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MODELING OF MATERIAL AND ENERGY INPUTS IN THE LIFE CYCLE OF A VEHICLE

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Abstract:

The paper focuses on the analysis of the environmental impacts related to the production, operation, decommissioning of vehicles as well as servicing and repairs of cars in real conditions of their use. The authors' presented mathematical model that was implemented in original numerical program EN-VEHICLE that enables the analysis of energy consumption and emission loads related to material inputs in the entire life cycle of a passenger car. It is a tool from the Life Cycle Assessment category that aims at the effective inclusion of environmental impacts in the decision-making process. The developed program allows for a quantitative interpretation of the calculation results in terms of the environmental safety in each of the phases and throughout the life cycle of the vehicle, taking into account the possibility of supplying the vehicle system with additional material streams derived from recycling and recovery, and introduced both during the construction phase and during the car operation phase. In the presented tool, linear algebra and matrix analysis were used in modeling the vehicle life cycle system. The results of the implementation of the mathematical model were presented in the form of a regression function that allows for approximation of selected empirical data. The regression analysis was used to verify the material characteristics. The tool can be used for a comprehensive comparative assessment of the environmental impact of a passenger car from different production periods. In addition, it can be used to forecast the environmental effects of changes in the material structure determining the production technology and, consequently, having a significant impact on the entire life cycle of the vehicle as well as energy and ecological parameters.

Keywords: *mathematical model, life cycle analysis, vehicle, environmental management*

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1. Introduction

Currently, road transport has a significant impact on the natural environment. According to estimates, it is responsible for 22% of human-made CO_2 emissions (UNECE 2021). The majority of this comes from passenger vehicles, contributing to 45% of global CO_2 emissions from transport (International Energy Agency 2022). Cars emit 40% of nitrogen oxides into the atmosphere (European Environment Agency 2021).

Life cycle analysis of transport means is the area of research related to environmental management. It allows the identification of the most important environmental aspects of the product or operation system, both by identifying and quantifying used materials and energy and waste released into the environment, and by assessing the impact of these materials, energy and waste on the environment. Currently, there is no doubt that road transport - next to the energy sector - is one of the greatest threats to the environment, related to both the systematic increase in global carbon dioxide CO_2 emissions and the increase in the amount of waste generated during the production, operation and disposal of end-of-life vehicles. Despite the fact that the process of using the car to the greatest extent pollutes the natural environment through the consumption of primary energy and the emission of pollutants and CO_2, the decisive importance for a comprehensive assessment of the environmental profile of the car is a vehicle impact on the environment throughout its whole "life". This means that it is not possible to focus solely on the assessment related to the use of the car, as the environmental impact before and after this period can also be significant. The impact of the vehicle on the environment must be assessed on the basis of the balance, which includes all of the processes "from cradle to grave". This is possible thanks to the Life Cycle Assessment (LCA). This approach to environmental assessment is necessary especially for the most complex high-tech product, such as a car. Almost all types of materials are used for its production, such as metals, polymers, glass and ceramics, as well as fabrics and others. The production of these materials and then the components needed for the mass production of cars on an unprecedented scale require the use of limited natural resources and energy in production processes, which is always accompanied by an environmental impact. The components are also supplied for the repair of cars in use. Production of these components absorb additional natural resources and energy. However, after a car is withdrawn from use, it is important whether its components can be recycled and reused in the production of new cars. The aim of the article is to develop a reliable mathematical model of environmental impacts related to the production, operation, decommissioning of vehicles as well as servicing and repairing cars in real conditions of their use.

The remainder of the paper is structured as follows. Section 2 contains a comprehensive overview of literature on the LCA methods in automotive industry. In section 3, the model of the life cycle of the vehicle divided into production, operation and decommissioning is presented. Section 4 contains the mathematical model of the product recycling and final waste subsystems and energy and material recovery streams, while in section 5, a numerical program implementing the problem is demonstrated. The verification of the presented life cycle model and its results for the selected cars are presented and discussed in section 6.

2. Literature review

The overwhelming majority of examples of the use of the LCA environmental life cycle assessment method in the automotive industry, published in the literature, relate to cases where the impact of different technologies and environmental consequences of using various materials are analyzed in order to determine the most favorable choice among various alternatives. Most attention is paid to the environmental optimization of car design and manufacturing processes (Chanaron 2007). The subject of the analysis are whole cars (Finkbeiner et al. 2006; De Medina 2006) or individual components or assemblies (Suzuki et al. 2005; Ribeiro et al. 2007; Gradin and Åström 2020). An example of the application of the LCA method to a comprehensive environmental analysis of a vehicle are works carried out by some automotive companies, including Mercedes and Volkswagen. The results of these works in the field of designing the two latest models of Mercedes cars (C-class and S-class) allowed to optimize significantly both structures in terms of the environment (Finkbeiner et al. 2006). Other research focuses on the assessment of the environmental impact of biofuels (Nitta 2011) or unconventional drives (Hawkins et al. 2013). It is important because with regard to electric cars, the environmental load may be transferred from the stage of vehicle use to the stage of its production or - much more often - production of energy carriers (Ma et al. 2012). Among the works related to the environmental optimization of car-related manufacturing processes, the inputs of energy and materials used to build a car are often analyzed (Ribeiro et al. 2007). Currently, due to the increasing pressure to reduce energy demand in the transport, special attention is paid to issues of selection of lightweight materials and the environmental impacts of such changes (Mayyas et al. 2012). Quantifying the impact of material substitution in car production is a difficult task and requires assessing the potential for weight reduction of materials, pollutant emissions during their production and recycling, as well as the impact of the use of these materials on the emission of pollutants during the life of the car (Gradin et al. 2013). Other studies have focused on car design to facilitate assembly (Petrov et al. 2001). Some of the authors (Schmidt et al. 2004; Lewicki 2009) focused on the environmental consequences of the process of managing end-of-life vehicles as a stage of ending the life cycle of a vehicle. The results of the LCA assessment were used, among others, for the environmental optimization of the recycling process (Gradin et al. 2013, Merkisz-Guranowska 2020). Such analyzes are carried out by both research centers and automotive concerns. Unfortunately, the obtained results are often difficult to verify and are not always consistent with each other (Raugei et al. 2015). This is often due to difficult access to detailed data, which in large part is available to producers. Due to the lack of all the necessary data and the complexity of the vehicle, the LCA of a car is most often performed with many simplifying assumptions, especially in comparative studies (Klocke et al. 2014, Mrozik et al. 2021). According to Petrov (2001), the LCA method used for comparative research should be simple enough, contain readily available data and be convenient in practical application. It is therefore proposed to focus on three main stages: production, use and remanufacturing, and to take into account the most important ecological factors, the impact on nature and people. Often the environmental impact is limited to energy consumption and global warming (Das 2011).

The conducted literature studies show that the environmental assessment of the effects of material substitution in the production of cars is a particularly difficult task. This applies in particular to the use of lightweight materials, as it requires an assessment of the car's weight reduction potential, taking into account the emission of pollutants during the production and recycling of materials, as well as the impact of the use of these materials on the emission of pollutants during the life of the car. It is therefore important to be able to compare different scenarios of the changes, as well as similar cars made by different manufacturers. The results of the literature analysis of the most important aspects of the problem of environmental car assessment lead to the conclusion that the calculation methods and procedures currently used in the world do not allow for efficient calculations for the whole car, especially in comparative studies of a large number of cars from different manufacturers. Difficulties in comparing the results of different studies are often the result of a lack of consistency in terms of scope, simplifying assumptions or methods of environmental impact assessment. This is often the result of difficult access to detailed data, which in large part is at the disposal of producers. In the case of a car, the need to obtain a large amount of data, which is usually confidential, is a significant problem and greatly affects the reliability of the assessment and the possibility of comparison with other tests. That is why LCA method used in comparative studies should be sufficiently simple and convenient in practical use, include accessible, easily verifiable data on the car and the processes associated with it. There is therefore a need for a methodology that would avoid too costly and complicated process of data collection and at the same time take into account the most important factors influencing nature and people. Therefore, the paper presents a proposal to build an LCA model of a car that meets the above-mentioned criteria.

3. Vehicle life cycle stages

3.1. Division into stages

The following period of time will be assigned to the life cycle of the vehicle: $T^C = [t_p^C, t_k^C]$. Such a period is expressed as a closed interval where *p* in subscript means the initial term, and *k* means the ending term.

The following stages are distinguished in the life cycle of the vehicle

- production,
- operation,
- decommissioning.

The durations of these steps are marked accordingly: T^I, T^{II}, T^{III}. These are the subsets of time T^C and their set theory sum is:

$$T^C = T^I \cup T^{II} \cup T^{III} \tag{1}$$

We assume that (Figure 1):
- the beginning of the stage production of the vehicle is concurrent with the beginning of the cycle of life and falls on a term equal to zero: $t_p^C = t_p^I = 0$,
- the end of the decommissioning stage is concurrent with the end of the life cycle: $t_k^C = t_k^{III}$,
- production end date and initial operation date and the end-of-life date and the initial decommissioning date coincide: $t_k^I = t_p^{II}$, $t_k^{II} = t_p^{III}$.

