ARCHIVES OF TRANSPORT
Volume 48, Issue 4, 2018

ISSN (print): *0866-9546*
e-ISSN (online): *2300-8830*
DOI: *10.5604/01.3001.0012.8368*

DESIGN OF BRAKE FORCE DISTRIBUTION MODEL FOR FRONT-AND-REAR-MOTOR-DRIVE ELECTRIC VEHICLE BASED ON RADIAL BASIS FUNCTION

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Abstract:

To achieve high-efficiency and stable brake of a front-and-rear-motor-drive electric vehicle (FRMDEV) with parallel cooperative braking system, a multi-objective optimal model for brake force distribution is created based on radial basis function (RBF). First of all, the key factors, which are the coefficient of brake force distribution between the front and rear shafts, the coefficient of brake force distribution at wheels, the coefficient of regenerative brake force distribution between front and rear axles, that influence the brake stability and energy recovery of the FRMDEV are analyzed, the fitness functions of brake stability and energy recovery are established. Secondly, the maximum allowed regenerative brake torque influenced by the state of charge of battery is confirmed, the correction model of the optimal distribution coefficient of regenerative brake force is created according to motor temperatures. Thirdly, based on HALTON sequence method, a two-factor database, vehicle velocity and brake strength, that characterizes vehicle operation is designed. Then an off-line response database of the optimal brake force distribution is established with the use of particle swarm optimization (PSO). Furthermore, based on hybrid RBF, the function model of the factor database and the response database is established, and the accuracy of the model is analyzed. Specially, the correlation coefficient is 0.995 and the predictive error variance is within the range between 0.000155 and 0.00018. The both indicate that the multi-objective distribution model has high accuracy. Finally, a hardware-in-loop test platform is designed to verify the multi-objective optimal brake force distribution model. Test results show that the real-time performance of the model can meet the demand of engineering application. Meanwhile, it can achieve both the brake stability and energy recovery. In comparison with the original brake force distribution model based on the rule algorithm, the optimized one proposed in this paper is able to improve the energy, recovered into battery, by 14.75%.

Key words:
front-and-rear-motor-drive electric vehicle, multi-objective optimization of brake force distribution, radial basis function, particle swarm optimization

To cite this article:

Sun, B., Zhang, T., Gao, S., Ge, W., Li, B., 2018. Design of brake force distribution model for front-and-rear-motor-drive electric vehicle based on radial basis function. Archives of Transport, 48(4), 87-98. DOI: https://doi.org/10.5604/01.3001.0012.8368

1. Introduction

Facing the pressure of energy crisis and environmental pollution, development of efficient and safe electric vehicle has become the highlight of automotive industry (China Automotive Technology Research Center, et al, 2015, Jafernik H. and Fellner R, 2015, Merkisz-Guranowska A. and Pielecha J., 2014). Different from the traditional vehicle, electric vehicle can recover all or part of the energy during brake operation owing to reasonable regenerative brake force control, which can significantly improve the energy utilization efficiency (Yamato M., 2005, Zhang J., et al. 2014.). Currently, researches on the optimization of brake force distribution mainly focus on the single-motor-drive electric vehicle, and brake force distribution strategy based on rule is the most commonly method (Xu W., et al, 2013, Ko J., et al., 2012, Zhang J., et al., 2009.).

Different from single-motor-drive electric vehicle (Sun B., et al., 2018, Sun B., et al., 2016.), FRMDEV can provide engineers more freedom in design (Mutoh N., et al., 2011.). Daxu Sun et al. put forward a brake force distribution model based on the ideal I curve to achieve brake stability of FRMDEV (Sun D., et al., 2016.). Chia-Cheng Wueng et al. proposed two strategies that can achieve the optimal brake stability and the maximum braking energy recovery of FRMDEV, respectively (Wueng C., et al., 2010.). Nobuyoshi Mutoh et al. designed a brake force distribution strategy based on the optimal slip-ratio control, which improved the brake stability of FRMDEV on wet and sliding road (Mutoh N., et al., 2007, Mutoh N., 2012.). In general, current studies about the brake force distribution of FRMDEV mainly focus on series hybrid brake system, brake force distribution for parallel hybrid brake system is still unclear. Moreover, the single-target brake control strategies introduced above are unable to balance both brake stability and energy recovery of FRMDEV.

Therefore, a multi-objective brake force distribution strategy for the parallel hybrid brake system of FRMDEV is studied in this paper. Firstly, the key factors that influence brake stability, energy recovery and brake pedal feel are analyzed, and then a multi-objective fitness function is proposed in section II. Secondly, a random database characterizing vehicle operation is designed and an off-line response database of the optimal brake force distribution is established in section III. Finally, in section IV, with the use of hardware-in-loop test platform, the proposed brake force distribution model is verified.

2. Fitness function modelling

2.1. Fitness Function of Brake stability

Brake force distribution between front and rear axles is the key factor that influences brake stability, which can be achieved when brake force between the two axles is distributed referring to the ideal I curve. Within the scope of the ECE regulations (Economic Commission for Europe, 2008.), the brake stability will reduce if the actual brake force distribution deviates from the ideal I curve. Consequently, in order to quantify the influence of brake force distribution between front and rear axles on brake stability, fitness functions as shown in formula 1 and 2 are modeled.

