vibration spectrum of signal No. 2 for the left wheel in the direction vertical to the vehicle movement; Rz\_1 - vibration spectrum of signal No. 1 for the right wheel in the direction vertical to the vehicle movement; the vehicle movement; Rz\_2 - vibration spectrum of signal No. 2 for the right wheel in the direction vertical to the vehicle movement; Rz\_2 - vibration spectrum of signal No. 2 for the right wheel in the direction vertical to the vehicle movement; Rz\_2 - vibration spectrum of signal No. 2 for the right wheel in the direction vertical to the vehicle movement; Rz\_2 - vibration spectrum of signal No. 2 for the right wheel in the direction vertical to the vehicle movement; Rz\_2 - vibration spectrum of signal No. 2 for the right wheel in the direction vertical to the vehicle movement; Rz\_2 - vibration spectrum of signal No. 2 for the right wheel in the direction vertical to the vehicle movement; Rz\_2 - vibration spectrum of signal No. 2 for the right wheel in the direction vertical to the vehicle movement.



Fig. 12. Vibration signal when changing the type of fastening on concrete sleepers

This railway line used concrete sleepers and S49 rails. Passing over SB-type fastenings resulted in a significant increase in amplitude levels, averaging approximately 15 m/s<sup>2</sup>, compared to K-type fastenings where amplitudes averaged around 6 m/s<sup>2</sup>. Visualization of these plots allows for the determination of dynamic value levels during passages over the selected infrastructure type in the context of fastening type. Passages over SB fastenings, known as resilient fastenings, generated amplitudes twice as high as those observed over rigid K-type fastenings. The analysis results emphasize a significant difference in acceleration levels depending on the type of fastenings used.

The third case of analysis in the context of track surface changes involved transitioning from concrete sleepers to wooden sleepers, which is the reverse scenario compared to case study number 1. In this analysis, samples were analyzed over time (Figure 13a), showing recorded amplitude levels during the transition of approximately 30 m/s<sup>2</sup> in the z-direction and 35 m/s<sup>2</sup> in the y-direction. Following the conclusions drawn from case study number 1, this case involved analyzing samples in two vibration directions – x and y. Subsequently, spectral analysis was performed (Figure 13b).

The spectral analysis was conducted over maximum frequency ranges tailored to the transducer recording capabilities as per the manufacturer's specifications. The results of this analysis revealed high vibration amplitudes up to 14000 Hz in the z-direction. In the y-direction, significantly greater dynamic activity was observed compared to the z-direction. Specifically, in the numerous recorded samples, high amplitude values were noted only at high frequencies ranging from 2000 to 9000 Hz. This information constitutes another component aiding in the identification and diagnostic assessment of railway track surface changes.

Understanding vibration levels specific to each type of surface can provide a basis for developing filters that distinguish signals from surface noise, thereby isolating signals for further defect analysis. Further investigation of this issue should include visual inspections of the track or material data obtained from cameras.

#### 5. Conclusion

The results of the research presented in the article lay the groundwork for further analysis in various aspects of railway infrastructure diagnostics, including the evaluation of rail and road crossings, as well as monitoring changes in track conditions. The utilization of vibration signals for detecting specific track damages has already been successfully implemented in systems mounted on vehicles, as exemplified by practices in Japan. The findings also underscore the effectiveness of using vibration signals in targeted diagnostic assessments through dedicated post-processing analyses.

The synthesis of the results from the conducted research demonstrates that each case of railway area diagnostics requires appropriate tools and analyses. In the case of rolling surface defects, specifically squats discussed in this article, the analysis of these defects involves identifying displacement signals vertical to the direction of the rail vehicle's movement at low frequencies up to approximately 200 Hz. The diagnostics of railway crossings involved the analysis of the power spectral density of vibration signals in the directions vertical and transverse to the rail vehicle's movement at frequencies around 200 Hz and higher, approximately 800 Hz. The case of surface changes shows the relevance of analyzing vibration signals in the frequency domain for the directions vertical and transverse to the rail vehicle's movement at frequencies as high as 15 000 Hz.

The use of vibrations for the analysis of railway track types is an advanced and efficient diagnostic method that ensures the safety and efficiency of railway infrastructure.

A crucial focus in future studies on vibration signals for assessing surface conditions will be to ascertain the dynamics of changes in diagnostic parameters, identifying the vibration directions that provide the most informative data. This analysis will serve as a guiding principle for future considerations. Additionally, subsequent analyses will verify how the arrangement of the mobile system set, considering the direction of travel of the measurement platform, influences the results and mitigates disturbances arising from specific configurations.



