THE ARCHIVES OF TRANSPORT

VOL. XXII NO 1 2010

10.2478/v10174-010-0003-4

Quasistatic Approach to Wheel – Rail Contact Problems with Elastic Graded Materials

Andrzej Chudzikiewicz*

Andrzej Myśliński**

Received November 2009

Abstract

Graded materials are generally two - phase composities with continously varying volume fraction. Numerous experiments indicate that used as the coatings attached to the conventional steel body and interfacial zones they can reduce the magnitude of mechanically and/or thermally induced stresses. In this paper the wheel - rail contact problem including friction and wear is considered. The rail is assumed to be covered with a coating. The mechanical properties of the coating material depend on its distance to the rail surface and are governed by power law. In the paper quasistatic approach to solve numerically this rolling contact problem is employed. This approach is based on the assumption that for the observer moving with the rolling wheel the displacement of the rail is independent on time. Finite element method is used as a discretization method. Numerical results are provided and discussed.

Keywords: rolling contact problem, elastic graded materials, quasistatic method

1. Introduction

Functionally graded materials [25, 26] are generally two – phase composities with continuously varing volume fractions. They are characterized by spatially varied microstructures created by non- uniform distributions of the reinforcement phase with different properties, sizes, and shapes, as well as, by interchanging the role

* Institute of Transport, Warsaw University of Technology, 00-662 Warszawa, Koszykowa 75 Str., Poland

** Systems Research Institute, Newelska 6 Str., 01-447 Warsaw, Poland

of reinforcement and matrix materials in a continuous manner. Such multi-phase materials cover a range of space and time scales, and are best understood by means of a comprehensive multi-scale multi-physics approach. Functionally graded material coatings are widely used in engineering structures where contact problem is a major concern. The results of experiments as well as numerical computations indicate [29, 31] these materials can reduce the magnitude of thermally and mechanically induced stresses following from material property mismatch and the likelihood of cracking due to excessive contact stress, increase the bonding strength and improve the structure performance as well as provide protection against adverse enviroments and extend their service life. Coating materialshave a broad range of applications [26, 31] including such structures as bearings, gears, machine tools or abradable seals in gas turbines. Graded materials constructed by material engineers are aimed to be more damage resistant than their conventional homogeneous counterparts.

The technical literature in contact mechanics is very extensive. The elastic or thermoelastic rolling contact problems involving homogeneous materials have been considered by many authors. For details see the references in monographs [9, 14, 20, 23, 27, 28, 30] or papers [3, 4, 7, 17, 22, 23]. The contact behaviour of functionally graded material structures has received increasing research effort in the recent years. The comprehensive review on the production techniques, thermo – mechanical behaviour and potential main applications of functionally graded materials is presented in [26]. In [1], on the basis of a large number of computational simulations, a general methodology for assessing instrumented indentation response of plastically graded materials is formulated. The quantitative interpretations of depth-sensing indentation experiments could be also performed.

Papers [8, 13, 15, 25, 31] deal with analytical and/or numerical solutions of indentation problems involving elastic graded materials. Among others, closed form analytical solutions to two – dimen- sional normal, sliding and rolling contact problems between a rigid cylinder and an elastic graded substrate are provided in [8]. The elastically graded substrate is modeled by the locally isotropic material with constant Poisson ratio and the elastic modulus varing with depth according to power law. In [25, 31] singular integral equation technique is employed to obtain formulae of analytical solutions. Moreover in [31] multilayered contact problem is considered and numerical results are provided.

Functionally graded material coating has been applied and investigated in [12] to reduce the sus- ceptibility of the brake and clutch system to thermoelastic instability phenomenon due to frictional heating at the interface. Coating material has ceramic and steel properties depending on the height of the coating layer. Heat flow in the coating layer is governed by the conduction equation with variable coefficients while heat flow in the frictional layers is assumed to be governed by the conduction – convection equation. The obtained numerical results indicate that the application of coating leads to increase of the critical speed of the brake disk.

The results of fields tests concerning the application of elastic graded coatings in wheel – rail contact problems are reported, among others, in papers [10, 11]. Laser supported cladding technology has been used in [10] to develop two – material rail to increase the productivity and reliability of rail infrastructure. Field test results confirm that two – material rail is rolling contact fatigue resistant and reduce squeal noise. In [29] shakedown diagrams for various wheel – rail contact situations at the test site are calculated and discussed. These diagrams are based on the results of dynamic train – track interaction simulation obtained due to GENSYS software and on the stresses calulated for the coated railheads using the commercial finite element code ABAQUS. It is shown that the coated rails with the correct rail profile can be used to prevent rolling contact fatigue and reduce wear for the current train traffic situation.

This paper deals with the contact between a rigid wheel and an elastic rail lying on a rigid foundation including Coulomb friction [28, 30] as well as wear phenomenon governed by Archard law [17, 21]. The railhead where wheel – rail contact occurs is assumed to be coated with elastic graded material which properties depend on its depth according to the power law.

The aim of this paper is to calculate and investigate contact pressure in the wheel – rail contact problem assuming the rail is covered with an elastic coating. Following [2, 3, 4, 6, 7, 18] we take special features of this rolling contact problem and use so – called quasistatic approach to calculate numerically normal contact pressure. The proposed approach is based on the assumption that for the observer moving with the rolling wheel the displacement of the rail is independent of time. Moreover we shall assume that the length of the rail is much bigger than the diameter of the wheel. In this approach the inertial term of displacement governing equation is replaced by the stationary term depending on wheel velocity and the derivative of displacement field with respect to spatial varaibles. This term is reflecting the dynamics of the body rather than completely neglected it as in the classical quasistatic formulation [9, 28]. Therefore under these assumptions the elastic wheel – rail contact problem with friction and wear is described by the system of coupled stationary equations rather than by the system of time dependent equations.