First of all, when brake force distribution dissatisfies the ECE regulations, the fitness function is designed as follow:

$$f(\lambda) = K_{\mathrm{pu}} \tag{1}$$

Where λ is the coefficient of brake force distribution between the front and rear shafts, which is defined as the ratio of the brake force at front axle to the total one. $f(\lambda)$ is the fitness function defined for evaluating brake stability, and better brake stability can be achieved if λ is closer to the ideal I curve. K_{pu} is the penalty coefficient to ensure brake stability.

Furthermore, when brake force distribution satisfies the ECE regulations, the fitness function is defined as:

$$f(\lambda) = \begin{cases} \dfrac{1}{\lambda}\dfrac{L_{\mathrm{r}} + zh_{\mathrm{g}}}{L_{\mathrm{r}} + L_{\mathrm{f}}} & \text{if}: \lambda \geq \dfrac{L_{\mathrm{r}} + zh_{\mathrm{g}}}{L_{\mathrm{r}} + L_{\mathrm{f}}} \\ \lambda \dfrac{L_{\mathrm{r}} + L_{\mathrm{f}}}{L_{\mathrm{r}} + zh_{\mathrm{g}}} & \text{if}: \lambda < \dfrac{L_{\mathrm{r}} + zh_{\mathrm{g}}}{L_{\mathrm{r}} + L_{\mathrm{f}}} \end{cases} \tag{2}$$

Where z is the braking intensity; h_{g} is the height of vehicle centroid; L_{f} is the distance from vehicle centroid to front axle; L_{r} is the distance from vehicle centroid to rear axle.

2.2. Fitness Function of Braking Energy Recovery

The characteristic of energy flow in FRMDEV is shown in figure 1. As the wind loss and rolling loss are relatively very small, the influence of them on energy recovery is negligible. As for the mechanical loss in transmission system, it mainly influenced by vehicle speed, therefore, the influence of it on energy recovery is not discussed (Sun B., et al., 2017.). The brake force distribution between the mechanical and regenerative systems at wheels has direct influence on energy recovery, which can be evaluated based on $f(\alpha)$ in formula 3. Moreover, energy loss during energy flow from wheels to battery influences energy recovery into battery, which can be quantified based on $f(\beta)$ in formula 3.

$$f\left(E_{\mathrm{re}}\right)=f\left(\alpha\right)+f\left(\beta\right) \tag{3}$$

Where E_{re} means the energy recovered into battery. α is the coefficient of brake force distribution at wheels, which is defined as the ratio of regenerative brake force to the total one. β is the distribution coefficient of regenerative brake force between front and rear axles, which is defined as the ratio of regenerative brake force distributed to front axle to the total one. $f(E_{\mathrm{re}})$ is the fitness function of energy recovery, which consists of $f(\alpha)$ and $f(\beta)$.

According to vehicle dynamics, $f(\alpha)$ can be derived as:

$$\begin{cases} f\left(\alpha\right)=\dfrac{\int_{t_0}^{t_1} F_{\mathrm{eb}}Vdt}{\int_{t_0}^{t_1}\left(F_{\mathrm{eb}}+F_{\mathrm{hb}}+F_{\mathrm{f}}+F_{\mathrm{w}}+sgn\left(\beta\right)\overline{F}_{\mathrm{mtr}}\right)Vdt} \\ 0\le F_{\mathrm{eb}}\le F_{\mathrm{eb\_max}} \end{cases} \tag{4}$$

Where V is vehicle speed; F_{eb} is the total regenerative brake force at wheels; $F_{\mathrm{eb\_max}}$ is the maximum allowed regenerative brake force; F_{hb} is the total mechanical brake force at wheels. F_{f} and F_{w} mean the rolling resistance and the wind resistance, the influence of them on energy recovery is not discussed as the braking process is very short. $\overline{F}_{\mathrm{mtr}}$ is the equivalent mechanical brake force of the non-work motor, which is the function of β as follow:

$$\begin{cases} sgn\left(\beta\right)=0 & \text{if}: 0<\beta<1 \\ sgn\left(\beta\right)=1 & \text{if}: \beta=1 \end{cases} \tag{5}$$

Where $sgn(\beta)$ is a symbolic function, as the single-rear-axle regenerative brake (β=0) has no advantage on brake stability and energy recovery, this situation is not discussed in this paper.

