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MODELLING AND ANALYSIS OF TRANSPORTATION NETWORKS USING COMPLEX NETWORKS: POLAND CASE STUDY

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Abstract: *In the paper a theoretical bases and empirical results deal with analysis and modelling of transportation networks in Poland using complex networks have been presented. Properties of complex networks (Scale Free and Small World) and network's characteristic measures have been described. In this context, results of empirical researches connected with characteristics of passenger air links network, express railway links network (EuroCity and InterCity) and expressways/highways network in Poland have been given. For passenger air links network in Poland results are compared with the same networks in USA, China, India, Italy and Spain. In the conclusion some suggestions, observations and perspective dealing with complex network in transportation networks have been presented.*

Key words: *transportation networks modelling, complex networks, network analysis, gravity measures in networks, transportation networks in Poland*

1. Introduction

Complex networks are specific graph and network models of real objects. Their significance is growing in recent years because of experimentation results on real topologies which confirm that real networks have some specific properties other than thought. Inspiration for complex networks is social networks[1]. Wide review of complex network applications is presented in (Bartosiak et al., 2011; Hage and Harary, 1995; Tarapata and Kasprzyk, 2009; Wang et al., 2011; Watts and Strogatz, 1998). Many of real networks (from different fields) is complex networks with specific properties (power node degree distribution, high value of clusterization, low value of average distance between each pair of nodes in a network; these properties have been shortly described in section 3). Some results (Barabási and Albert, 1999; Berche et al., 2012; Cheung and Gunes, 2012; Von Ferber et al., 2009; Wang et al., 2011; Xie and Levinson, 2009; Zanin and Lillo, 2013) confirm that transportation networks belong to this group. For example, the goals of the paper (Berche et al., 2012) are to present criteria that allow to a priori quantify the attack stability of real world correlated networks of finite size and to check how these criteria correspond to analytic results available for infinite uncorrelated networks in several major cities of the world. In the Cheung and Gunes (2012) authors analyze the air transportation network in the U.S. to better understand its characteristics. For this, they measure several complex network features and make some interesting conclusions. Authors of the paper (Von Ferber et al., 2009) use complex network concepts to analyze statistical properties of urban public transport networks in several major cities of the world. Authors of the paper (Wang et al., 2011) use a complex network approach to examine the network structure and nodal centrality of individual cities in the air transport network of China. The paper (Xie and Levinson, 2009) explores the topological evolution of surface transportation networks, using empirical evidence and a simulation model validated on that data. A self-organization property of surface transportation networks has been shown. Authors of the paper (Zanin and Lillo, 2013) review some recent approaches to air transport, which make extensive use of theory of complex networks. They discuss possible networks that can be defined for the air transport and they focus their attention to networks of airports connected by flights. Moreover, they discuss the results of some recent papers investigating the dynamics on air transport network, with emphasis on passengers traveling in the network and epidemic spreading mediated by air transport.

[11] Social networks describe relations (interactions) between members of some socialites.

On the Fig.1a the air transportation network in USA is presented. Nodes represent cities (airports) and edges represent air communication between two nodes. This network is an example of a complex network holding some properties described in section 3. However, on the Fig.1b a part of street network in Sao Paulo (Brasil) is presented. We can see two characteristic parts of the network: first one with complex network features (outskirts of the city) and second one as regular graph[2] (center of the city).