In the vehicle system, there are subsystems which are assemblies, subassemblies and functional elements. Thus, the vehicle system consists of n subsystems forming a 3-level hierarchical structure. From the point of view of further analysis, only the lowest-level subsystems are relevant, the masses of which add up to the total mass of the vehicle.

3.2. Production stage

Each subsystem installed in a vehicle has a specific mass. If we distinguish n subsystems, the initial structure of the vehicle vector will be called:

$$z^I = \begin{pmatrix} z_1^I \\ \vdots \\ z_n^I \end{pmatrix} \tag{2}$$

where z_i^I is the mass of the i -th subsystem at the time of starting operation.

The total mass of the vehicle will be then calculated from the dependence:

$$M = \sum_{i=1}^{n} z_i^I \tag{3}$$

This mass may change along with the consumption of e.g. operating fluids, however, it was assumed that keeping the vehicle in a serviceable condition consists in restoring its mass structure to its initial state.

A thorough analysis of the vehicle's production may allow the identification of the dynamics of the formation of the initial structure in the machining and assembly processes. Then the production dynamics function depending on time will be obtained:

$$z^I : \mathbb{R}_+ \cup \{0\} \to {\mathbb{R}_+}^n, \quad z^I(t) = \begin{pmatrix} z_1^I(t) \\ \vdots \\ z_n^I(t) \end{pmatrix} \tag{4}$$

If the production dynamics function is known, then the components of the initial structure of the vehicle are the integrals of the corresponding components of this function over the time interval $[t_k^I, t_k^I]$:

$$\forall i = 1, \dots, m: \quad \int_{t_p^I}^{t_k^I} z_i^I(t)dt = z_i^I \tag{5}$$

We assume that the vehicle at the moment of t_k^I is fully equipped and has all the necessary operating fluids, including a full fuel tank.

Additionally, we assume that the state of the system at the start of its production is described by the null vector:

$$z^I(t_p^I) = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix} = \theta \tag{6}$$

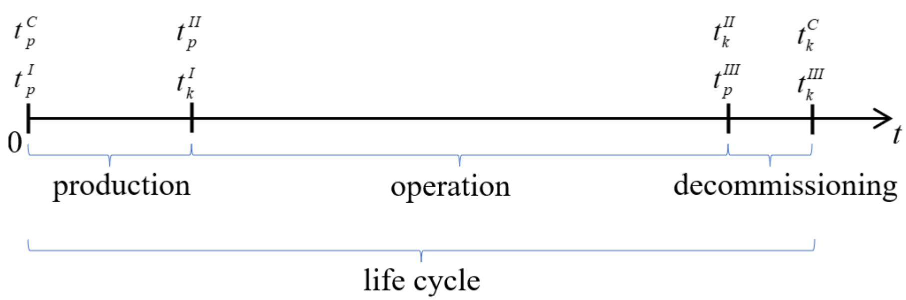


Fig. 1. Stages of the life cycle of the vehicle and marking the limiting terms

3.3. Operation stage

The vehicle system should maintain its initial mass structure $z^I(t_k^I) = z^I(t_p^{II})$ throughout the operation stage. This means that consumable subsystems are either supplemented (e.g. fuel) or replaced (e.g. tires). Consumption of subsystems can take place through:

– loss in time (fuel, windshield washer fluid) or
– deterioration of properties (engine oil, tires, timing belt, accident damage).

Of course, such a sharp division into two categories is a simplification. For example, brake pads are replaced due to deterioration of their properties, but it is because of weight loss. Similarly corroded body parts lose both properties and weight at the same time. Both the defects filling and replacement of a subsystem will be referred to as the supply, while the quantity of the delivered subsystem will be referred to as the supply value. We define the following function of the dynamics of operation, analogous to (4):

$$z^{II}: \mathbb{R}_+ \cup \{0\} \to \mathbb{R}_+{}^n, \quad z^{II}(t) = \begin{pmatrix} z_1^{II}(t) \\ \vdots \\ z_n^{II}(t) \end{pmatrix} \tag{7}$$

where $z_i^{II}(t)$ is the value of the supply of the *i*-th subsystem at the moment of *t*.

Very often, the exact date of the subsystem supply cannot be determined because the assembly process takes a certain time. The final date of the assembly time is then assumed. Similarly, when we assume the vector of the initial structure of the vehicle (2) instead of the production dynamics function (4).

The dynamics of operation takes the form of pairs: (supply value, term). In general, the number of such pairs for any subsystem is not limited. The following typical methods of supplying subsystems during the vehicle operation can be distinguished:

1. zero power supply - no power supply throughout the operation stage,
2. cyclic power supply - constant power supply values at a fixed time cycle,
3. accidental power supply - a one-time power supply with a variable value and time (results of random failures, accidents).

In practice, the subsystems are powered in many other ways, but the above-mentioned situations are the most typical.

Allocation of the subsystem to one of the above categories can be done *ex post* or on the basis of forecasts. Collecting a reliable statistical sample may allow for the identification of subsystems that we will assign with an acceptable probability to the non-supplied group. These can include some interior parts of the body, engine block, crankshaft, etc. Such subsystems can be assigned a zero operation dynamics function: $z_i^{II}(t_p^{II} \le t \le t_k^{II}) = 0, \ \forall i = 1, \dots, n.$

Cyclically powered subsystems can be assigned the following functions:

$$z_i^{II}(t) = \begin{cases} \hat{z}_i^{II}, & when\ t = \Delta_i^{II} \cdot k \\ 0, & when\ t \ne \Delta_i^{II} \cdot k \end{cases}, \quad k \in \mathbb{N} \cap \left(0, \frac{T^{II}}{\Delta_i^{II}}\right) \tag{8}$$

where $\hat{z}_i^E$ is the power value of the *i*-th subsystem, Δ_i^E is the time at which the power is supplied (the length of a power cycle).

The value of the cycling power and the length of the cycle may be determined on the basis of the technical characteristics of the given vehicle or the statistical analysis of a random sample. Such a sample should contain information on the age of the vehicle, in which the worn-out subsystems had to be replaced.

Accidental power supply can only be estimated on the basis of a statistical analysis of the consequences of accidents. Forecasts of total random supplies can be made in this way, but the estimation of dates is pointless here. An accident causing damage to a specific subsystem can occur at any time during the operational phase.

3.4. Decommissioning stage

The beginning of the decommissioning phase means that the vehicle's subsystems are no longer powered. So there is:

$$z^{III}(t) = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix} = \theta, \quad t_p^{III} \le t \le t_k^{III} \tag{9}$$

The division of the life cycle into three stages is not very precise, especially as regards the operational stage. It may last several dozen years. For this reason, we will divide it into time intervals of equal length equal to the interval τ. We assume that the data is:

- initial term t_p^{II} and the operational time of the vehicle TII,
- initial production date t_p^I,
- the length of the time interval τ such that $\frac{T^{II}}{\tau} \in \mathbb{N}$,
- functions of dynamics of production and operation (4), (7).

Despite defining the functions of production and operation dynamics as specified on the set of non-negative real numbers, in practice they are defined on non-negative rational numbers. This is due to the natural discretization of time by measuring it in fixed units. The detail of the analysis, of course, depends on the units adopted, but formally important is the fact that we deal with time as a countable variable, after which we can sum up the values of the function.

We consider a sequence of terms:

$$\hat{t} = (\hat{t}_j)_{j=1}^{q}, \quad q = \frac{T^{II}}{\tau} + 2,$$
$$\hat{t}_j = \begin{cases} t_p^{II} + (j-1)\tau, & when \quad j < q \\ t_k^C, & when \quad j = q \end{cases} \tag{10}$$

As a result we obtain: $\hat{t} = (t_p^{II}, t_p^{II} + \tau, t_p^{II} + 2\tau, t_p^{II} + 3\tau, \ldots, t_p^{II} + T^{II} - \tau, t_p^{II} + T^{II}, t_k^C)$.

The power supply matrix of the system is called the matrix:

$$Z = (z_{ij})_{n\times q} \tag{11}$$

whose components are determined as follows:

$$\forall i = 1, \ldots, n:$$
$$z_{ij} = \begin{cases} \sum_{t\in[t_p^P, t_p^E]} z_i^I(t), & when \quad j = 1 \\ \sum_{t\in(\hat{t}_{j-1}, \hat{t}_j]} z_i^{II}(t), & when \quad j = 2,3,\ldots,q \\ 0, & when \quad j = q \end{cases} \tag{12}$$

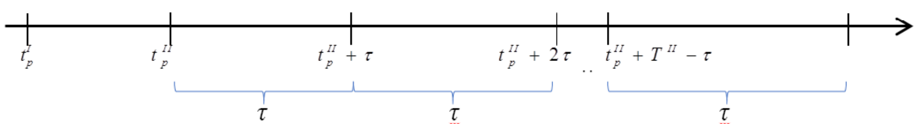
The first column of the Z matrix contains the initial numbers of subsystems that constitute the structure of the vehicle commencing operation. These quantities can also be determined on the basis of a continuous function of the production dynamics using the integral (5) or they can be defined immediately as the initial structure of the vehicle (2).

In the following columns of the Z matrix (for j=2,...,q-1) each component z_{ij} is the amount of the i-th subsystem that is delivered to the vehicle in the time interval $(\hat{t}_{j-1}, \hat{t}_j]$.