Fig. 13. Spectrum signal when changing the surface with concrete sleepers to wooden sleepers, where Lz - vibration spectrum for the left wheel in the direction vertical to the vehicle movement; Rz - vibration spectrum for the right wheel in the direction vertical to the vehicle movement; Ly - vibration spectrum of signal for the left wheel in the direction horizontal to the vehicle movement; Ry - vibration spectrum of signal for the right wheel in the direction horizontal to the vehicle movement; Ry - vibration spectrum of signal for the right wheel in the direction horizontal to the vehicle movement; Ry - vibration spectrum of signal for the right wheel in the direction horizontal to the vehicle movement.

The proposed mobile measurement system offers a solution to challenges related to railway line diagnostics and enhances the efficiency of diagnostic processes. Future efforts will concentrate on developing a streamlined measurement system that is compatible with and installable on commercial trains. This system aims to detect damages promptly during regular transport operations. Therefore, upcoming steps will also focus on expanding the dataset with signals recorded at speeds ranging from 80 to 160 km/h.

The use of vibration signals to detect squat defects is an efficient method because it allows for continuous monitoring of rail conditions without the need to stop railway traffic. However, to obtain accurate results, it is necessary to use advanced signal analysis techniques. By utilizing mathematical models and machine learning techniques, the diagnostic system can classify identified signal changes as potential squat defects. This may include comparing vibration patterns with a database of known defects and their characteristics.

#### Acknowledgments

Financial support of these studies from Gdansk University of Technology by the DEC-3/1/2022/IDUB/I3b/Ag grant under the ARGEN-TUM TRIGGERING RESEARCH GRANTS - 'Excellence Initiative - Research University' program is gratefully acknowledged.

Acknowledgements to Prof. A. Wilk, Project Manager of "InnoSatTrack," and the "InnoSatTrack" Project Team for their technical support.

#### References

- 1. Arcieri, G., Hoelzl, C., Schwery, O., Straub, D., Papakonstantinou, K. G., & Chatzi, E. (2024). POMDP inference and robust solution via deep reinforcement learning: An application to railway optimal maintenance. *Machine Learning*, 1-29. https://doi.org/10.1007/s10994-024-06559-2.
- Arcieri, G., Rigoni, T., Hoelzl, C., Haener, D., Chatzi, E. (2024). Zurich, E. T. H., & SBB, S. F. R. Ground penetrating radar for moisture assessment in railway tracks: An experimental investigation. *Journal of Physics Conference Series*. https://doi.org/10.58286/29747.
- Avci, O., Abdeljaber, O., Kiranyaz, S., Hussein, M., Gabbouj, M., & Inman, D. J. (2021). A review of vibration-based damage detection in civil structures: From traditional methods to Machine Learning and Deep Learning applications. *Mechanical systems and signal processing*, 147, 107077. https://doi.org/10.1016/j.ymssp.2020.107077.
- Barbosa, B., Serrador, A., & Casaleiro, J. (2023). Condition Monitoring Maintenance in Train Bogie: A Low-Cost Acceleration Sensor Proposal. *In 2023 International Conference on Control, Automation and Diagnosis (ICCAD)* (1-4). IEEE. https://doi.org/10.1109/ICCAD57653.2023.10152337.
- Carnevale, M., La Paglia, I., & Pennacchi, P. (2021). An algorithm for precise localization of measurements in rolling stock-based diagnostic systems. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit,* 235(7), 827-839. https://doi.org/10.1177/0954409720965798.
- 6. Chrzan, M. (2022). Effect of uniform time on the transmission of signals in rail open systems. *Archives of Transport*, 61(1), 39-49. https://doi.org/10.5604/01.3001.0015.8150.
- Chudzikiewicz, A., & Kostrzewski, M. (2013). Analiza sygnałów wibroakustycznych w procesie monitorowania stanu zawieszenia pojazdów szynowych oraz toru. *Pojazdy Szynowe*, 1(2013), 10-17. https://doi.org/10.53502/RAIL-139165.
- 8. De Rosa, A., Alfi, S., & Bruni, S. (2019). Estimation of lateral and cross alignment in a railway track based on vehicle dynamics measurements. *Mechanical Systems and Signal Processing*, 116, 606-623. https://doi.org/10.1016/j.ymssp.2018.06.041.
- Di Summa, M., Griseta, M. E., Mosca, N., Patruno, C., Nitti, M., Renò, V., & Stella, E. (2023). A review on deep learning techniques for railway infrastructure monitoring. *IEEE Access*. https://doi.org/10.1109/ACCESS.2023.3309814.
- Faccini, L., Karaki, J., Di Gialleonardo, E., Somaschini, C., Bocciolone, M., & Collina, A. (2023). A methodology for continuous monitoring of rail corrugation on subway lines based on axlebox acceleration measurements. *Applied Sciences*, 13(6), 3773. https://doi.org/10.3390/app13063773.