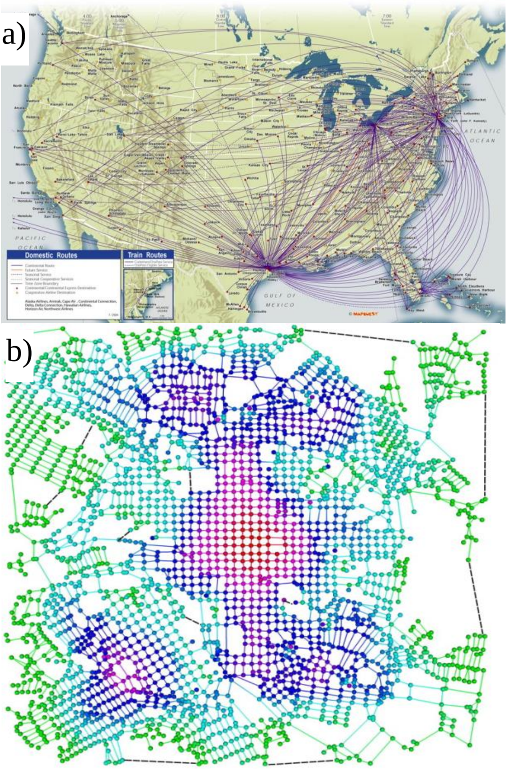

Fig. 1. (a) Air transportation network in USA (Lima, 2004) and (b) a part of street network in Sao Paulo (Brasil) (Travençolo and Costa, 2008)

The goal of this paper is to examine whether selected transportation networks in Poland have features specific for complex networks. Confirmation of this hypothesis would allow us to predict evolution of the networks and use the conclusions resulting from other researches relating to complex networks.

The paper is organized as follows. In the section 2 and 3 we present basic network measures (characteristics) and properties of complex networks. Section 4 contains results of research of passenger air links network, express railway links network (EuroCity and InterCity) and expressways/highways network in Poland in context of complex network properties.

2. Centrality measures in networks

Let assume that structure of the complex network is described by the directed (or undirected) graph $G = \langle V, E \rangle$, where V describes set of nodes and E describes set of arcs (or edges), $|V|=N$, $|E|=M$.

We can address the question of: "What is the most important or central node in a given network?" Centrality measures (defined below) are the most basic and frequently used methods for analysis of complex networks.

Normalized degree dc_i of the i-th node:

$$dc_i = \frac{k_i}{N-1} \tag{1}$$

where: k_i describes degree of the i-th node in the graph G. This measure gives the highest score of influence to the node with the largest number of first-neighbours (normalized over the maximum number of neighbours this node could have). The higher dc_i value, the better (the i-th node is more important or more central).

Eccentricity ec_i of the i-th node (Hage and Harary, 1995):

$$ec_i = \max_{j \in V} d_{ij} \tag{2}$$

where: d_{ij} – length of the shortest path in G between the i-th and the j-th node (number of arcs (edges) on the shortest path from i do j). The lower ec_i value, the better (the i-th node is more important or more central).

Radius (*graph centrality*) rc_i of the i-th node (Brandes, 2001):

[2] In regular graph, each node has the same degree (count of neighbours nodes directly connected by links with considered node).

$$rc_i = \frac{1}{\max_{j \in V} d_{ij}} = \frac{1}{ec_i} \quad (3)$$

The higher rc_i value, the better. It means that the best is such i-th node with the smallest value of the longest of the shortest path from i to each node (the i-th node is more important or more central).
Closeness cc_i of the i-th node (Brandes, 2001):

$$cc_i = \frac{N-1}{\sum_{j \in V} d_{ij}} \quad (4)$$

The higher cc_i value, the better (the i-th node is more important or more central). In the other words, the more central a node is the lower its total distance from all other nodes.
Betweenness (*load*) bc_i of the i-th node (Brandes, 2001)[3]:

$$bc_i = \sum_{l \in V} \sum_{k \neq l \in V} \frac{p_{l,i,k}}{p_{l,k}} \quad (5)$$

where: $p_{l,i,k}$ – count of the shortest paths in G between l and k nodes visiting the i-th node, $p_{l,k}$ – count of the shortest paths in G between l and k nodes. Betweenness centrality quantifies the number of times a node acts as a bridge along the shortest path between two other nodes. The higher bc_i value, the better (the i-th node is more important or more central).
Clusterization gc_i of the i-th node (Latapy, 2008; Watts and Strogatz, 1998):