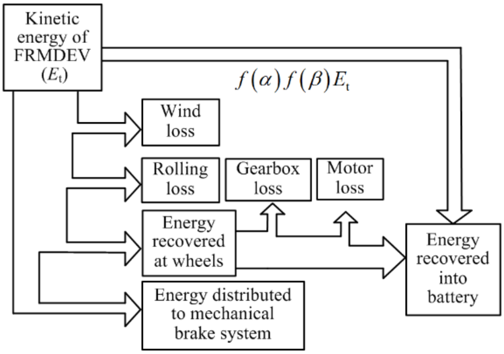


Fig. 1. Characteristic of energy flow during the brake process of FRMDEV

Furthermore, different from the series hybrid brake system, as for the parallel one, since the mechanical brake force is distributed according to a given γ curve, the engagement of regenerative brake under any brake pedal degree means higher brake strength than the required one, which actually impairs the feeling of brake pedal. Consequently, for FRMDEV with parallel hybrid brake system, the optimization of $f(\alpha)$ is limited by the feeling of brake pedal. Given the above analysis, $F_{\mathrm{eb\_max}}$ can be deduced as:

$$\begin{cases} F_{\mathrm{eb\_max}}=\dfrac{\sqrt{\Delta}-\left(2mgz+L_r\right)}{2h_{\mathrm{g}}} \\ \Delta=\left(2mgzh_{\mathrm{g}}+L_{\mathrm{r}}\right)^2-4mgzh_g\left(L_{\mathrm{r}}+mgzh_{\mathrm{g}}-\gamma L\right) \end{cases} \tag{6}$$

Where γ is the given distribution coefficient of mechanical brake force, which is defined as the ratio of mechanical brake force at front wheels to the total one. m is vehicle mass; g is gravity acceleration; L is the distance between front and rear axles.

For a given brake condition, β actually influences the transfer efficiency of regenerative energy from wheels to battery if α is constant. In order to quantify the effect of β on energy transfer efficiency, $f(\beta)$ is deduced as follow:

$$f(\beta)= \\ =\frac{g_{fm}\left(N_{fm},\beta T_{eb},t_{m\_fm}\right)+g_{rm}\left(N_{rm},(1-\beta)T_{eb},t_{m\_rm}\right)}{\max\left[\eta_s(\beta)\right]} \quad (7)$$

Where $g_{fm}(N_{fm}, \beta T_{eb}, t_{m\_fm})$ and $g_{rm}(N_{rm}, (1-\beta)T_{eb}, t_{m\_rm})$ are the efficiency models for front and rear motors respectively, which are the function of motor speed, torque and temperature. N_{fm} and N_{rm} mean speeds of front motor and rear motor; T_{eb} is the total regenerative brake torque; t_{m_fm} and t_{m_rm} are the temperatures of front motor and rear motor; $\eta_s(\beta)$ is the system efficiency model of the dual motors, which can be tested with the use of the test platform shown in figure 2 (Sun B., et al., 2017.).

Fig. 2. Test platform for the dual-motor system
1-Control unit of motor 1, 2-Motor 1, 3-Host computer, 4-Electric dynamometer, 5-Industrial personal computer, 6-Gearbox, 7-Motor 2, 8- Control unit of motor 2.

Based on the test platform shown in figure 2, a single motor efficiency model is established as follow:

$$g_m(N,T,t_m)=1-\frac{\vec{A}\vec{U}}{9.55|T|N} \quad (8)$$

Where $\vec{A}$ is the power factor vector; $\vec{U}$ is the power vector based on Gauss RBF. Referring to the measured data, they are deduced as:

$$\vec{A}=1000\begin{bmatrix}1.800\\3.341\\21.415\\\vdots\\-3.632\\-0.885\end{bmatrix}^T \qquad \vec{U}=\begin{bmatrix}\phi_1\\\phi_2\\\phi_3\\\vdots\\\phi_{24}\\\phi_{25}\end{bmatrix} \quad (9)$$

Where ϕ_i is the Gauss basis function, which can be expressed as:

$$\phi_i=\exp\left(-\frac{\left\|\vec{x}-\vec{C_i}\right\|^2}{2b_i^2}\right) \quad (10)$$

Where b_i represents the base width of the i network node. $\vec{x}$ is the dimensionless input vector, which is defined as $[\ N\ , T\ \ , \tilde{t}\ ]^T$. $\vec{C_i}$ is the dimensionless central vector of the i network node, which is defined as $[\ N_i\ , T_i\ , t_i\ ]^T$.

The dimensionless motor speed, torque, and temperature are designed as:

$$\begin{cases} N_i=2\dfrac{N_i-N_{min}}{N_{max}-N_{min}}-1 \\ T_i=2\dfrac{T_i-T_{min}}{T_{max}-T_{min}}-1 \\ t_i=2\dfrac{t_{m\_i}-t_{m\_min}}{t_{m\_max}-t_{m\_min}}-1 \end{cases} \quad (11)$$

Where N_{min} means the minimum motor speed; N_{max} is the maximum motor speed; T_{max} is the maximum motor torque; T_{min} is the minimum motor torque; t_{m_max} means the maximum motor temperature; t_{m_min} is the minimum motor temperature.

3. Multi-objective distribution model of brake force

As shown in figure 3, method for modeling multi-objective strategy of brake force distribution is designed. First of all, based on HALTON sequence method, a input database characterizing FRMDEV operation is constructed, which is defined as the factor database. Then, for any given brake condition of the factor database, a output database consisting of multi-objective brake force distribution is optimized, with the use of off-line PSO, which is defined as the response database. Finally, based on hybrid RBF, the dominant function relationship between the factor database and the response database is established.