The quasistatic method allows to solve wheel – rail contact problems at a lower mathematical and numerical effort than standard methods. The method has been already used to solve different wheel – rail contact problems [3, 4, 6] with the homogeneous materials. The comparison [5, 6] of this method to the Fastsim algorithm [14] or Green function approach [7] indicates that the calculated contact pressure or temperature values and distributions inside the rail are very close to the calculated ones by others methods.

In this study, the elastic model of wheel – rail contact problem with friction and wear in the frame- work of two-dimensional linear elasticity theory [9, 27] is formulated. This physical phenomenon is described by general coupled time dependent system. Under the mentioned earlier assumptions this nonstationary system is transformed into equivalent stationary system in the framework of the so – called quasistatic formulation. The numerical results are provided and discussed.

2. Rolling Contact Problem

Consider deformations of an elastic rail lying on a rigid foundation (Fig. 1). The rail has constant height h and occupies domain $\Omega \in R^2$ with the boundary Γ. The strip is assumed to consists from two elastic layers denoted by Ω_c and Ω_s such that $\Omega = \Omega_c \cup \Omega_s$. Ω_c and Ω_s denote the coating and substrate layers, respectively. The thickness of each layer is h_c and h_s respectively. Moreover $h_c < h_s$ and $h = h_c + h_s$. By Γ_c and Γ_s we denote the boundaries of Ω_c and Ω_s respectively. The boundary Γ of strip Ω is assumed to consist from two parts Γ_0 and Γ_C such that $\Gamma = \Gamma_0 \cup \Gamma_C$. Obviously the boundary Γ is equal to sum of the boundaries Γ_c and Γ_s minus their intersection. A wheel rolls along the upper surface Γ_C of the strip Ω. The wheel has radius r_0, rotating speed ω and linear velocity V. The axis of the wheel is moving along a straight line at a constant altitude h_0 where $h_0 < h + r_0$, i.e. the wheel is pressed in the elastic strip. We assume that the lenght of the strip is much bigger than the radius of the wheel. Therefore it is assumed that the head and tail ends of the strip are clamped. No mass forces in the strip are assumed.

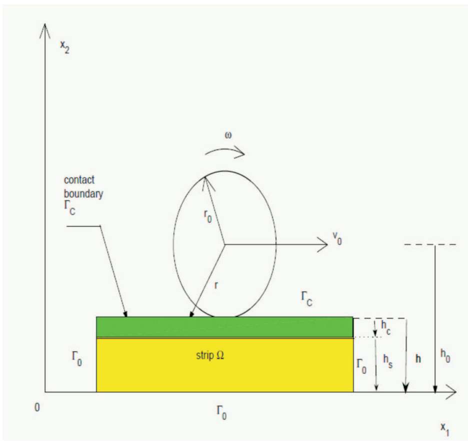

Fig. 1. Wheel rolling over the rail

2.1. Material Properties in Functionally Graded Material

The material property of the wheel – rail model is composed of steel and ceramic like in the coating layer. The steel material is assumed to possess homogeneous properties through the whole volume while the coating material has a variation property along its height in its domain. The graded material coating of rail is assumed to be processed in such a way that the property grading is smooth. It implies that the discontinuities in the material property distribution are eliminated and stresses throught the composite layer are bounded.

The determination of material distribution in the graded layer has been reported in many papers [12]. One of the method to select the material properties distribution is to divide the domain into multi-layers with constant properties. The material property of graded layer is a function of height and is described in each layer by the following equation

$$P = P(x) = P_c + (P_s - P_c)\left(\frac{h_s + h_c - x_2}{h_c}\right)^n, \quad h_s \le x_2 \le h_s + h_c \tag{1}$$

where $x = (x_1, x_2) \in \Omega$, $P = P(x)$, denote the material property dependent on height x_2 and P_c, P_s are the ceramic property and the steel property, respectively. n denotes the nonhomogenity parameter of the graded material. P may be used for the elastic modulus.

Distribution model (1) indicates that the composition would vary continuously from 100% steel material near the interface to 100% ceramic near the surface. Thus the material is purly steel at the core part and gradually move and approaches the ceramic properties at the upper surface of coating layer. The inner material distribution of graded layer is determined by the parameter n. For n= 1 the material of graded layer is linearly distributed. For n >1 the coating layer is more ceramic rich while for 0 < n <1 is more steel – rich. Remark n= 0 denotes the homogeneous rail material, i.e., there is no coating. The variation of the elasic modulus and the thermal conductivity in the graded material coating layer for three values of n is shown on Fig. 2.

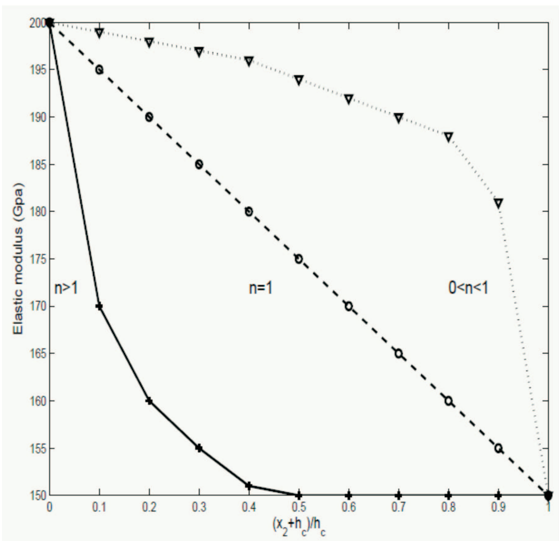

Fig. 2. The variation of the elastic modulus for different values of n

3. Elastic Model

The elastic rolling contact problem is described by the displacement governing system of equations. Let us denote by $u = (u_1, u_2)$, $u = u(x,t)$, $x \in \Omega$, $t \in (0,T)$, $T > 0$, a displacement of the strip. Since the rail consists from the coating layer Ω_c and the substrate layer Ω_s we denote by u_c and u_s the displacements of these layers, respectively. Therefore $u = u_c$ in Ω_c and $u = u_s$ in Ω_s.