The last column represents the vehicle liquidation stage. The system is not powered then, so this column is zero. We introduce it to simplify further calculations. Finally, the supply matrix is as follows (below the timeline shows the stages to which each column relates):

supply of production | supply of operation | supply of scrapping

$$Z = \left(\begin{array}{c|c|c|c|c|c} \sum_{t\in[t_p^P, t_p^E]} z_1^I(t) & \sum_{t\in(t_p^E, t_p^E+\tau]} z_1^{II}(t) & \sum_{t\in(t_p^E+\tau, t_p^E+2\tau]} z_1^{II}(t) & \cdots & \sum_{t\in(t_p^E+T^E-\tau, t_p^E+T^E]} z_1^{II}(t) & 0 \\ \sum_{t\in[t_p^P, t_p^E]} z_2^I(t) & \sum_{t\in(t_p^E, t_p^E+\tau]} z_2^{II}(t) & \sum_{t\in(t_p^E+\tau, t_p^E+2\tau]} z_2^{II}(t) & \cdots & \sum_{t\in(t_p^E+T^E-\tau, t_p^E+T^E]} z_2^{II}(t) & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \sum_{t\in[t_p^P, t_p^E]} z_n^I(t) & \sum_{t\in(t_p^E, t_p^E+\tau]} z_n^{II}(t) & \sum_{t\in(t_p^E+\tau, t_p^E+2\tau]} z_n^{II}(t) & \cdots & \sum_{t\in(t_p^E+T^E-\tau, t_p^E+T^E]} z_n^{II}(t) & 0 \end{array}\right) \tag{13}$$



The size of the interval τ determines the number of columns in the supply matrix and, consequently, the detail of the analysis. In practice, a period from one to several years is assumed as τ. In extreme cases, it can be assumed $\tau = T^{II}$, which means to collectively capture the system's power supplies during operation and assign these values to a term $t_k^{II} = t_p^{II} + T^{II}$. The supply matrix in this extreme case will be a three-column one.

4. Recycling processes

4.1. Types of forms of vehicle elements development

Recycling is understood as separating and processing one of the vehicle's subsystems for further use. This use may take place in the same vehicle, different vehicles, or in a completely different system. It is worth noting that recycling begins during the production of the vehicle. Processing into useful raw materials is subject, among others, to material waste generated during production, recycling can also be subjected to packaging, used tools, etc.
There are three types of recovery of the subsystem dismounted from the vehicle:

1. rebuilding to original operational properties (remanufacturing)- product recycling,
2. processing into raw materials or materials - material recycling,
3. thermal energy recovery - energy recovery.

The six distinguished forms of development differ from the point of view of their share in the balance of raw materials and energy of the vehicle system. The recycling demand for energy and possible additional raw materials should be included in the input streams of the P system. At the same time, recovered raw materials, materials and energy are the components of the system's output streams.
When considering the life cycle of a vehicle in a wider population of vehicles in the context of the long-term functioning of the automotive industry, two cycles will be noticed: product and energy-material (Figure 2).

4.2. Product recycling

Remanufacturing of the subsystem may be aimed at producing a replacement part for a vehicle or a prefabricated element for the production process of both the vehicle and other products. Product recycling also includes the re-use of the subsystem without undergoing remanufacturing. The need for a remanufacturing may be dependent on the demands on the properties of the subsystem. For example, the subsystem re-use in the same vehicle will require a remanufacturing (mechanical repair, improvement of the appearance) but not necessarily in another vehicle (if the subsystem is fit for further use).
The standards of vehicle supply in subsystems recovered without being processed from other vehicles or after the remanufacturing process will be marked as: $\widehat{ZRP}$. The standards for the production and operation stages will be considered separately: $\widehat{ZRP}^I$, $\widehat{ZRP}^{II}$. They can differ significantly because $\widehat{ZRP}^I$ is determined by manufacturers of the car subsystems in the form of a long-term recycling policy, while $\widehat{ZRP}^{II}$ is often determined individually by vehicle owners who use components obtained from damaged vehicles on their own. Both $\widehat{ZRP}^I$ and $\widehat{ZRP}^{II}$ take the form of an n-element vector (n types of subsystems):

$$\widehat{ZRP}^I = \begin{pmatrix} \widehat{zrp}_1^I \\ \widehat{zrp}_2^I \\ \vdots \\ \widehat{zrp}_n^I \end{pmatrix}, \quad \widehat{ZRP}^{II} = \begin{pmatrix} \widehat{zrp}_1^{II} \\ \widehat{zrp}_2^{II} \\ \vdots \\ \widehat{zrp}_n^{II} \end{pmatrix} \tag{14}$$

where $\widehat{zrp}_i^I$ is the quantity of the i-th subsystem that is installed to the vehicle during production but comes from product recycling while $\widehat{zrp}_i^{II}$ is the amount of the same subsystem derived from product recycling mounted as a complement to the amount consumed, damaged, etc. at the stage of life.
On the basis of the standards (14) and streams of total supplies (matrix (13)) the streams of total system supplies from product recycling are obtained:

$$ZRP = \begin{pmatrix} zrp_{1,1} & \cdots & zrp_{1,q} \\ \vdots & \ddots & \vdots \\ zrp_{n,1} & \cdots & zrp_{n,q} \end{pmatrix} = \\ = (diag(\widehat{zrp}_1^I, \dots, \widehat{zrp}_n^I) \cdot Z^{\langle 1 \rangle} \quad ⬚ \\ diag(\widehat{zrp}_1^{II}, \dots, \widehat{zrp}_n^{II}) \cdot Z^{\langle 2,\dots,q \rangle}) \tag{15}$$

where $diag(\dots)$ denotes a diagonal matrix whose diagonal is formed by the elements in brackets.
Of course, the last column of the matrix (15) is zero, which results from the lack of power to the system at the stage of decommissioning.

Table 1. Classification of forms of development of vehicle subsystems

No.	Processing method	Process effect	Purpose
1	lack	replacement part	another vehicle
2	remanufacturing		the same or a different vehicle
3	lack	prefabricated element	production of a new product (e.g. vehicle)
4	remanufacturing		
5	processing	raw material or production material	
6	energy recovery	thermal energy	

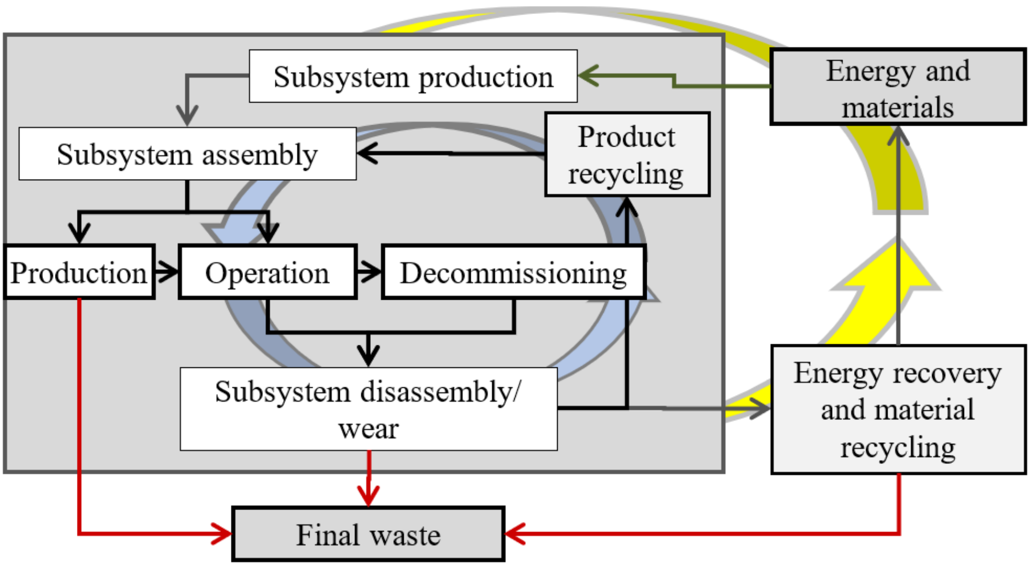


Fig. 2. Product and energy and material cycle

It is also worth noting that the value of the supply stream from product recycling of each subsystem may not be greater than the total sum of supplies, so the following conditions must be met:

$$\forall i = 1,\dots,n;\ j = 1,\dots,q: \quad 0 \leq zrp_{i,j} \leq z_{i,j} \tag{16}$$

<u>Standard coefficients of product recycling of vehicle subsystems</u>

Standards for the number of subsystems intended for product recycling during operation constitute a certain proportion of mass units of the subsystems dismantled from the vehicle after use. The standards for the decommissioning stage of the vehicle will be related to its initial structure (2):

$$\widehat{RP}^{II} = \begin{pmatrix} \widehat{rp}_1^{II} \\ \widehat{rp}_2^{II} \\ \vdots \\ \widehat{rp}_n^{II} \end{pmatrix}, \quad \widehat{RP}^{III} = \begin{pmatrix} \widehat{rp}_1^{III} \\ \widehat{rp}_2^{III} \\ \vdots \\ \widehat{rp}_n^{III} \end{pmatrix} \tag{17}$$

The coefficients of supplies from product recycling and from recycling of used subsystems assume values from the range of [0,1]. They can be expressed as a percentage.