3.1. Database Design of Random Brake Condition

As shown in figure 4, in order to ensure the real-time performance and model accuracy of the brake force distribution, a hierarchical brake force distribution strategy is designed. First of all, as SOC mainly acts as the role to limit the charging torque and discharging torque of battery, it is designed into the upper level strategy to correct the maximum allowed regenerative brake torque. Secondly, the middle level strategy is a two-factor model based on the hybrid RBF. As motor temperatures influence the efficiency of the dual-motor system, they are designed into the lower control strategy to correct β output from the middle layer model.
According to the performance of battery, a correction model designed for limiting the maximum regenerative brake torque is derivated. When SOC is large, the maximum charging torque is limited to protect battery. When SOC is small, the battery can be charged with high power. When SOC is within the given range, the maximum charging torque increases with the decrease of SOC.

$$T_{\mathrm{cha\_max\_soc}} = \\ = \begin{cases} \dfrac{rk_{\mathrm{c}}P_{\mathrm{cha\_max}}}{f_{\mathrm{d}}V} & \mathrm{SOC} \le 0.3 \\ \dfrac{0.95-\mathrm{SOC}}{0.65}\dfrac{rk_{\mathrm{c}}P_{\mathrm{cha\_max}}}{f_{\mathrm{d}}V} & 0.3 < \mathrm{SOC} < 0.85 \\ 0.1\dfrac{rk_{\mathrm{c}}P_{\mathrm{cha\_max}}}{f_{\mathrm{d}}V} & \mathrm{SOC} \ge 0.85 \end{cases} \quad (12)$$

Where $T_{\mathrm{cha\_max\_soc}}$ is the maximum allowed charging torque, which is corrected on the basis of SOC. k_{c} is a conversion factor of unit; r is the wheel radius; $P_{\mathrm{cha\_max}}$ is the maximum allowed charging power; f_{d} is the ratio of the transmission system.
Moreover, the peak torque of motor ($T_{\mathrm{m\_max}}$) also influences the maximum allowed regenerative brake torque. Consequently, while taking both $T_{\mathrm{cha\_max\_soc}}$ and $T_{\mathrm{m\_max}}$ into consideration, the maximum allowed regenerative brake torque is derivated as:

$$T_{\mathrm{eb\_max\_cr}} = \\ = \min\left(\left|\frac{rF_{eb\_\max}}{\eta_t f_d}\right|, \left|T_{\mathrm{cha\_max\_cr}}\right|, \left|T_{\mathrm{fm\_max}} + T_{\mathrm{rm\_max}}\right|\right) \quad (13)$$

Where η_{t} is the mechanical efficiency of the transmission system; $T_{\mathrm{fm\_max}}$ is the peak torque of the front motor; $T_{\mathrm{rm\_max}}$ is the peak torque of the rear motor.

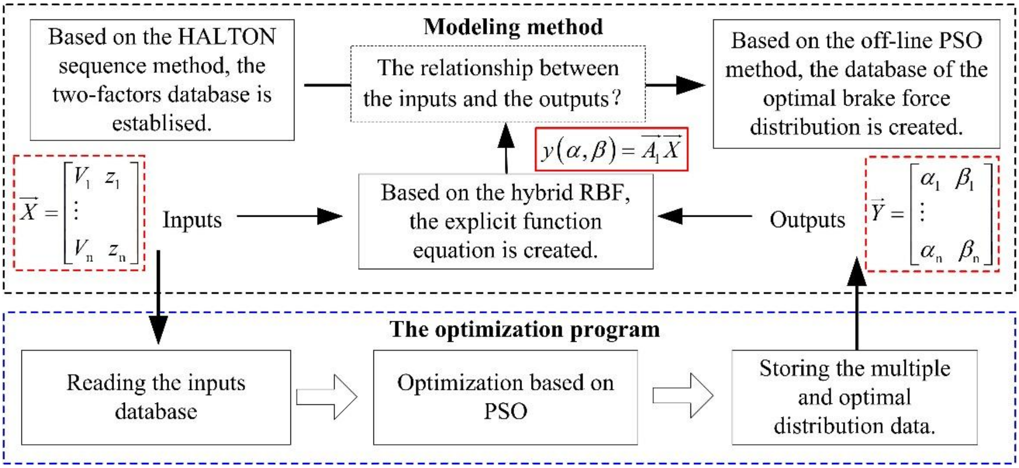


Fig. 3. Multi-objective optimal brake force distribution strategy

Referring to the loss model of the dual-motor system used in FRMDEV [16], the optimal distribution coefficient of regenerative brake force is corrected as follow:

$$\beta_{cr} = \frac{R_r}{R_r + R_f} \tag{14}$$

Where R_f is the resistance of front motor; R_r is the resistance of the rear one. As the motor resistance is the function of motor temperature, β_{cr} is derived according to the measured data.