Assume wheel and rail are brought into contact under the action of the static wheel load. The contact area and the contact pressure distribution are usually calculated using Hertz's theory [7, 27]. Here we use two dimensional elastic linear model and Coulomb friction model to determine contact area and stress distribution. The displacement u of the strip Ω is governed by the system of the following equations [3, 22]:

$$\rho(x_2)\frac{\partial^2 u_c}{\partial t^2} = A^* D(x_2) A u_c \quad \text{in } \Omega_c \times (0,T) \tag{2}$$

$$\rho_s \frac{\partial^2 u_s}{\partial t^2} = A^* D A u_s \quad \text{in } \Omega_s \times (0,T) \tag{3}$$

Mass density $\rho(x_2)$ depends on x_2 in Ω_c according to (1). $\lambda(x_2)$ and $\gamma(x_2)$ are Lame coefficients [15, 16] depending on x_2 in Ω_c. Their dependence on x_2 is governed by (1). Constants ρ_s, λ_s, γ_s denote mass density, and Lame coefficients in domain Ω_s. Recall [15] Lame coefficients λ and γ are related with Young modulus E and Poisson's ratio ν by relations

$$\lambda \frac{E\nu}{(1+\nu)(1-2\nu)}, \qquad \gamma = \frac{E}{2(1+\nu)}$$

The matrices A, B and coefficient matrix $D = D(x_2)$ are defined as follows [6]

$$A = \begin{bmatrix} \frac{\partial}{\partial x_1} & 0 \\ 0 & \frac{\partial}{\partial x_2} \\ \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_1} \end{bmatrix}, \qquad B = \begin{bmatrix} n_1 & 0 \\ 0 & n_2 \\ n_2 & n_1 \end{bmatrix}, \qquad D = \begin{bmatrix} \lambda + 2\gamma & \lambda & 0 \\ \lambda & \lambda + 2\gamma & 0 \\ 0 & 0 & \gamma \end{bmatrix} \tag{4}$$

In (4) $n = (n_1, n_2)$ denotes outward normal versor to the boundary Γ of the domain Ω. A^* denotes a transpose of A. Along the boundary $\Gamma_0 = \Gamma \backslash \Gamma_C$ the strip is assumed to be clamped, i.e.,

$$u_c = u_s = 0 \quad \text{on } \Gamma_0 \times (0,T) \tag{5}$$

Displacement continuity condition is assumed on the interface between layers

$$u_s = u_c = 0 \quad \text{on } \Gamma_c \cap \Gamma_s \tag{6}$$

In the contact zone the surface traction vector F is determined by:

$$B^* D(x_2) A u_c = F \quad \text{on } \Gamma_C \times (0, T) \tag{7}$$

At the initial moment $t=0$ the displacement and velocity of points of the rail domain is given, i.e.,

$$u_c(0) = \bar{u}_{0c} \quad u_c'(0) = \bar{u}_{1c} \quad \text{in } \Omega_c \tag{8}$$

$$u_s(0) = \bar{u}_{0s} \quad u_s'(0) = \bar{u}_{1s} \quad \text{in } \Omega_s \tag{9}$$

where for $j=$ "c" or $j=$ "s" we denote $u_j' = du_j/dt$, $\bar{u}_{0j}$, $\bar{u}_{1j}$ are given functions.

By $\sigma = (\sigma_{11}, \sigma_{22}, \sigma_{12})$ and F we denote the stress tensor in domain Ω and surface traction vector on the boundary Γ respectively. The surface traction vector $F = (F_1, F_2)$ on the boundary Γ_C is a priori unknown and is given by conditions of contact and friction. Under the assumptions that the strip displacement is suitable small the contact conditions take a form [10, 15, 30]:

$$u_{c2} + g_r + w \leq 0, \quad F_2 \leq 0 \quad (u_{c2} + g_r + w)F_2 = 0 \quad \text{on } \Gamma_C \times (0, T) \tag{10}$$

$$|F_1| \leq \mu |F_2|, \quad F_1 \frac{du_1}{dt} \leq 0, \quad (|F_1| - \mu |F_2|) \frac{du_1}{dt} = 0 \quad \text{on } \Gamma_C \times (0, T) \tag{11}$$

where μ is a friction coefficient and u_{c2} denotes vertical component of u_c. For the sake of simplicity we assume here this friction coefficient is constant. In general it may be dependent on temperature or sliding velocity $\frac{du_1}{dt}$ [3, 4]. Under suitable assumptions the gap between the bodies is equal to $g_r = h - h_0 + \sqrt{r_0^2 - (u_1 + x_1)^2}$. Relations (10) and (11) describe nonpenetration and Coulomb friction conditions, respectively.