In practice, there may be a situation where product recycling during the life cycle of a vehicle will be imposed in the form of coefficients by applicable law or it will be a forecast based on a statistical analysis of a sufficiently large sample of vehicle life cycles.

Discrete streams of product recycling of vehicle subsystems

The quantities of vehicle subsystems directed after use (or during the use) for product recycling will be presented as discrete streams:

$$RP = \begin{pmatrix} rp_{1,1} & \cdots & rp_{1,q} \\ \vdots & \ddots & \vdots \\ rp_{n,1} & \cdots & rp_{n,q} \end{pmatrix} = \\ = \begin{pmatrix} 0 & & \\ \vdots & diag(\widehat{rp}_1^{II}, \dots, \widehat{rp}_n^{II}) \cdot Z^{\langle 2,\dots,q-1 \rangle} & \\ 0 & & \\ & diag(\widehat{rp}_1^{III}, \dots, \widehat{rp}_n^{III}) \cdot Z^{\langle 1 \rangle} \end{pmatrix} \quad (18)$$

Wherein the following conditions are met:

$$\forall i = 1, \dots, n; \; j = 1, \dots, q: \quad 0 \le rp_{i,j} \le z_{i,j} \quad (19)$$

which show that:

- during operation, product recycling may include such numbers of subsystems that will be delivered to it in a given period of time (as part of maintaining the initial structure of the system),
- during decommissioning, only the quantities constituting the initial structure can be recycled.

Knowing the appropriate values of the matrices (15) and (18), we can determine for each subsystem the value of the product recycling balance:

$$BRP = (brp_i)_{i=1}^{n} = \left(\frac{\sum_{j=1}^{q} (zrp_{ij} - rp_{ij})}{\sum_{j=1}^{q} z_{ij}} \right)_{i=1}^{n} \quad (20)$$

Since for each i and j zrp_{ij}, rp_{ij} take the values from the interval $[0, z_{ij}]$so the numerator of the quotient (20) always belongs to the range $[-\sum_{j=1}^{q} z_{ij}, \sum_{j=1}^{q} z_{ij}]$. Ultimately every indicator brp_i takes values from the interval $[-1,1]$wherein the following properties occur:

1. If $brp_i = -1$ then:
   a. the entire mass of the i-th subsystem supplied to the vehicle is a new product, i.e.: $\sum_{j=1}^{q} zrp_{ij} = 0$ and
   b. the entire mass of the i-th subsystem leaving the system is subject to product recycling, i.e.: $\sum_{j=1}^{q} rp_{ij} = \sum_{j=1}^{q} z_{ij}$.

Considering the global car industry in the long run, we notice that it is impossible to maintain the *brp* index at the level of positive values in the subsequent life cycles of vehicles. The advantage of the remanufactured amount of the subsystem installed in vehicles over the amount directed for remanufacturing will result in the depletion of the resources of the previously produced subsystems and, consequently, the necessity to:

- increasing the production of new subsystems and/or
- increasing product recycling of scrapped subsystems.

From the point of view of energy and material recycling analysis, it is important to identify the actual quantities of the subsystems that are produced and scrapped. The production of subsystems concerns both the vehicle production and operation stages, while the scrapping of subsystems takes place at the operational and decommissioning stages of the vehicle (Figure 3).

The actual production volumes of the subsystems will be determined as the amount of supplies minus the amounts of product recycling:

$$Prod = \begin{pmatrix} prod_{1,1} & \cdots & prod_{1,q} \\ \vdots & \ddots & \vdots \\ prod_{n,1} & \cdots & prod_{n,q} \end{pmatrix} = Z - ZRP \quad (21)$$

where Z – system supply matrix (formulas (11) - (13)), *ZRP* – matrix of product recycling streams feeding the production of subsystems (formula (15)). Certainly, there is: $Prod^{\langle q \rangle} = (0)_{n \times 1}$.

The actual amounts of subsystem scrapping are the difference between the amount of total feeds and the amounts directed for product recycling (formula 18):

- the number of scrapped subsystems at the vehicle production stage is assumed to be zero:
  $$Zlom^{\langle 1 \rangle} = (0)_{n \times 1} \quad (22)$$
- the amount of scrapped subsystems at the operational stage of the vehicle:
  $$Zlom^{\langle 2,\dots,q-1 \rangle} = Z^{\langle 2,\dots,q-1 \rangle} - RP^{\langle 2,\dots,q-1 \rangle} \quad (23)$$
- the number of scrapped subsystems at the stage of vehicle decommissioning:
  $$Zlom^{\langle q \rangle} = Z^{\langle 1 \rangle} - RP^{\langle q \rangle} \quad (24)$$

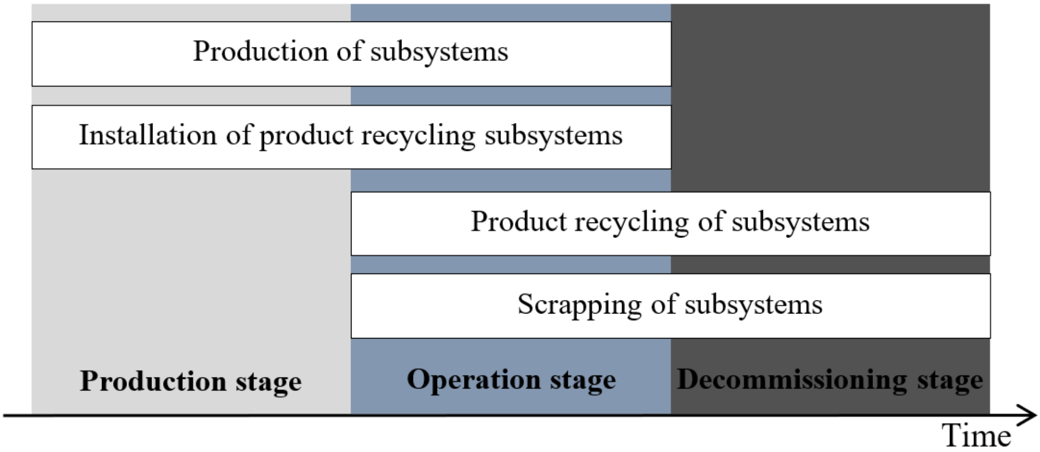


Fig. 3. Processes realized during the life cycle of the vehicle

4.3. Energy and material recovery

End-of-life vehicle subsystems are subject to energy and material recycling (hereinafter the collective term was used: energy and material recycling). As a result, the so-called output factors are recovered.

During the production and operation stages, waste generated in the processes of subsystem production may be subjected to energy and material recycling.

If we distinguish *m* factors, the values of factor recovery can be determined in relation to the unit production of the subsystems:

$$\hat{R}^{I,II} = \begin{pmatrix} \hat{r}_{1,1}^{I,II} & \cdots & \hat{r}_{1,n}^{I,II} \\ \vdots & \ddots & \vdots \\ \hat{r}_{m,1}^{I,II} & \cdots & \hat{r}_{m,n}^{I,II} \end{pmatrix} \quad (25)$$

where $\hat{r}_{i,j}^{I,II}$ is the amount of the *i*-th factor recovered during the production of the mass unit of the *j*-th subsystem.

It should be remembered that standards (29) may change with the development of production optimization methods, which lead, inter alia, to a better use of production materials.

For the operation and decommissioning stages, the following matrix of standards for energy and material recycling will be created:

$$\hat{R}^{II,III} = \begin{pmatrix} \hat{r}_{1,1}^{II,III} & \cdots & \hat{r}_{1,n}^{II,III} \\ \vdots & \ddots & \vdots \\ \hat{r}_{m,1}^{II,III} & \cdots & \hat{r}_{m,n}^{II,III} \end{pmatrix} \quad (26)$$

where $\hat{r}_{i,j}^{II,III}$ is the amount of the *i*-th factor recovered in the process of energy and material recycling from the used mass unit of the *j*-th subsystem.

Based on the matrix of standards (30) and (29), the recycling flows for a specific vehicle system with a fixed feed matrix can be determined (see formulas (11) - (13)).

For a fixed factor, the recycling flow will be presented as a *p*-element sequence of its amounts assigned to the life cycle stages. The operation stage will be assigned separate values for successive time intervals with the length of the interval τ.

Energy and material recycling streams will be written as rows of the matrix:

$$R = \begin{pmatrix} r_{1,1} & \cdots & r_{1,q} \\ \vdots & \ddots & \vdots \\ r_{m,1} & \cdots & r_{m,q} \end{pmatrix} \quad (27)$$

where the first column contains the amount of factors recovered during the production of the vehicle, the columns: 2, ..., *q*-1 contain the amounts of factors recovered during operation, while the last column contains the amounts of factors recovered during vehicle decommissioning.