- 11. Faghih-Roohi, S., Hajizadeh, S., Núñez, A., Babuska, R., & De Schutter, B. (2016). Deep convolutional neural networks for detection of rail surface defects. *In 2016 International joint conference on neural networks (IJCNN)* 2584-2589. IEEE. https://doi.org/10.1109/IJCNN.2016.7727522.
- 12. Firlik, B., Czechyra, B., & Chudzikiewicz, A. (2012). Condition monitoring system for light rail vehicle and track. *Key Engineering Materials*, 518, 66-75. https://doi.org/10.4028/www.scientific.net/KEM.518.66.
- 13. Hoelzl, C., Arcieri, G., Ancu, L., Banaszak, S., Kollros, A., Dertimanis, V., & Chatzi, E. (2023). Fusing expert knowledge with monitoring data for condition assessment of railway welds. *Sensors*, 23(5), 2672. https://doi.org/10.3390/s23052672.
- Hoelzl, C., Dertimanis, V., Ancu, L., Kollros, A., & Chatzi, E. (2023). Vold–Kalman filter order tracking of axle box accelerations for track stiffness assessment. *Mechanical Systems and Signal Processing*, 204, 110817. https://doi.org/10.1016/j.ymssp.2023.110817.
- Hoelzl, C., Dertimanis, V., Landgraf, M., Ancu, L., Zurkirchen, M., & Chatzi, E. (2022). On-board monitoring for smart assessment of railway infrastructure: A systematic review. The Rise of Smart Cities, 223-259. https://doi.org/10.1016/B978-0-12-817784-6.00015-1
- Hoelzl, C., Keller, L., Simpson, T., Stoura, C., Kossmann, C., & Chatzi, E. (2024). Data-Driven Railway Vehicle Parameter Tuning using Markov-Chain Monte Carlo Bayesian updating. *In Journal of Physics: Conference Series* 2647 (18), 2024. IOP Publishing. https://doi.org/10.1088/1742-6596/2647/18/182024.
- 17. Kostrzewski, M., & Melnik, R. (2021). Condition monitoring of rail transport systems: A bibliometric performance analysis and systematic literature review. *Sensors*, 21 (14), 4710. https://doi.org/10.3390/s21144710.
- La Paglia, I., Carnevale, M., Corradi, R., Di Gialleonardo, E., Facchinetti, A., & Lisi, S. (2023). Condition monitoring of vertical track alignment by bogie acceleration measurements on commercial high-speed vehicles. *Mechanical Systems and Signal Processing*, 186, 109869. https://doi.org/10.1016/j.ymssp.2022.109869.
- La Paglia, I., Corradi, R., & Carnevale, M. (2022). Continuous monitoring of rail vehicle dynamics by means of acceleration measurements. *In Proceedings of the Fifth International Conference on Railway Technology: Research, Development and Maintenance, Civil-Comp Press*, Edinburgh, UK, 300 (1). https://doi.org/10.4203/ccc.1.27.4.
- Licow, R. (2018). Ocena uszkodzeń powierzchni tocznej szyn kolejowych za pomocą zjawisk wibroakustycznych, [Doctoral dissertation, Poznan University of Technology]. Poznan University of Technology Repository. https://sin.put.poznan.pl/dissertations/details/d106.
- Mori, H., Tsunashima, H., Kojima, T., Matsumoto, A., & Mizuma, T. (2010). Condition monitoring of railway track using in-service vehicle. *Journal of Mechanical Systems for Transportation and Logistics*, 3(1), 154-165. https://doi.org/10.1299/jmtl.3.154.
- 22. Muñoz, S., Ros, J., Urda, P., & Escalona, J. L. (2021). Estimation of lateral track irregularity through Kalman filtering techniques. *IEEE Access*, 9, 60010-60025. https://doi.org/10.1109/ACCESS.2021.3073606.
- 23. Naganuma, Y., Kobayashi, M., & Okumura, T. (2010). Inertial measurement processing techniques for track condition monitoring on shinkansen commercial trains. *Journal of Mechanical Systems for Transportation and Logistics*, 3(1), 315-325. https://doi.org/10.1299/jmtl.3.315.
- Nielsen, J. C., & Li, X. (2018). Railway track geometry degradation due to differential settlement of ballast/subgrade–Numerical prediction by an iterative procedure. *Journal of Sound and Vibration*, 412, 441-456. https://doi.org/10.1016/j.jsv.2017.10.005.
- 25. Nielsen, J. C., Berggren, E. G., Hammar, A., Jansson, F., & Bolmsvik, R. (2020). Degradation of railway track geometry–Correlation between track stiffness gradient and differential settlement. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, 234(1), 108-119. https://doi.org/10.1177/0954409718819581.
- 26. Office of Rail Transport in Poland. (2023). Data on rail transport. https://www.utk.gov.pl (Access from 15.06.2024).