$$\beta_{cr} = \frac{5.063 + 0.0215 t_{m\_rm}}{10.126 + 0.0215\left(t_{m\_rm} + t_{m\_fm}\right)} \tag{15}$$

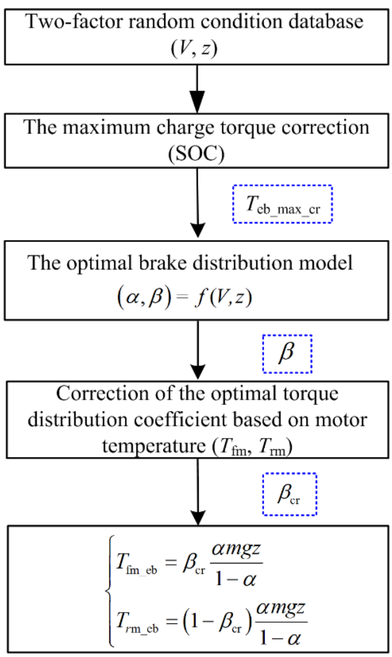


Fig. 4. Hierarchical distribution strategy for regenerative brake

Randomness, capacity and filling performance are the key factors required to characterize the brake operation of FRMDEV. Owing to HALTON sequence method, a two-factor database of brake condition is designed, which can meet the requirements discussed above.

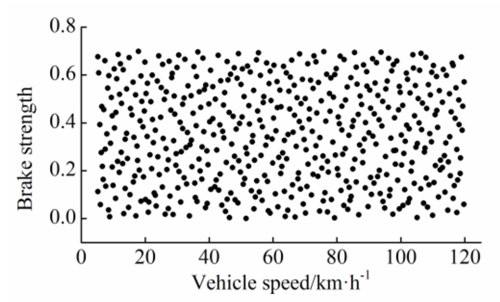


Fig. 5. Two-factor database of brake operation based on HALTON sequence method

$$\vec{X}_2 = \begin{bmatrix} V_1 & z_1 \\ \vdots & \\ V_i & z_i \\ \vdots & \\ V_{500} & z_{500} \end{bmatrix} = \begin{bmatrix} 62.501 & 0.467 \\ \vdots & \\ 74.685 & 0.337 \\ \vdots & \\ 6.600 & 0.218 \end{bmatrix} \tag{16}$$

As shown in figure 5 and formula 16, vehicle speed and brake strength are designed as the two factors. The capacity of the database is designed as 500 initially. As regenerative brake energy is very small under the operations of low brake speed ($V \leq 5$km·h^{-1}) and higher brake strength ($z > 0.7$), they are not discussed in this paper.

3.2. Off-line Optimization Based on PSO

In order to reduce the cost of optimization, as shown in figure 6, an off-line optimization program based on PSO is designed. Details of it are as follows .

First of all, according to the flying range of the particles (the optimization variables), a random particle swarm is formatted. Secondly, in each iteration step, its fitness value is calculated for any particle. Then, the new fitness value of the particle is compared with two factors, one is the optimum fitness value that it has experienced, the other one is the optimum fitness value experienced by the particle group. Furthermore, base on the comparison results, the speed and direction of the particle are updated. Finally, repeating the iteration step discussed above, the optimal particle can be obtained.

As shown in formula 17, λ is the function of α and β. Consequently, in order to reduce the size and dimension of the optimization population (optimization variables), only α and β are designed particles.

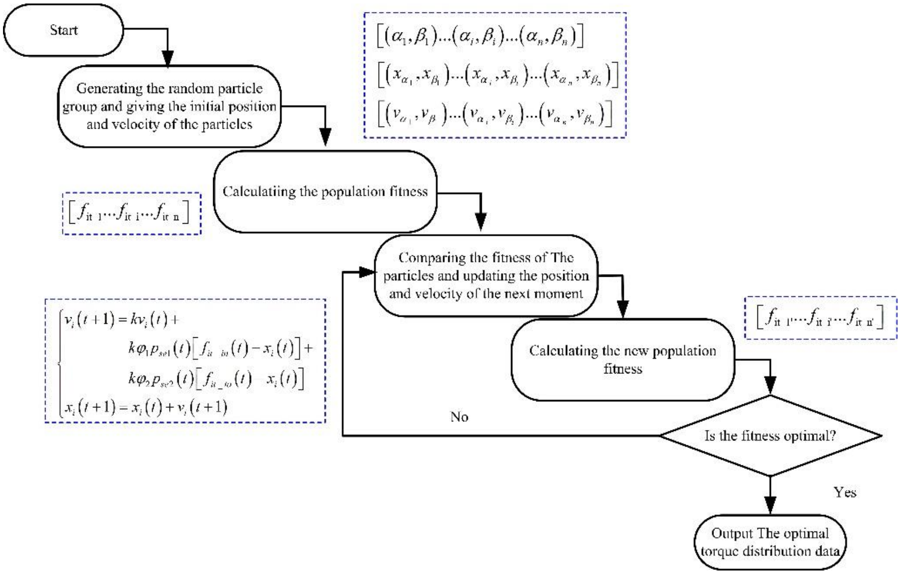


Fig. 6. Off-line optimization program for brake force distribution based on PSO

$$\lambda = \frac{\alpha\beta + \gamma}{1+\alpha} \tag{17}$$

The flight ranges of the designed particles are related to their effective values. According to the fitness function models designed above, the ranges of particle α and β are deduced as:

$$\begin{cases} 0 \le \alpha \le \dfrac{f_d \eta_t T_{eb\_max\_cr}}{mgzr + f_d \eta_t T_{eb\_max\_cr}} \\ 0 < \beta \le 1 \end{cases} \tag{18}$$