Individual columns of the matrix *R*, are determined as follows (see formulas (25) - (30)):

$$\forall j = 1, \dots, q: \\ R^{\langle j \rangle} = \hat{R}^{I,II} \cdot Prod^{\langle j \rangle} + \hat{R}^{II,III} \cdot Zlom^{\langle j \rangle} \quad (28)$$

For the operation stage, there is information about the distribution of power into individual time intervals with the length of the time interval τ. Each interval is assigned a corresponding column of matrix *Z*. Thus, it is also possible to determine the factor recovery streams for any *j*-th period of time.
For the production and decommissioning stages (the first and the *q*-th columns of the matrix *R*, respectively), formula (32) is reduced by one of the sum components. This is due to the fact that the last column of the matrix *Prod* and the first column of the matrix *Zlom* are zero.
As a result, it is possible to write a method of calculating any matrix element *R* (i=1,...,m; j=1,…,q):

$$r_{i,j} = \hat{r}_{i,1}^{I,II} \cdot prod_{1,j} + \hat{r}_{i,2}^{II} \cdot prod_{2,j} + \dots + \hat{r}_{i,n}^{II} \cdot prod_{n,j} + \hat{r}_{i,1}^{II,III} \cdot zlom_{1,j} + \hat{r}_{i,2}^{II,III} \cdot zlom_{2,j} + \dots + \hat{r}_{i,n}^{II,III} \cdot zlom_{n,j} \tag{29}$$

$(\rho^{I} \quad \rho^{II} \quad \rho^{III})$ will denote the matrix with three rows and *m* columns, where each column consists of the total amounts of factors recovered in the following stages of the vehicle's life cycle:

$$\forall i = 1, \dots, m: \begin{cases} \rho_i^{I} = r_{i,1} \\ \rho_i^{II} = \sum_{j=2}^{q-1} r_{i,j} \\ \rho_i^{III} = r_{i,q} \end{cases} \tag{30}$$

The amounts of factors that are recovered in the processes of energy and material recycling from a vehicle during the entire life cycle will be determined as follows:

$$\forall i = 1, \dots, m: \quad \rho_i = \rho_i^{I} + \rho_i^{II} + \rho_i^{III} = \sum_{j=1}^{q} r_{i,j} \tag{31}$$

4.4. Final waste

Some of the vehicle's subsystems are not recycled after their end of life, but are stored or released to the environment as final waste. Final waste is also generated during the production of subsystems in the form of unused materials, spent fuels, etc. From an ecological point of view, the content of certain types of materials, raw materials and forms of energy in waste is important. If *m* factors are distinguished and their amounts constituting waste per unit of mass of each manufactured and dismantled subsystem are known, then the following matrices of standards can be created:

1. production waste:

$$\hat{O}^{I,II} = \begin{pmatrix} \hat{o}_{1,1}^{I,II} & \cdots & \hat{o}_{1,n}^{I,II} \\ \vdots & \ddots & \vdots \\ \hat{o}_{m,1}^{I,II} & \cdots & \hat{o}_{m,n}^{I,II} \end{pmatrix} \tag{32}$$

   where $\hat{o}_{i,j}^{I,II}$ is the amount of *i*-th factor contained in waste generated during production of mass unit of *j*-th subsystem. These standards also apply to the operational phase when replacement subsystems are manufactured,

2. scrapping waste:

$$\hat{O}^{II,III} = \begin{pmatrix} \hat{o}_{1,1}^{II,III} & \cdots & \hat{o}_{1,n}^{II,III} \\ \vdots & \ddots & \vdots \\ \hat{o}_{m,1}^{II,III} & \cdots & \hat{o}_{m,n}^{II,III} \end{pmatrix} \tag{33}$$

   where $\hat{o}_{i,j}^{II,III}$ is the amount of *i*-th factor released as waste due to wear of the subsystem (e.g. fuel combustion) and storage in whole or in part of the subsystem removed from the vehicle as used or damaged.

The above standards do not take into account waste generated during the product and energy and material recycling processes. The waste streams leaving the system will be presented analogously to the recycling streams (see matrix (31)) in the form of the final waste matrix:

$$O = \begin{pmatrix} o_{1,1} & \cdots & o_{1,q} \\ \vdots & \ddots & \vdots \\ o_{m,1} & \cdots & o_{m,q} \end{pmatrix} \tag{34}$$

where individual columns are determined according to the following formulas (similar to the formula 28):

$$\forall j = 1, \dots, q: \quad O^{\langle j \rangle} = \hat{O}^{I,II} \cdot Prod^{\langle j \rangle} + \hat{O}^{II,III} \cdot Zlom^{\langle j \rangle} \tag{35}$$

As a result, it is possible to describe the method of calculating any element of the final waste matrix (i=1,...,m; j=1,…,q):

$$o_{i,j} = \hat{o}_{i,1}^{I,II} \cdot prod_{1,j} + \hat{o}_{i,2}^{II} \cdot prod_{2,j} + \dots + \hat{o}_{i,n}^{II} \cdot prod_{n,j} + \hat{o}_{i,1}^{II,III} \cdot zlom_{1,j} + \hat{o}_{i,2}^{II,III} \cdot zlom_{2,j} + \dots + \hat{o}_{i,n}^{II,III} \cdot zlom_{n,j} \tag{36}$$

For the i-th factor, the total amount of waste during vehicle operation can be determined by adding up after the *i*-th row of the waste matrix:

$$\theta_i^{II} = \sum_{j=2}^{q-1} o_{i,j} \quad (37)$$

Performing analogous summations over the full range of variability j allows to obtain information about the number of factors that are absorbed or emitted by the system during the entire life cycle:

$$\forall i = 1,\dots,m: \quad \theta_i = \theta_i^{I} + \theta_i^{II} + \theta_i^{III} = \sum_{j=1}^{q} o_{i,j} \quad (38)$$

$(\theta^{I} \quad \theta^{II} \quad \theta^{III})$ will denote the matrix of total amounts of factors recovered in successive stages of the vehicle life cycle:

$$\forall i = 1,\dots,m: \begin{cases} \theta_i^{I} = o_{i,1} \\ \theta_i^{II} = \sum_{j=2}^{q-1} o_{i,j} \\ \theta_i^{III} = o_{i,q} \end{cases} \quad (39)$$

The amount of factors which is the final waste leaving the vehicle system during the entire life cycle will be determined as follows:

$$\forall i = 1,\dots,m: \ \theta_i = \theta_i^{I} + \theta_i^{II} + \theta_i^{III} = \sum_{j=1}^{q} o_{i,j} \quad (40)$$

The amount of any waste factor can be related to the weight of the vehicle:

$$\frac{\theta_i}{M} = \frac{\sum_{j=1}^{q} o_{i,j}}{\sum_{i=1}^{n} z_i^{I}} \quad (41)$$

5. Description of the numerical program

The mathematical model was implemented in the EN-VEHICLE program. The structure of the EN-VEHICLE program fully reflects the structure of the material and energy-ecological assessment model of motor vehicles, considering their entire life cycle. It consists of six subprograms representing individual blocks vehicle, as listed below:

- Subprogram EN-VEHICLE-PP - block PP;
- Subprogram EN-VEHICLE-WP - block WP;
- Subprogram EN-VEHICLE-EP - block EP;
- Subprogram EN-VEHICLE-RP - block RP;
- Subprogram EN-VEHICLE-OS - block OS;
- Subprogram EN-VEHICLE-UT - block UT.

Similar to the assessment model, the individual subprograms are interconnected through mathematically and informatively represented streams of energy, production resources, as well as environmental loads and waste.

The EN-VEHICLE program is written in the Java language and is built of a set of "objects" (elements that combine data and methods) communicating with each other to perform and visualize calculations. During the implementation of the developed model, efforts were made to utilize as many object-oriented criteria as possible. The program's input data regarding streams and cumulative energy expenditures and environmental loads are stored using XML (Extensible Markup Language) technology. XML is an extensible markup language created to provide flexibility in data formatting. Elements are organized into a hierarchical tree structure, where individual elements are "nested" within previous ones. This organization is achieved through tags and attributes. Each element consists of a start tag, content, and an end tag. Tags are the basic component of XML, characterized by two features: attributes and content. Attributes are "name-value" pairs found inside start tags after the element's name. Content is placed between the opening and closing tags and can include nested elements or character data. Each attribute and content must meet specific requirements for the document to be correctly validated against the standard.

To run the EN-VEHICLE program, the Java Virtual Machine (JVM) is required and must be installed on the computer. Necessary software and information about installing the virtual machine on different operating systems are available on the Sun company's website: http://java.sun.com.

After entering input data files in the appropriate directories, the program can be launched. Since the program is designed to automatically perform computational procedures upon startup, the duration of this operation depends on the computational capabilities of the computer and may take from several seconds to several minutes. Upon running, the EN-VEHICLE program creates an object of the

"Vehicle" class. Data for this object is then loaded from input files, which contain mass streams (construction phase, operation, scrapping). The program also loads data related to databases of cumulative energy expenditures and environmental loads. After the data loading and grouping stage, the program calculates energy expenditure and emission streams for all phases of the vehicle's life. Subsequently, calculations of indicators are performed, and the results are exported to output files. The final stage involves presenting the calculation results in numerical values grouped in appropriate tables, which can be copied to a spreadsheet.

The program window consists of two main panels. The left panel displays the tree structure of the vehicle model (its elements and sub-elements). The main panel contains the program's calculation results, including cumulative energy expenditures, cumulative material expenditures, and cumulative environmental loads.