$$gc_i = \frac{2E_i}{k_i(k_i - 1)},\ k_i > 1 \quad (6)$$

where: E_i – count of arcs (edges) between firs-neighbours of the i-th node.
This measure describes "probability" that the first neighbours[4] of the i-th node are their first neighbours, too. In the other words, this is the relation of all triangles[5] existing in a network to all potential triangles in a network. The higher gc_i value, the better (the i-th node is more important or more central).
Measures (1)-(6) describe characteristics of graph (network) nodes.
Let's define basic characteristics for whole graph (network).
Average shortest paths length L in a network (the lower L value, the better) - Watts and Strogatz, (1998):

$$L = \frac{1}{N(N-1)} \sum_{i \neq j \in V} d_{ij} \quad (7)$$

Clusterization coefficient C of a network (the higher C value, the better)) - Watts and Strogatz, (1998):

$$C = \frac{1}{N} \sum_{i \in V} gc_i \quad (8)$$

Diameter D of a network (the lower D value, the better) - Hage and Harary (1995):

$$D = \max_{i \in V} ec_i \quad (9)$$

Radius R of a network (the lower R value, the better) - Hage and Harary (1995):

$$R = \min_{i \in V} ec_i \quad (10)$$

Average nodes degree $\bar{k}$ of a network (the higher $\bar{k}$ value, the better)) - Watts and Strogatz, (1998):

$$\bar{k} = \frac{1}{N} \sum_{i \in V} k_i \quad (11)$$

3. Basic properties of complex networks

Identifying and measuring properties of real networks is a first step towards understanding their topology, structure and dynamics. The next step is to develop a mathematical model, which typically

[3] In undirected graphs value of this measure is divided by 2 (between each pair of nodes x, y and y, x the same count of shortest paths exist).
[4] Commonly, first-neighbours of the i-th node are such nodes which directly link (by an arc or an edge) the i-th node.
[5] Triangle in a graph (network) is such a part of the graph (subgraph) consisting of three nodes, that each of the nodes is linked (by an arc or an edge) with each other (a clique with 3 nodes).

takes a form of an algorithm for generating networks with the same statistical properties. Apparently, networks derived from real data (most often are spontaneously growing) have a low average d_{ij}, power law degree distributions ($P(k) \sim k^{-\gamma}$, where γ is a constant, k is degree of node in a network), occurrence of hubs (nodes with much higher degrees than the average node degree), tendency to form clusters and many other interesting features. Three very interesting models, which capture these features, have been introduced recently: *Random Graph, Small World* and *Scale Free*.

Figure 2 presents networks generated using these models. The leftmost picture in Fig.2 shows Erdős' and Rényi's model of the *Random Graph* presented in Erdős and Rényi (1959). It is unfortunately an inaccurate model of real networks due to the lack of features that the remaining two models have. The middle picture shows an example of Watts and Strogatz, (1998) *Small World* network. It is characterized by a high clustering coefficient and a small average shortest path length. A graph is considered small-world if C is significantly higher than C of a random graph constructed on the same node set, and if the graph has got approximately the same average shortest path length L ($L \sim \frac{\ln N}{\ln \bar{k}}$ and it weakly depend of network size) as its corresponding random graph. The rightmost picture shows an example of Barabási and Albert (1999) *Scale Free* network which has some additional values in comparison with networks generated using the *Small World* model. It is characterized by a node degree distribution that follows a power law (as it has been written: $P(k) \sim k^{-\gamma}$, in real *Scale Free* networks γ<3). Power degree distribution decides that majority of nodes (so called *authorities*) has low degree and minority of them (so called *hubs*) has high degree. *Scale Free* network is very resistance to random attacks but is very non-resistant to targeted attacks (Tarapata and Kasprzyk, 2010). From the point of view of communication in a network, it is very fast but propagation of faults, viruses, accidents is fast, too. It has been the most accurate model since many empirically observed networks appear to be *Scale Free*, including social networks, Internet, WWW, citation networks, bionetworks, etc.