Based on the fitness function models derivated above, the fitness value of any particle (α_i, β_i) can be calculated at any given moment. As shown in formula 19, taking the t moment for example. In order to ensure the particles converge to the minimum fitness value, which is equivalent to the positive maximum fitness value, the fitness function is designed as follow:

$$f_{it\_i}(t) = -\left[f\left(\frac{\alpha_i \beta_i + \gamma}{1+\alpha_i}\right) + f(\alpha_i) + f(\beta_i) \right] \tag{19}$$

As for the fitness value calculated by formula 19 (at the t moment), it needs to be compared with the best positions that the particle and the group have experienced. According to the comparison results, the velocity and direction of the particle at the next moment (t+1) are updated as follow:

$$\begin{cases} v_i(t+1) = kv_i(t) + k\varphi_1 p_{se1}(t)\left[f_{it\_in}(t) - x_i(t)\right] \\ \qquad + k\varphi_2 p_{se2}(t)\left[f_{it\_to}(t) - x_i(t)\right] \\ x_i(t+1) = x_i(t) + v_i(t+1) \end{cases} \tag{20}$$

Where $v(t)$ represents the particle velocity at the t moment; $x(t)$ means the particle position at the t moment; φ_1 and φ_2 are the learning factors used to adjust the particle to the best position; p_{se1} and p_{se2} are the independent pseudorandoms to ensure the randomness of particle search; k is the convergence factor; $f_{it\_in}(t)$ and $f_{it\_to}(t)$ are the optimal fitness values of the particle and the population at t moment, respectively.

3.3. Database of Brake Force Distribution

As shown in figure 7, taking a random brake operation for example (V=74.685km·h^{-1}, z=0.337), the fitness value converges to -1.997, which means the distribution strategy of brake force is able to balance both brake stability and energy recovery, when α is 0.0819 and β is 0.498.

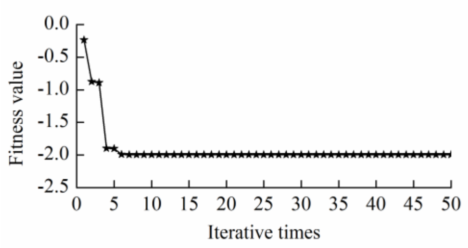
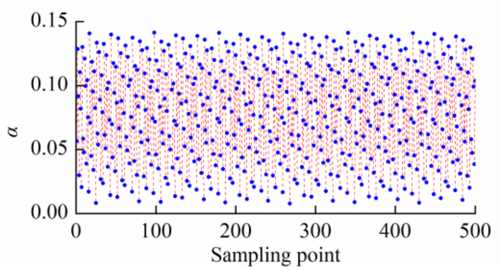
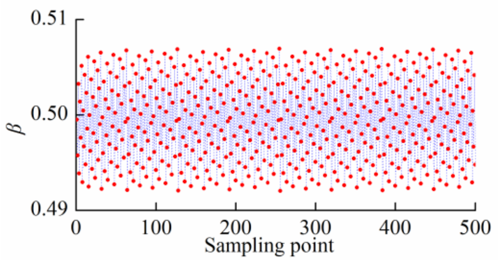


Fig.7. Fitness value under the given operation

As shown in figure 8 and formula 21, owing to the off-line optimization program designed above, the optimal database of brake force distribution is confirmed. The optimized results show that the values of β are concentrated within the range of 0.5±0.008 under different braking conditions. Moreover, for any given braking operation, within the optimized range of β, the differences of energy transfer loss are little for different β. Consequently, the value of β for any braking operation is designed as 0.5, which is helpful for simplifying the control model of brake force distribution.



(a) The optimal α under the given operation database



(b) The optimal β under the given operation database

Fig. 8. The optimized results of α and β

$$\vec{Y}=\begin{bmatrix}\alpha_1 \ \beta_1\\ \vdots\\ \alpha_i \ \beta_i\\ \vdots\\ \alpha_{500} \ \beta_{500}\end{bmatrix}=\begin{bmatrix}0.0566 & 0.499\\ \vdots & \\ 0.0819 & 0.496\\ \vdots & \\ 0.104 & 0.498\end{bmatrix} \tag{21}$$