Function $w = w(x, t)$ in (10) denotes the additional distance between the bodies due to wear [17, 21, 27] of the contacting surfaces. For details concerning wear phenomenon and models see [17, 27]. We assume wear on the boundary $\Gamma_C \times (0, T)$ is governed by Archard law [21],

$$\frac{dw}{dt} = kVF_2 \tag{12}$$

$w = w(x, t)$ is an internal state variable to model the wear process taking place at the contact interface [17, 27]. $k > 0$ denotes a given dimensional wear coefficient. In the considered model the wear is described as an increase in the gap in the normal direction between the contacting bodies. For generalizations of wear models see [21].

4. Quasistatic Formulation

Taking into account the special features of the contact problem (2)–(12) one can reformulate it in the framework of the quasistatic approach [3]. This approach is based on the main assumption that for the observer moving with a wheel with the constant linear velocity V its displacement does not depend on time. Moreover let us assume:

(i) the lenght of the strip is much bigger than the radius of the wheel,

(ii) for the observer moving with the wheel the displacement of the strip does not depend on time,

(iii) the velocity of the wheel is small enough,

(iv) the wear debris disappear immediately at the point where it is formed influencing the contact conditions by increasing the gap between the contacting bodies only.

Since we consider the rail which has finite length rather than infinte length assumption (i) is the minimal requirement to formulate displacement equations (2), (3). Assumption (ii) is essential to transform the original contact problem into quasistatic one. The observer does not distinguish between points of the upper surface of the rail. Assumption (iii) is introduced to ensure the positive definitness of the stiffeness matrix, i.e., the existence of solutions to the contact problem. Numerically this assumption allows to cover wide range of velocities appearing in the operational systems. The last assumption simplifies the wear phenomenon. Contact models assuming the existence of intermediate layer of wear particles between contacting surfaces are still under development and it is not known whether they possess a solution.

To transform equations (2), (3) into quasistatic form let us introduce the new cartesian coordinate system O'x'$_1$x'$_2$ hooked in the middle of the wheel. Then new system O'x'$_1$x'$_2$ and the original one Ox$_1$x$_2$ are related by:

$$x_1' = x_1 - Vt, \qquad x_2' = x_2 \tag{13}$$

Since by assumption (ii) u(x'$_1$x'$_2$) does not depend on time we obtain:

$$\frac{\partial u}{\partial t}(x_1', x_2') = \frac{\partial u}{\partial t}(x_1 - Vt, x_2, t) = 0 \tag{14}$$

By elementary differentiation (13)–(14) imply

$$\frac{\partial u}{\partial t} = -V\frac{\partial u}{\partial x_1} \quad \text{and} \quad \frac{\partial^2 u}{\partial t^2} = V^2\frac{\partial^2 u}{\partial x_1^2} \tag{15}$$

Let Ω denotes now the moving part of the strip seen by the observer. Taking into account (13) and using (15) we can transform system (2)–(12) into quasistatic form. This problem has the following form: find u_c, u_s, θ_c, θ_s depending on spatial variables only satisfying displacement governing equations

$$A^*D(x_2)Au_c - \rho(x_2)V^2\frac{\partial^2 u_c}{\partial x_1 \partial x_2} = 0 \quad \text{in} \quad \Omega_c \tag{16}$$

$$A^*DAu_s - \rho_s V^2\frac{\partial^2 u_s}{\partial x_1 \partial x_2} = 0 \quad \text{in} \quad \Omega_s \tag{17}$$

Let us transform the boundary conditions (5)–(12) into equivalent ones using (13)–(15). Strip clamped condition (7) takes the form

$$u_c = u_s = 0 \quad \text{on} \quad \Gamma_0 \tag{18}$$

Displacement continuity condition (6) becomes

$$u_s = u_c \quad \text{on} \quad \Gamma_c \cap \Gamma_s \tag{19}$$

In the contact zone the surface traction vector F in condition (7) is determined by

$$B^*D(x_2)Au_c = F \quad \text{on} \quad \Gamma_c \tag{20}$$

Contact conditions (10)–(11) take the following equivalent form

$$u_{c2} + g_r + w \leq 0, \quad F_2 \leq 0 \quad (u_{c2} + g_r + w)F_2 = 0 \quad \text{on } \Gamma_C \tag{21}$$

$$|F_1| \leq \mu|F_2|, \quad -0F_1\frac{\partial u_1}{\partial x_1} \leq 0, \quad (|F_1| - \mu|F_2|)\frac{\partial u_1}{\partial x_1} = 0 \quad \text{on } \Gamma_C \tag{22}$$

Under assumption (iv) the formed wear debris disappear immediately without interfering with con- tact conditions apart from changing the gap between the wheel and the rail. Moreover wear condition (12) takes the form

$$\frac{\partial w}{\partial x_1} = -kF_2 \tag{23}$$

There are also given initial conditions (8), (9) where the given functions are assumed to be zero.

In order to solve numerically quasistatic system (16)–(23) and ensure the existence [10, 15, 16] of solutions to this system we have to consider it as a problem with the prescribed friction. It means this problem has to be replaced by the regularized one. Let $\varepsilon > 0$ denotes a regularization parameter. We use in numerical algorithm the following formula relating tangential F_1 and normal F_2 tractions on the contact boundary Γ_C[15]

$$F_1 = F_1(\varepsilon, F_2, u_1) = -\mu|F_2|\arctan\left[\frac{1}{\varepsilon}(V\frac{\partial u_1}{\partial x_1})\right] \tag{24}$$

Remark the proposed quasistatic approach based on the assumptions (i)-(vi) consists in replacing the time derivatives terms in equations (2), (3) by the stationary

Table 1

Material properties of coating and steel layers

Properties	Ω_c	Ω_s
Elastic modulus E(GPa)	151	200
Poisson ratio ν	0.24	0.30
density ρ (kg/m^3)	3260	7800

terms depending on the wheel ve- locity and spatial derivatives of displacement. These terms still reflect the dynamics of the moving body rather than completely neglect it as in the classical quasistatic formulation [9]. Therefore the nonstationary original system (2)–(12) is transformed into the stationary one (16)–(24).