Tree structures are commonly used in computer science, naturally representing data hierarchies (physical and abstract objects, concepts, etc.) and are mainly used for this purpose. Trees facilitate and accelerate searches and allow for easy manipulation of sorted data. They are well-suited for representing finite sets, which are typically created in computer applications by adding, removing, and checking if an element is in the set (typical database operations).

A special element of the tree is the root (an element without a parent or superior element). Additional concepts related to the tree include leaves (elements without children), node degree (the number of a node's descendants), and sibling elements (elements sharing the same parent as a given node). The key properties of a node include storing data, having a reference to its parent node (in the case of the root, this value is zero), and having a list of its children (a list of references to nodes that are its descendants).

The figure 4 depicts a graphical representation of the EN-VEHICLE program.

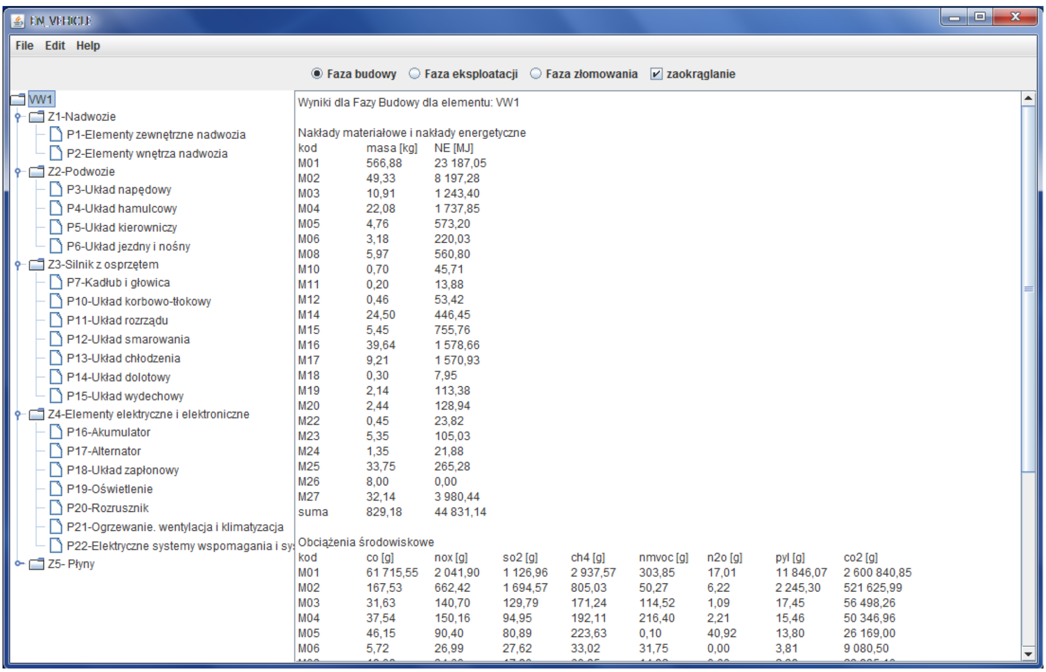


Fig. 4. Graphic illustration of the program

6. Life cycle model verification

The verification of the mass structure model of vehicles during the vehicle construction phase, whose coefficients were determined in the identification process based on empirical data encompassing 33 vehicles from various manufacturers belonging to segments B and C, produced between 1984 and 2013, was brought to a comparison with the course of determined approximation functions (33), (35), (37), (39) with data specifying the percentage shares of analysed material groups. Subsequently, to verify whether the model can be applied to vehicles from other segments, the results were examined for four vehicles belonging to segments A, D, E, and F: Mercedes-Benz A 140 (segment A), Ford Mondeo Mk3 1.8 (segment D), Citroën C6 3.0i V6 (segment E), and BMW 7 Series (E23) 732i (segment F). Data related to the weight of the vehicles and the share of materials were collected during the disassembly of the vehicles at the dismantling station. The necessary characteristics for the analysis of material groups and their environmental impacts were obtained from available databases: Gemis and LCA Plastics Europe Report, as well as from supporting computational software: SimaPro and Greet.

The accuracy of the model was evaluated based on the maximum values of the percentage difference in the share of selected materials described by regression functions and determined from empirical data for these four vehicles. The graphical illustration of the results obtained using the model is presented in Figure 5-8.

The relative differences in the results of this comparison in the examined vehicle production periods have been estimated as follows:

1. When determining the average percentage share of steel, cast iron, and iron mass in motor vehicles (material group M1) – $\Delta M1 \leq 1\%$;
2. When determining the average percentage share of aluminum and its alloys mass in motor vehicles (material group M2) – $\Delta M2 \leq 2\%$;
3. When determining the average percentage share of plastics and rubber mass in motor vehicles (material group M3:M13, M16) – $\Delta(M3{:}M13\ M16) \leq 1\%$;
4. When determining the average percentage share of non-ferrous metals: copper, zinc, tin, nickel, magnesium mass in motor vehicles (material group M17) – $\Delta M17 \leq 2\%$.

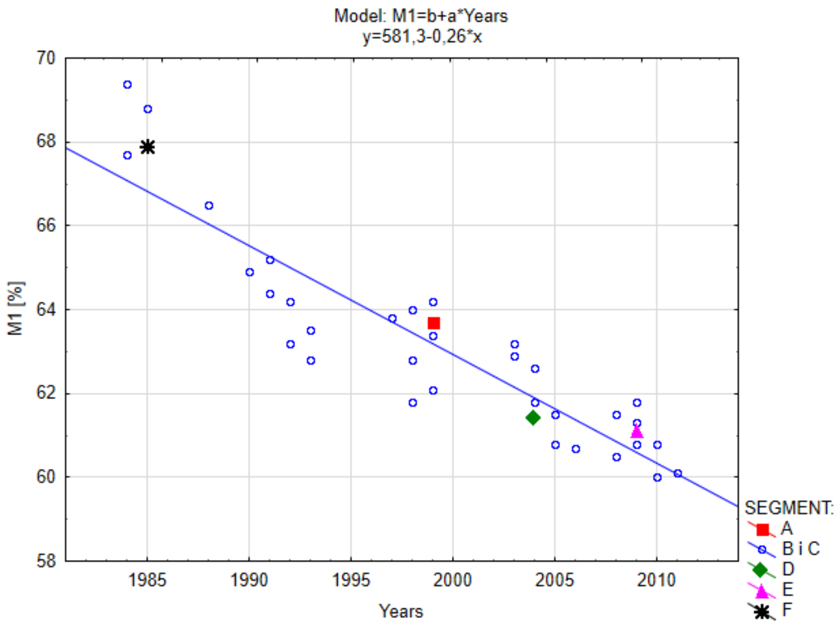


Fig. 5. The share of steel, cast iron, and ductile iron in analysed passenger cars in successive production periods

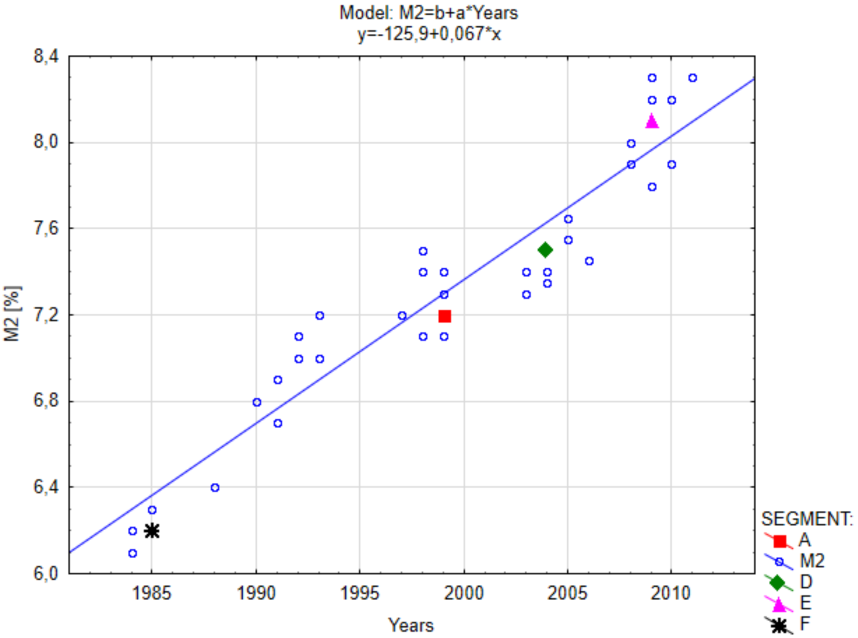


Fig. 6. The share of aluminum and its alloys in the analysed passenger cars in successive production periods