3.4. Brake Force Distribution Model Based on RBF

In order to establish the explicit relationship between the factor database and the response database, the least square method, the stepwise regression analysis and the hybrid RBF are used. The optimal brake force distribution model strategy is derived as follow:

$$\begin{cases} y(\alpha,\beta)=\vec{A}_1\vec{U}_1\\ \beta=0.5\\ \vec{A}_1=\begin{bmatrix}0.181\\0.125\\0.129\\ \vdots\\ 0.112\\0.089\end{bmatrix}^{\mathrm{T}} \quad \vec{U}_1=\begin{bmatrix}\phi_{1\_\alpha}\\ \phi_{2\_\alpha}\\ \phi_{3\_\alpha}\\ \vdots\\ \phi_{24\_\alpha}\\ \phi_{25\_\alpha}\end{bmatrix}\end{cases} \tag{22}$$

Where ϕ_{i_α} is the Gauss basis function, which can be expressed as:

$$\phi_{i\_\alpha}=\exp\left(-\frac{\left\|\vec{x}_{\_\alpha}-\vec{C}_{i\_\alpha}\right\|^2}{2b_{i\_\alpha}^{\ 2}}\right) \tag{23}$$

Where $\vec{x}_{\_\alpha}$ represents the dimensionless input vector, which is defined as $[V, \tilde{z}]^{\mathrm{T}}$. $\vec{C}_{i\_\alpha}$ is the dimensionless central vector of the i network node, which is defined as $[V_{i\_\alpha}, z_{i\_\alpha}]^{\mathrm{T}}$.

The dimensionless vehicle speed and brake strength are designed as:

$$\begin{cases} V_{i\_\alpha}=2\dfrac{V_i-V_{min}}{V_{max}-V_{min}}-1\\ z_{i\_\alpha}=2\dfrac{z_i-z_{min}}{z_{max}-z_{min}}-1\end{cases} \tag{24}$$

Where V_{min} is the minimum vehicle speed, and V_{max} meams the maximum one. z_{max} is the maximum brake strength, and z_{min} is the minimum one.
In order to verify the model accuracy of the brake force distribution, the correlation coefficient and the predictive error of the model are calculated based on the regression analysis. Results show that the correlation coefficient of the model is 0.995. Moreover, as shown in figure 9, the predictive error variance of the model is controlled within the range between 0.000155 and 0.00018. Both the correlation coefficient and the predictive error indicate that the multi-objective distribution model for brake force control has high accuracy.

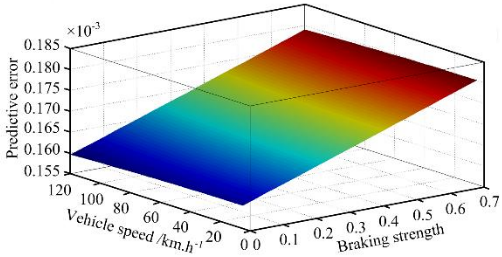


Fig.9. Model accuracy of brake torque distribution based on RBF

4. Verification

4.1. Hardware-in-Loop Test Platform

In order to verify the real-time performance of the predictive brake force distribution model based on hybrid RBF, a hardware-in-loop test platform, as shown in figure 10, is designed.

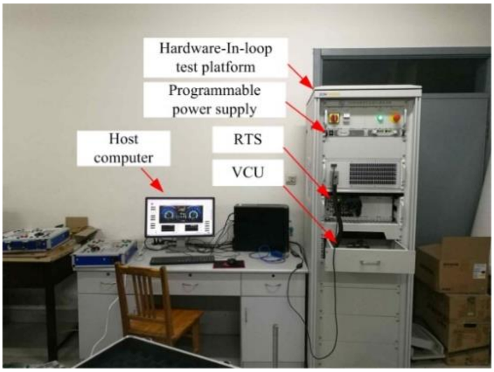


Fig.10. Hardware-in-loop test platform

Details about the verification are presented in figure 11. Specially, first of all, the control strategy is developed with the use of the Simulink software. Secondly, the signal interfaces, the control strategy, the vehicle model and so on are designed based on the Simulink/MotoHawk. Then, with the use of MotoTune, the control strategy code is compiled and downloaded to the vehicle control unit (VCU). Furthermore, based on the NI Simulator (Real-Time Simulator, RTS), the dynamic model of the FRMDEV is run in real time, and then the status parameters of the FRMDEV and the key components are fed back to the VCU and the host computer. Some remarks about the FRMDEV were designed in our previous research (Sun B., et al., 2016). Finally, owing to the hardware-in-loop test platform the control strategy is verified, calibrated and improved.