5. Numerical Algorithm

Let us briefly present the algorithms for solving contact problem (16)–(24). The finite element method is used to discretize this problem. The discretized contact problem is reformulated as a quadratic optimization problem in terms of tangent and normal contact tractions. In order to solve this auxiliary optimization problem one has to approximate inverse stiffness matrix of the discretized system. This matrix is calculated using collocation approach. Newton method [30] is employed to calculate regularized tangent traction. Linearization optimization method [24] is used to solve auxiliary quadratic optimization problem and to find tangent and normal tractions. Having calculated these tractions one can calculate by back substitution displacement and stresses in the whole strip as well as the wear. For algorithm details see references [6, 18].

6. Numerical Results and Discussion

A series of simulations are conducted to calculate and to investigate, governed by system (16)–(24), the influence of the elastic grading on the stress distribution in the contact area. Polygonal domain Ω occupied by the rail has a form

$$\Omega = \{(x_1, x_2) \in R^2 : x_1 \in (-2, 2), x_2 \in (0, 1)\} \tag{25}$$

Using mesh generator domain Ω has been divided into 320 triangles. The contact boundary is modelled by 39 segments. The coating layer where the wheel – rail contact may occur has been covered with fine mesh while the coarse mesh has been used to divide the steel layer. The size of surface layer is made large enough to ensure that stresses in wheel – rail contact zone are not affected by the boundary between the fine and coarse meshes. Multi – point constraints are applied on the boundary between layers and coarse and fine meshes. The ratio h_s/h_c= 4 is chosen. The layer properties are assembled in Table 1. Other data are as follows: the velocity

V= 10 m/s, radius of the wheel r_0= 0.46 m, the wear constant k= 0.5 · $10^{-6}MPa^{-1}$. The friction coefficient μ is equal to 0.45. The penetration of the wheel is taken as δ = 0.1 · 10^{-3}m. The regularization parameter ε in (24) is set to 0.001. Functions $\bar{u}_{0j}$ and $\bar{u}_{1j}$, for j= "c" or j= "s", in (8)–(9) are selected as equal to 0.

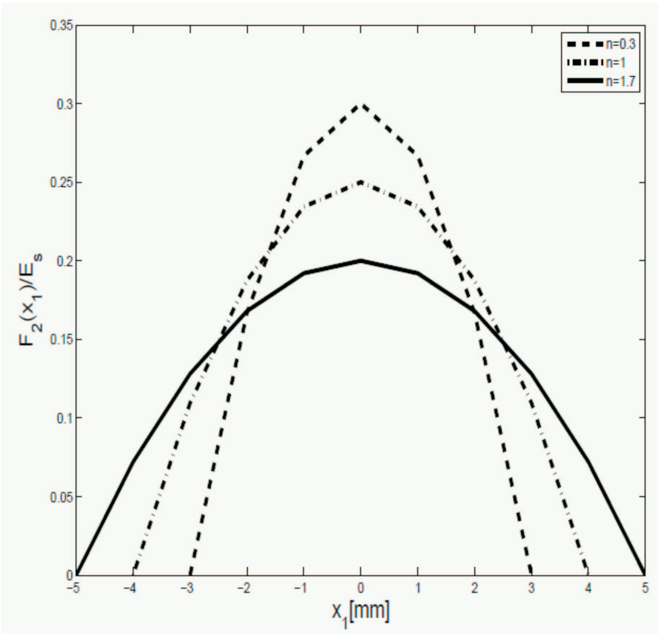

Fig. 3. Normal contact pressure distribution as a function of n

Fig. 3 compares the normal contact pressure distributions on the contact surface for different values of the nonhomogenity gradient n. Three values of n have been employed n= 0.3, 1, 1.7. In all cases the maximum contact pressure of F_2 occurs at the center of the contact zone. Tangent traction F_1 has different shapes in front and behind of the rolling wheel. As n increases and the coated surface becomes more flexible the maximal contact pressure value is slightly increasing at a cost of shrinking the contact zone length. The obtained maximal values are lower than in pure homogeneous case, i.e., n= 0.

Fig. 4 studies the effect of thickness ratio h_s/h_c on the normal contact pressure distribution. In this case nonhomogenity gradient n is equal to 1. Computations are performed for h_s/h_c = 4 and 1.5. It can be seen that a thicker surface coating slightly increases the maximum contact pressure and narrows the contact zone.

Stresses σ_{11} and σ_{22} as well as σ_{12} reach their maximum values at the points of the contact surface and then decay rapidly down to the bottom of the rail. The distribution of these stresses along the line x_1= 0, i.e. in x_2 direction is displayed on Fig. 5-7. The shear stress σ_{12} attains significantly lower values than the other stresses. The maximum value of all three stress components are effectively slightly