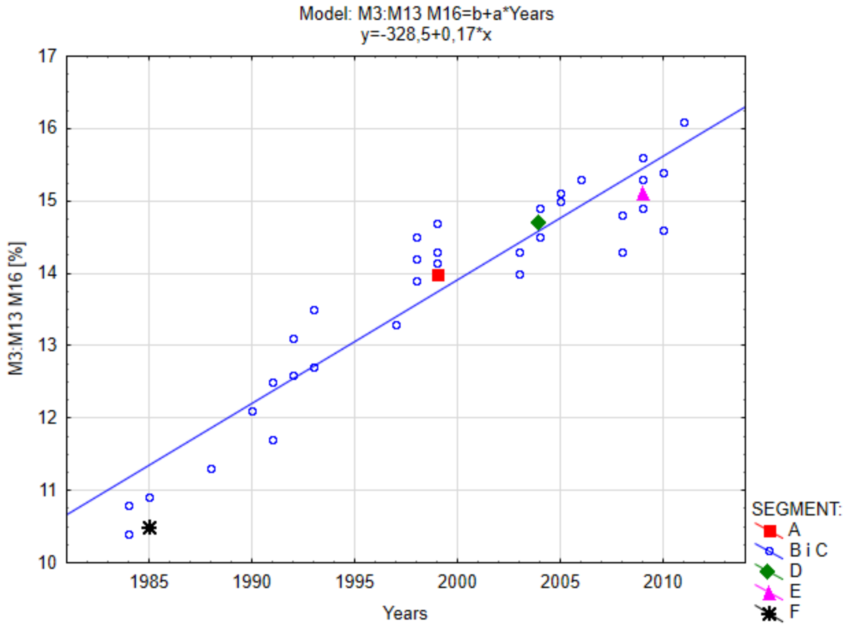


Fig. 7. The share of plastics and rubber in the analysed passenger cars in successive production periods

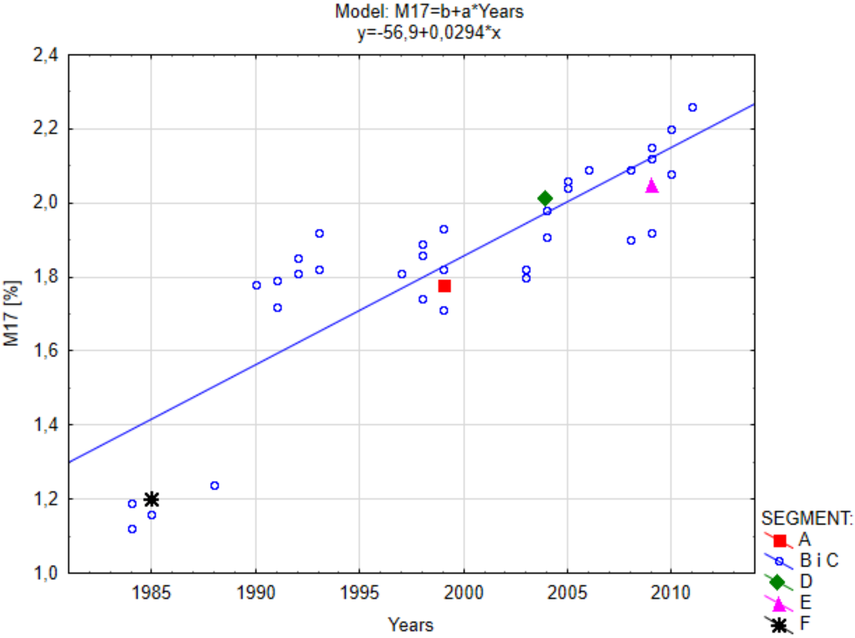


Fig. 8. The share of non-ferrous metals in the analysed passenger cars in successive production periods

Additionally, the model describing the changes in vehicle mass in successive production periods was verified. For this purpose, the relationship curve (31) was compared with the curve of the approximation function for the mass of the BMW 7 Series with a gasoline engine, belonging to segment F. The mass of vehicles of this brand in successive production years was as follows:

- Model E23 732i, production years: 1979–1986, mass: 1540 kg;
- Model E32 730i, production years: 1986–1994, mass: 1600 kg;
- Model E38 730i, production years: 1994–1998, mass: 1725 kg;
- Model E65 730i, production years: 2003–2005, mass: 1790 kg;
- Model F01 730i, production years: from 2009, mass: 1860 kg.

The following model representing the changes in vehicle mass in successive production periods along with the estimation errors was obtained:

$$\underset{}{Mass} = \underset{(2,0)}{13,0} \cdot Years - \underset{(3995,0)}{24336,7} \quad [\mathrm{kg}] \tag{42}$$

The mass of BMW 7 Series vehicles increases on average by about 13±2.0 kg each year.

Graphical illustration of the verification of the models for changes in vehicle mass is presented in Figure 9. It can be noticed that there is a significant similarity between the regression function curves determined for the BMW 7 Series vehicle (segment F) and for vehicles belonging to segments B and C. The relative changes in mass for these groups of vehicles in successive production periods are at a similar level. This is confirmed by the similar value of the regression models' direction coefficients (14.3 for vehicles from segments B and C, and 13.0 for the BMW car from segment F).

The presented verification results confirm that the determined regression models describing changes in mass (31), (40), and material structure (33), (35), (37), (39) accurately reflect the state of development and qualitative changes in the construction of passenger cars belonging to different segments and manufactured by various producers from 1984 to 2013. The general nature of the obtained mathematical description indicates the possibility of forecasting changes in the material structure of vehicles independently of their total mass in the subsequent years of production, provided that there is no significant change in their manufacturing technology.

7. Conclusions

The main goal of the paper was to develop a proprietary numerical program enabling the analysis of energy consumption and emission loads associated with material inputs across the entire life cycle of a passenger car. This developed program facilitates a quantitative interpretation of calculation results concerning environmental safety in each phase and throughout the vehicle's life cycle, considering the potential incorporation of additional material streams from recycling and recovery during both the construction and operational phases of the car.
The developed tool employs matrix calculus and regression function analysis to establish relationships. Specifically, the regression functions depict the correlation between a vehicle's production year and its mass. This model and numerical program can be used for comparative analyses of the environmental impacts of various vehicles manufactured over the years. The determined characteristics of changes in the ecological profile of vehicles stem from alterations in their material structure.
However, achieving such analytical capability necessitates the development of an extensive database comprising technical and material characteristics of vehicles, as well as statistics on their defects. Therefore, in subsequent studies, the model presented in the article can also forecast the environmental effects of changes in the material structure influencing production technology. Consequently, these changes significantly impact the entire life cycle of the vehicle and its energy and ecological parameters.

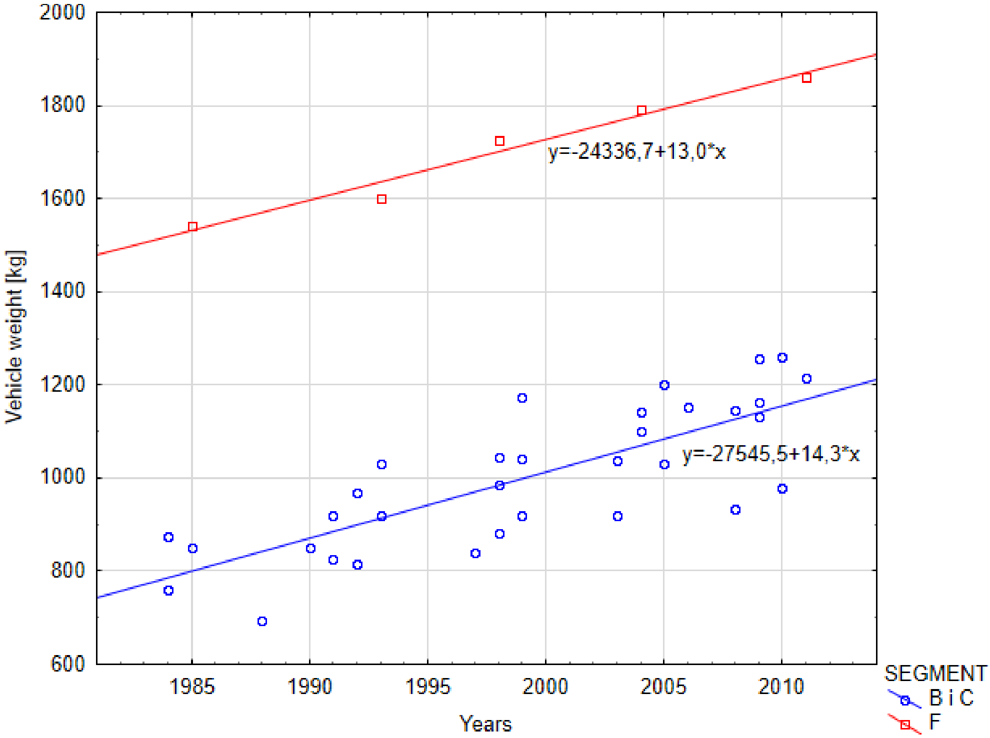


Fig. 9. Weight of passenger cars from segments B, C, and F in relation to their production years