- ONO, H., TSUNASHIMA, H., TAKATA, T., & OGATA, S. (2023). Development and operation of a system for diagnosing the condition of regional railways tracks. *Mechanical Engineering Journal*, 10(3), 22-00239. https://doi.org/10.1299/mej.22-00239.
- Rodriguez, A., Sanudo, R., Miranda, M., Gomez, A., & Benavente, J. (2021). Smartphones and tablets applications in railways, ride comfort and track quality. Transition zones analysis. *Measurement*, 182, 109644. https://doi.org/10.1016/j.measurement.2021.109644.
- 29. Rosano, G., Massini, D., Bocciolini, L., Zappacosta, C., Di Gialleonardo, E., Somaschini, C., ... & Pugi, L. (2024). Diagnostics of the railway track-Possibility of development through the measurement of accelerations and contact forces La diagnostica dell'armamento ferroviario-Possibilità di sviluppo attraverso la misura di accelerazioni e forze di contatto. *INGEGNERIA FERROVIARIA*, 79(2), 81-102.
- Sadri, M., Lu, T., & Steenbergen, M. (2020). Railway track degradation: The contribution of a spatially variant support stiffness-Global variation. *Journal of Sound and Vibration*, 464, 114992. https://doi.org/10.1016/j.jsv.2019.114992.
- 31. Sadri, M., Lu, T., & Steenbergen, M. (2019). Railway track degradation: The contribution of a spatially variant support stiffness-Local variation. *Journal of Sound and Vibration*, 455, 203-220. https://doi.org/10.1016/j.jsv.2019.05.006.
- 32. Stoura, C. D., Dertimanis, V. K., Hoelzl, C., Kossmann, C., Cigada, A., & Chatzi, E. N. (2023). A Model-Based Bayesian Inference Approach for On-Board Monitoring of Rail Roughness Profiles: Application on Field Measurement Data of the Swiss Federal Railways Network. *Structural Control and Health Monitoring*, 2023(1), 8855542. https://doi.org/10.1155/2023/8855542.
- 33. Tsunashima, H., Naganuma, Y., & Kobayashi, T. (2014). Track geometry estimation from car-body vibration. *Vehicle System Dynamics*, 52, 207-219. https://doi.org/10.1080/00423114.2014.889836.
- Tsunashima, H., Ono, H., Takata, T., & Ogata, S. (2023). Development and Operation of Track Condition Monitoring System Using In-Service Train. *Applied Sciences*, 13(6), 3835. https://doi.org/10.3390/app13063835.
- 35. Uhl, T., Mendrok, K., & Chudzikiewicz, A. (2010). Rail track and rail vehicle intelligent monitoring system. *Archives of Transport*, 22, 495-510. https://doi.org/10.2478/v10174-010-0030-1.
- 36. UNE EN 13848-1:2020 Railway applications Track Track geometry quality Part 1: Characterization of track geometry.
- Wang, H., Berkers, J., van den Hurk, N., & Layegh, N. F. (2021). Study of loaded versus unloaded measurements in railway track inspection. *Measurement*, 169, 108556. https://doi.org/10.1016/j.measurement.2020.108556.
- Wang, C., Xiao, Q., Liang, H., Liu, Y., & Cai, X. (2006). On-line monitoring of railway deformation using acceleration measurement. *In 2006 6th World Congress on Intelligent Control and Automation* 2, 5828-5832. IEEE. https://doi.org/10.1109/WCICA.2006.1714194.
- Westeon, P. F., Ling, C. S., Roberts, C., Goodman, C. J., Li, P., & Goodall, R. M. (2007). Monitoring vertical track irregularity from in-service railway vehicles. *Proceedings of the institution of mechanical engineers, Part F: Journal of Rail and Rapid Transit*, 221(1), 75-88. https://doi.org/10.1243/0954409JRRT65.