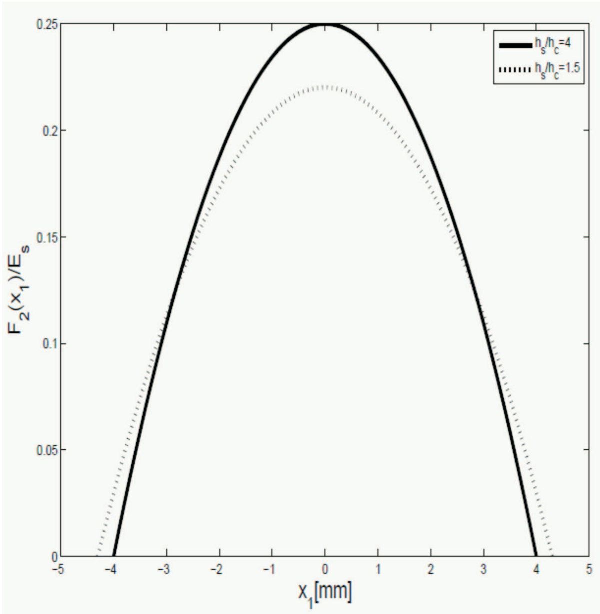

Fig. 4. The dependance of normal contact pressure on thickness ratio h_s/h_c

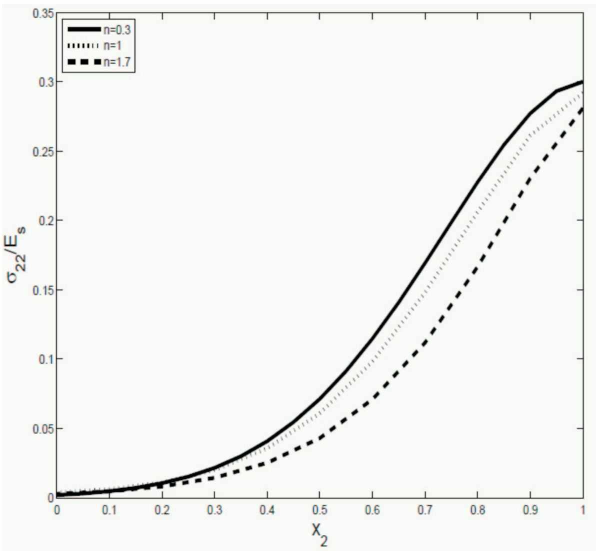

Fig. 5. Through – thickness stress σ_{22}at x_1= 0

lowered by increasing the nonhomogenity index n. The variation in the gradient index n has very moderate effect on stress distributions.

Fig. 8-10 display the longitudinal distribution of stresses σ_{22}, σ_{11} and σ_{11} along

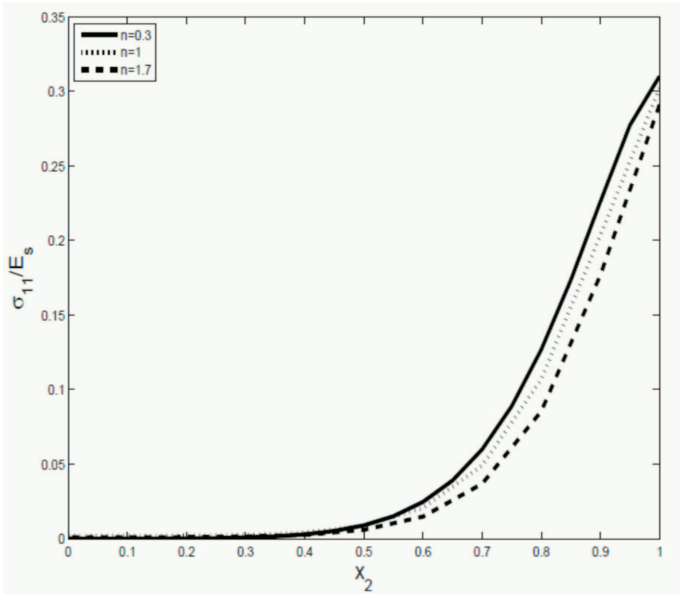

Fig. 6. Through – thickness stress σ_{11} at x_1= 0

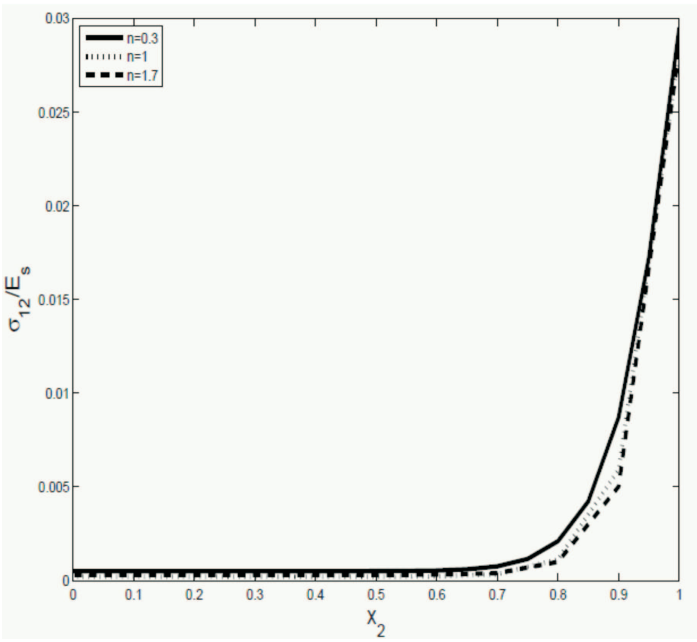

Fig. 7. Through – thickness stress σ_{12} at x_1= 0

the contact interface x_2= 1. Results show that the variation in the gradient index *n*has very moderate effect on stress distributions.