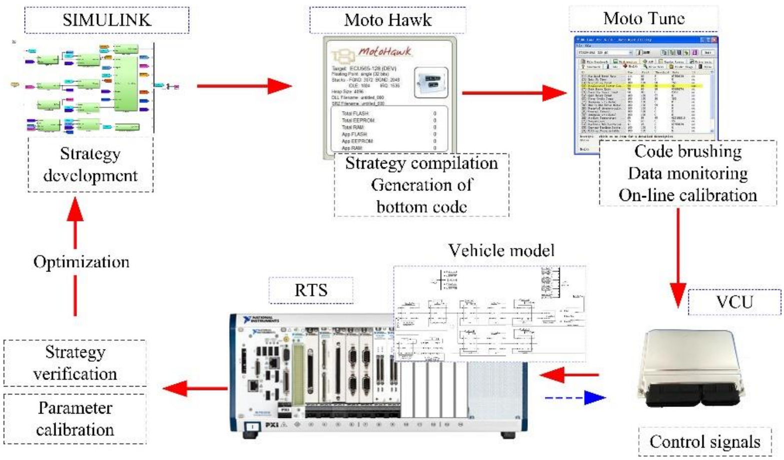
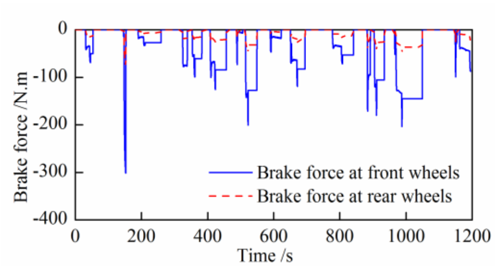


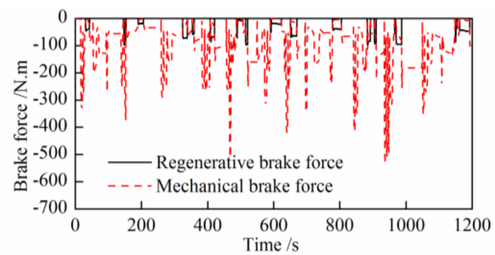
Fig.11. Validation process of the control strategy

4.2. Test Results

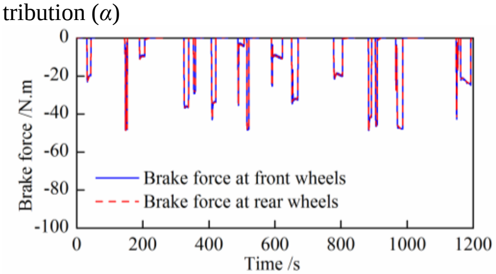
For the FRMDEV parallel hybrid brake system, the multi-objective optimal brake force distribution model is tested under the typical urban conditions. In the tested model, the initial SOC of battery is set to 0.4. As the tested results shown in figure 12, the predictive brake force distribution model based on the hybrid RBF is verified with perfect real-time performance and is able to meet the requirements of engineering application.



(a) Brake torque distribution (λ) between front and rear shafts



(b) Regenerative and mechanical brake torque distribution (α)



(c) Regenerative brake torque distribution (β)

Fig.12. Test results of the torque distribution model based on RBF

Furthermore, according to the theoretical analysis of the FRMDEV discussed above, the developed brake force distribution model can achieve the optimal total brake force distribution between the front and the rear shafts, the optimal brake force distribution between the mechanical and regenerative systems, and the regenerative brake force distribution between the front and the rear shafts. Consequently, the multi-objective brake force distribution model developed in this paper can be used to control the brake force of the FRMDEV.

As the energy recovery shown in figure 13, the energy recovered by the two motors is small than that at the wheels due to the influence of mechanical and electrical losses caused by transimission and dual-motor system. Moreover, about 31.76% of the energy recovery at wheels is wasted by the battery system. In comparison with the original brake force distribution model based on the rule algorithm, the optimized one proposed in this paper is able to improve the energy, recovered into battery, by 14.75%.

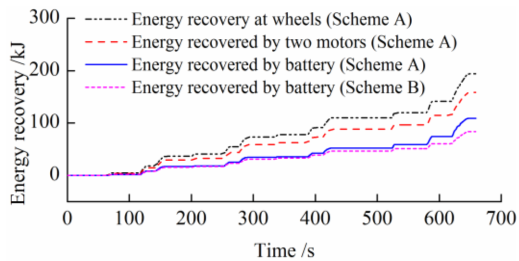


Fig. 13. Energy recovery

Where scheme A is the optimized brake force distribution control strategy proposed in this paper, scheme B is the traditional control strategy based on rule algorithm.

5. Conclusions

In this paper, to achieve high-efficiency and stable brake of the front-and-rear-motor-drive electric vehicle (FRMDEV), a multi-objective optimization model for brake force distribution is s proposed and verified. Conclusions are as follows:

(1) The mathematical method of hybrid RBF can be used to develop brake force distribution model with prefect real-time performance, high accuracy and optimal energy recovery.

(2) In comparison with the original brake force distribution model based on the rule algorithm, the optimized one proposed in this paper is able to improve the energy, recovered into battery, by 14.75%.

(3) Further research about the development of multiple energy sources (Lithium and supercapacitor or flywheel battery system) is still required to reduce the energy loss caused by the single energy source (Lithium power battery system used in this paper).

Acknowledgement
This work was supported by the National Natural Science Foundation Project of China (51805301, 51505263), the Scientific Research Project of Universities in Shandong (J18KA021), and the Zibo City-Shandong University of Technology Cooperative Projects, China (2017ZBXC165). The corresponding author of this paper is Bo Li, whose e-mail is njustlibo@126.com.

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