Nomenclature

I, II, III	– designation of the next stages of the vehicle life cycle,
$BRP = (brp_i)_{i=1}^{n}$	– vector of subsystem product recycling balance,
M	– total mass of the vehicle,
m	– number of distinguished energy and material factors used/recovered during the production/recycling of subsystems,
n	– number of subsystems in the vehicle,
$\hat{O}^{I,II} = \left(\hat{o}_{i,j}^{I,II}\right)_{m\times n}, \hat{O}^{II,III} = \left(\hat{o}_{i,j}^{II,III}\right)_{m\times n}$	– matrices of final waste standards during the production and decommissioning of subsystems, respectively,
$O = \left(o_{i,j}\right)_{m\times q}$	– final waste stream matrix,
q	– number of time intervals distinguished in the life cycle of the vehicle,
$Prod = \left(prod_{i,j}\right)_{n\times q}$	– matrix of production streams of new subsystems ($prod_{i,j}$ – supply of the *i*-th vehicle subsystem in the *j*-th time interval of the life cycle),
$\hat{R}^{I,II} = \left(\hat{r}_{i,j}^{I,II}\right)_{m\times n}, \hat{R}^{II,III} = \left(\hat{r}_{i,j}^{II,III}\right)_{m\times n}$	– matrices of energy and material recycling standards during the production and decommissioning of subsystems, respectively,
$R = \left(r_{i,j}\right)_{m\times q}$	– matrix of energy and material recycling streams,
$\widehat{RP}^{II} = \left(\widehat{rp}_i^{II}\right)_{n\times 1}, \widehat{RP}^{III} = \left(\widehat{rp}_i^{III}\right)_{n\times 1}$	– subsystem mass standards, which are routed after consumption for product recycling during the use and decommissioning phase, respectively,
$RP = \left(rp_{i,j}\right)_{n\times q}$	– matrix of subsystem streams directed to product recycling after consumption,
T^I, T^{II}, T^{III}	– duration of stages,
$\left(\hat{t}_j\right)_{j=1}^{q}$	– a sequence of terms assigned to the appropriate (*j*-th) columns of the discrete flux matrix (including *Z*, *Prod*, *RP*, *O*, ...),
$t_k^I, t_k^{II}, t_k^{III}$	– final terms of stages,
$z_i^I(t), z_i^{II}(t), z_i^{III}(t)$	– power supply of the *i*-th vehicle subsystem in successive stages as a function of time,
$Z = \left(z_{i,j}\right)_{n\times q}$	– matrix of system supply streams ($z_{i,j}$ – supply of the *i*-th vehicle subsystem in the *j*-th time interval of the life cycle),
$Zlom = \left(zlom_{i,j}\right)_{n\times q}$	– matrix of subsystem decommissioning streams ($zlom_{i,j}$ – scrapped quantity of the *i*-th vehicle subsystem in the *j*-th time interval of the life cycle),
$\widehat{ZRP}^I = \left(\widehat{zrp}_i^I\right)_{n\times 1}, \widehat{ZRP}^{II} = \left(\widehat{zrp}_i^{II}\right)_{n\times 1}$	– mass standards vectors of subsystems that are fitted to a vehicle during the production and operational stages respectively, but come from product recycling,
$ZRP = \left(zrp_{i,j}\right)_{n\times q}$	– matrix of supply streams for the production of subsystems from product recycling,
$\theta^I = \left(\theta_i^I\right)_{m\times 1}, \theta^{II}, \theta^{III}$	– vectors of the total quantity of factors scrapped as final waste in the next stages of the vehicle life cycle,
$\theta = (\theta_i)_{m\times 1}$	– vectors of the total amounts of factors scrapped as final waste over the entire life cycle of the vehicle,
$\rho^I = \left(\rho_i^I\right)_{m\times 1}, \rho^{II}, \rho^{III}$	– vectors of the total amount of factors recovered in the successive stages of the life cycle of the vehicle,
$\rho = (\rho_i)_{m\times 1}$	– vectors of the total amounts of factors recovered over the entire life cycle of the vehicle,
τ	– length of time interval of the operational phase of the vehicle.

References

[1] Chanaron, J-J. (2007). Life cycle assessment practices: Benchmarking selected European Automobile Manufactures. International Journal of Product Lifecycle Management, 2(3), 290–311. https://doi.org/10.1504/IJPLM.2007.016293.
[2] Das, S. (2011). Life cycle assessment of carbon fiber-reinforced polymer composites. International Journal of Life Cycle Assessment, 16(3), 268–282. http://dx.doi.org/10.1007/s11367-011-0264-z.
[3] De Medina, H.V. (2006). Eco-design for materials selection in automobile industry. In: CIRP International Conference on Life Cycle Engineering, 13th, Leuven, Belgium, 299-304.
[4] European Environment Agency. Nitrogen Oxides (Nox) Emissions. Available online: https://www.eea.europa.eu/data-and-maps/indicators/eea-32-nitrogen oxides-nox-emissions-1/assessment. 2010-08-19.0140149032-3 (accessed on 25 October 2021).
[5] Finkbeiner, M., Hoffmann, R., Ruhland, K., Liebhart, D., & Stark, B. (2006). Application of Life Cycle Assessment for the Environmental Certificate of the Mercedes-Benz S-Class. International Journal of Life Cycle Assessment, 11(4), 240–246.
[6] Gradin, K.T., & Åström, A.H. (2020). Comparative life cycle assessment of car disc brake systems - case study results and method discussion about comparative LCAs. International Journal of Life Cycle Assessment, 25, 350–362. https://doi.org/10.1007/s11367-019-01704-9.
[7] Gradin, K.T., Luttropp, C., & Björklund, A. (2013). Investigating improved vehicle dismantling and fragmentation technology. Journal of Cleaner Production, 54(1), 23–29. https://doi.org/10.1016/j.jclepro.2013.05.023.
[8] Hawkins, T.R., Singh, B., Majeau-Bettez, G., & Strømman, A.H. (2013). Comparative Environmental Life Cycle Assessment of Conventional and Electric Vehicles. Journal of Industrial Ecology, 17(1), 53–64. https://doi.org/10.1111/j.1530-9290.2012.00532.x.
[9] International Energy Agency, https://www.iea.org/data-and-statistics/charts/transport-sector-co2-emissions-by-mode-in-the-sustainable-development-scenario-2000-2030 (accessed on 25 April 2022)
[10] International Organization for Standardization (ISO), 2006a. 14040–Environmental Management. Life Cycle Assessment. Principles and Framework.
[11] International Organization for Standardization (ISO), 2006b. 14044–Environmental Management. Life Cycle Assessment. Requirements and Guidelines.
[12] Klocke, F., Kampker, A., Döbbeler, B., Maue, A., & Schmieder, M. (2014). Simplified life cycle assessment of a hybrid car body part. Procedia CIRP, 15, 484–489. https://doi.org/10.1016/j.procir.2014.06.056.
[13] Lewicki, R. (2009). End-of-life vehicles in the light of environmental benefits identified in the products life cycle. Scientific Problems of Machines Operation and Maintenance, 1(57), 87–99.
[14] Ma, H., Balthasar, F., Tait, N., Riera-Palou, X., & Harrison, A. (2012). A new comparison between the life cycle greenhouse gas emissions of battery electric vehicles and internal combustion vehicles. Energy Policy, 44, 160–173. https://doi.org/10.1016/j.enpol.2012.01.034.
[15] Mayyas, A., Qattawi, A., Omar, M., & Shan, D. (2012). Design for sustainability in automotive industry: A comprehensive review. Renewable and Sustainable Energy Review, 16(4), 1845–1862. https://doi.org/10.1016/j.rser.2012.01.012.
[16] Merkisz-Guranowska, A. (2020). A comparative study on end-of-life vehicles network design. Archives of Transport, 54(2), 107-123; DOI: 10.5604/01.3001.0014.2971.
[17] Mrozik, M., Merkisz-Guranowska, A. (2021). Environmental assessment of the vehicle operation process. Energies, 14, 76. https://dx.doi.org/10.3390/en14010076.
[18] Nitta, S., & Moriguchi, Y. (2011). New methodology of life cycle assessment for clean energy vehicle and new car model, SAE Technical Paper 2011-01-0851. https://doi.org/10.4271/2011-01-0851.
[19] Petrov, R., Vasilyev, M., Kabanova, O., & Kirillov, V. (2001). Comparative analysis of dismantlability for LADA Cars. SAE Technical Paper, 2001-01-3755. https://doi.org/10.4271/2001-01-3755.
[20] Raugei, M., Morrey, D., Hutchinson, A., & Winfield, P. (2015). A coherent life cycle assessment of a range of light weighting strategies for compact vehicles. Journal of Cleaner Production, 108, 1168–1176. http://dx.doi.org/10.1016/j.jclepro.2015.05.100.

[21] Ribeiro, C., Ferreira, J.V., & Partidário, P. (2007). Life Cycle Assessment of a multi-material car component. International Journal of Life Cycle Assessment, 12(5), 336–345. https://doi.org/10.1065/lca2006.12.304.
[22] Schmidt, W.P., Dahlqvist, E., Finkbeiner, M., Krinke, S., Lazzari, S., Oschmann, D., et al. (2004). Life Cycle Assessment of lightweight and end-of-life scenarios for generic compact class passenger vehicles. International Journal of Life Cycle Assessment, 9(6), 405–416. https://doi.org/10.1007/BF02979084.
[23] Suzuki, T., Odai, T., Hukui, R., & Takahashi, J. (2005). LCA of passenger vehicles lightened by recyclable carbon fiber reinforced plastics. In: Proceedings of the International Conference on Life Cycle Assessment, San Jose, Costa Rica, April 25−28, 2005. http://jt.o.oo7.jp/publications/050425.pdf.
[24] Unece United Nations Economic Commission For Europe, Climate Change and Sustainable Transport. Available online: http://www.unece.org/?id=9890 (accessed on 26 October 2022).