All computations are performed for the constant friction coefficient. It is shown

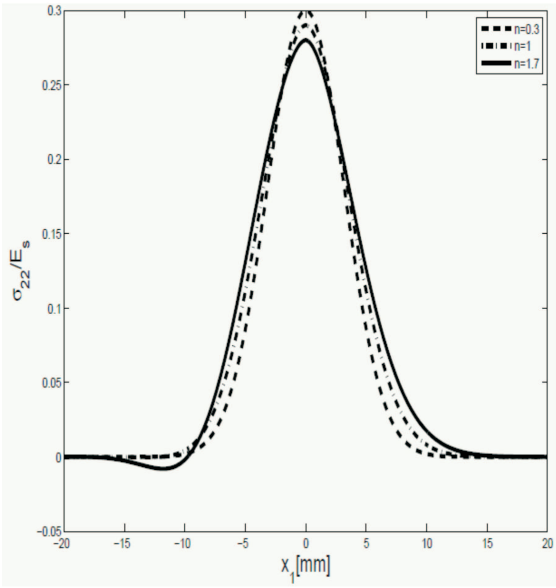

Fig. 8. Longitudinal stress σ_{22} on x_2= 1

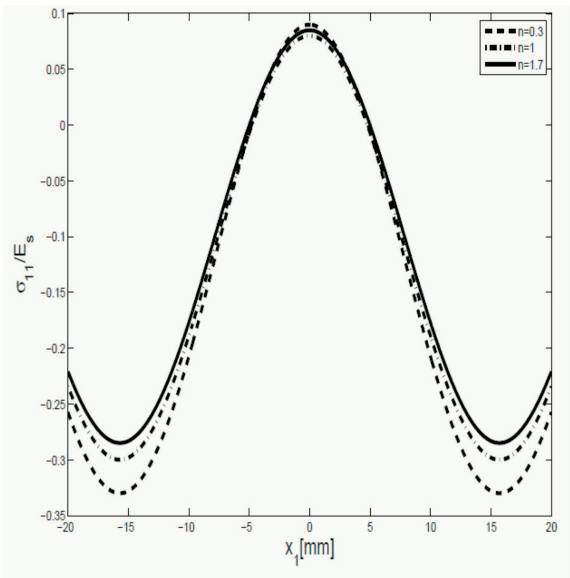

Fig. 9. Longitudinal stress σ_{11} on x_2= 1

in [3, 4, 5] that as the temperature or sliding velocity increase the friction coefficient decrease. Therefore one can expect that in a case of temperature dependent friction coefficient and nonhomogeneous material the maximal value of contact pressure and temperature are lower than in a case of constant friction coefficient.

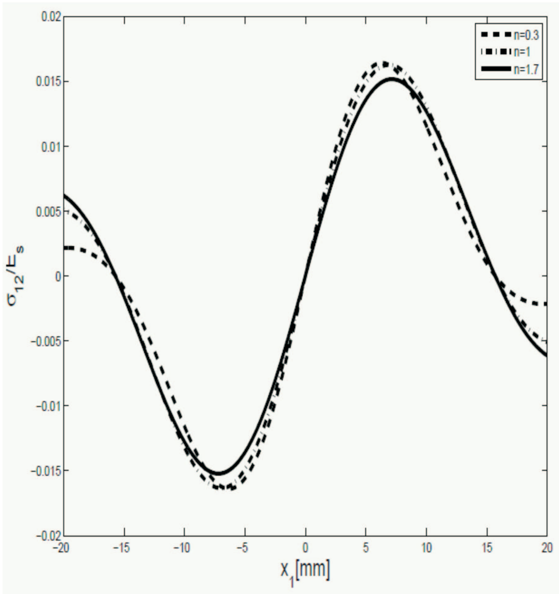

Fig. 10. Longitudinal stress σ_{12} on x_2= 1

7. Conclusions

The elastic rolling contact problem where the properties of the elastic upper coating layer of the rail surface are dependent on its depth is solved numerically using the quasistatic approach. The material properties of the graded layer are assumed to be governed by the power equation. Such selection of the model ensures [1, 8, 9, 23] the existence of solutions to the rolling contact problem.

The obtained numerical results indicate that the elastic graded layer reduce the values of the nor- mal contact stress in the contact zone. Since nonhomogenity index n is assumed to change in a narrow interval the dependence of the obtained stress on the nonhomogenity index n is rather mild. In order to obtain strong dependence one has to take grater values of n. Remark also, using the quasistatic approach we can observe dynamic phenomena of the rolling wheel. The proposed algorithm converges very quickely. Its speed of convergence depends on the choice of the regularization parameter ε value. For ε very small we obtain much more accurate results than for big values of ε at a cost of increase in computational time.

Since we confine to the elastic contact model with the elastic coating layer the computations indicate that the changes in the highest contact pressure or temperature value and their distribution are moderate with the comparison to the homogeneous material. Following [1] one can consider plastic coating layers on the rail surface.

The aim of the application of the coating material is to reduce the contact pressure and the rolling contact fatigue. The analysis of two – material rails performed in

[29] indicates that even for coated rails there are cases where the shakedown limit is exceeded. It implies that optimisation of the rail profile or material properties with combination of coating approach is required to reduce the contact pressure or generated temperatures. For the optimisation results concerning contact systems see [19].

Acknowledgement

The authors express their gratitude for the financial support by the Polish Ministry of Sciences and Highest Education within the framework of the project No: N N509 0823 33.

References

1. Choi I.S., Dao M., Suresh S.: Mechanics of indentation of plastically graded materials I: Analysis, Journal of Mechanics and Physics of Solids, 2008, Vol. 56, Issue 1, pp. 157-171.
2. Chudzikiewicz A., Myśliński A.: On wheel – rail contact problem with material properties varing with depth, submitted to Proceedings of Applied Mathematics and Mechanics.
3. Chudzikiewicz A., Myśliński A.: Rolling contact problem with the generalized Coulomb friction, Proceedings of Applied Mathematics and Mechanics, 2006, Volume 6, Issue1, pp. 299–300.
4. Chudzikiewicz A., Myśliński A.: Thermoelastic wheel – rail contact problem with temperature dependent friction coefficient, CD-ROM Proceedings of III European Conference on Com- putational Mechanics, Solids, Structures and Coupled Problems in Engineering, C.A. Mota Soares et.al. (eds.), Lisbon, Portugal, 5-9 June 2006, CIMNE, 2006.
5. Chudzikiewicz A., Myśliński A., Nagórski Z., Piotrowski J.: Comparison of numerical methods for solution of thermoelastic wheel – rail contact problems with friction, CD-ROM Proceedings of XVI International Conference on Computer Methods in Mechanics, June 21-24, 2005, Częstochowa, Poland, ISBN: 83-921605-7-6, 2005.
6. Chudzikiewicz A., Żochowski A., Myśliński A.: Quasistatic versus Kalker approach for solving rolling contact problems, The Archive of Transport, 1992, Vol. 4, pp. 103-120.
7. Ertz M., Knothe K.: A comparison of analytical and numerical methods for the calculation of temperatures in wheel – rail contact, Wear, 2002, vol. 253, pp. 498–508.
8. Giannakopoulos A.E., Pallot P.: Two dimensional contact analysis of elastic graded materials, Journal of Mechanics and Physics of Solids, 2000, Vol. 48, Issue 8, pp. 1597-1631.
9. Han W., Sofonea M.: Quasistatic contact problems in viscoelasticity and viscoplasticity, AMS and IP, 2002, pp. 104-110.
10. Hiensch M., Larsson P.O., Nilsson O., Levy D., Kapoor A., Franklin F., Nielsen J., Ringsberg J.W., Josefson B.L.: Two-material rail development: field test results regarding rolling contact fatigue and squeal noise behaviour, Wear, 2005, Vol. 258, No 7-8, pp. 964-972.
11. Hansaka M., Mamada S., Sato K.: Development of rail noise isolating material, Quarterly Report of RTRI, 2007, Vol. 48, No. 4, pp. 215-220.
12. Jang Y. H., Ahn S.: Frictionally-excited thermoelastic instability on functionally graded material, Wear, 2007, Vol. 262, pp. 1102-1112.
13. Lee D., Barber J.R., Thouless M.D.: Indentation of an elastic half space with material properties varing with depths, International Journal of Engineering Science, 2008, in press.
14. Kalker J.J.: Three dimensional elastic bodies in rolling contact. Kluwer Academic Publishers, 1990.

15. Ke L.L., Wang Y.S.: Fretting contact with finite friction of a functionally graded coating with arbitraly varing elastic modulus. Part I: normal loading, Part II tangential loading, The Journal of Strain Analysis for Engineering Design, 2007, Vol. 42, No 5, pp. 293-304 (Part I), pp. 305-313 (Part II).
16. Mahmoud F.F., Al Shafei A.G.: A quasistatic analysis for thermoviscoelastic contact problems, The Journal of Strain Analysis for Engineering Design, 2008, Vol. 43, No 7, pp. 655-672.
17. Meng H.C., Ludema K.C.: Wear models and predictive equations: their form and content, Wear, 1995, pp. 443-457.
18. Myśliński A., Żochowski A.: A numerical analysis of rolling contact problems using quasistatic variational formulation, Computers & Structures, 1991, Vol. 40, pp. 1261-1266.
19. Myśliński A.: Level set method for optimization of contact problems, Engineering Analysis with Boundary Elements, 2008, Vol. 32, pp. 986-994.
20. Oden J.T., Kikuchi N.: Contact problems in elasticity: A study of variational inequalities and finite element method, SIAM, Philadelphia, 1988.
21. Paczelt I., Mróz Z.: Optimal shape of contact interfaces due to wear in the steady relative motion, Int. J. Solids Struct., 2007, Vol. 44, pp. 895-925.
22. Paulk V., Zastrau B.: 2D rolling contact problem involving frictional heating, International Journal of Mechanical Sciences, 2002, Vol. 44, Issue 12, pp. 2573-2584.
23. Panagiotopoulos P.D.: Inequality Problems in Mechanics and Applications, Birkhaüser, 1985. [24] B.N. Pschenichnyj: The Linearization Method, Optimization, 1987, Vol. 18, pp. 179-196.
24. Serkan D.: Contact Mechanics of Graded Materials: Analysis Using Singular Integrated Equations, in Multiscale and Functionally Graded Materials 2006, AIP Conference Proceedings, 2008, Vol. 973, pp. 820-825.
25. Suresh S.: Graded materials for resistance to contact deformation and damage, Science, 2001, Vol. 292, pp. 2447-2451.
26. Sextro W.: Dynamical Contact Problems with Friction, Springer, Berlin, 2007, pp. 160-163. [28] M.
27. Shillor M., Sofonea J., Telega J.: Models and Analysis of Quasistatic Contact: Variational Methods, Springer, Berlin, 2004, pp. 174-175.
28. Ringsberg J.W., Franklin F.J., Josefson B.L., Kapoor A., Nielsen J.C.O.: Fatigue evaluation of surface coated railway rails using shakedown theory, finite element calculations, and lab and field trials, International Journal of Fatigue, 2005, Vol. 27, No 6, pp. 680-694.
29. Wriggers P.: Computational Contact Mechanics, Second Edition, Springer, Berlin, 2006, pp. 368-376.
30. Yang J., Ke L.L.: Two – dimensional contact problem for a coating – graded layer – substarate structure under a rigid cylindrical punch, International Journal of Mechanical Sciences, 2008, Vol. 50, pp. 985-994.