4. Characteristics of selected transportation networks in Poland

In this section we present results for exploring three transportation networks in Poland: passenger air links network, express railway links network (EuroCity (EC) and InterCity (IC)) and expressway/highway network. To model these networks and calculate values of their characteristics the Gephi (Gephi: webpage) open source software for exploring and manipulating graphs and networks has been used.

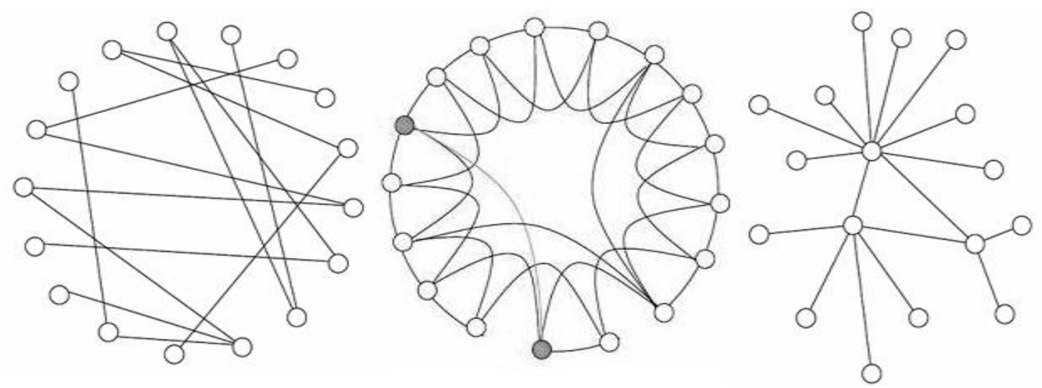

Fig. 2. From the left – a *Random Graph*, a *Small World* network and a *Scale Free* network
Source: Bartosiak et al. (2011).

Fig. 3. Network S_1 of passenger air links in Poland, March 2013
Source: own work on the basis of Eurolot (LOT) and SprintAir data.

The network S_1 from Fig. 3 (nodes – airports, edges – air links) with centrality measures presented in the Table 1 consists of 10 nodes and 12 edges (on each edge the name of carrier is described). Node degree distribution in this network (Fig.4) is similar to power distribution: there is single central node (Warszawa), which has high degree and majority of nodes has low degree. Moreover, node Warszawa has the highest value of betweenness centrality; it means that air communication between two different nodes in this network, in majority cases, need Warszawa node for change (small number of direct connections between two nodes in the network). There is one more interesting feature of betweenness in this network: there are 7 nodes with bc_i=0. Taking into account formula (5), it means that there is no pair of nodes in the network which need as a change airport each of these 7 nodes (airports). Value of eccentricity measure equal 1 for Warszawa means that the longest of the shortest paths from Warszawa to all cities is equal 1 (Warszawa has direct air connection to each city).

Table 1. Centrality measures of nodes in the network S_1 from Fig.3

Node *i*	*Degree* k_i	*Normalized degree* dc_i	*Eccentricity* ec_i	*Radius* rc_i	*Closeness* cc_i	*Betweenness* bc_i	*Clusterization* gc_i
Bydgoszcz	1	0.11	2	0.5	1.89	0	0
Gdańsk	3	0.33	2	0.5	1.67	0.5	0.67
Katowice	1	0.11	2	0.5	1.89	0	0.17
Kraków	3	0.33	2	0.5	1.67	0.5	0.67
Poznań	2	0.22	2	0.5	1.78	0	1
Rzeszów	1	0.11	2	0.5	1.89	0	0
Szczecin	1	0.11	2	0.5	1.89	0	0
Warszawa	9	1	1	1	1	32	0.08
Wrocław	2	0.22	2	0.5	1.78	0	1
Zielona Góra	1	0.11	2	0.5	1.89	0	0

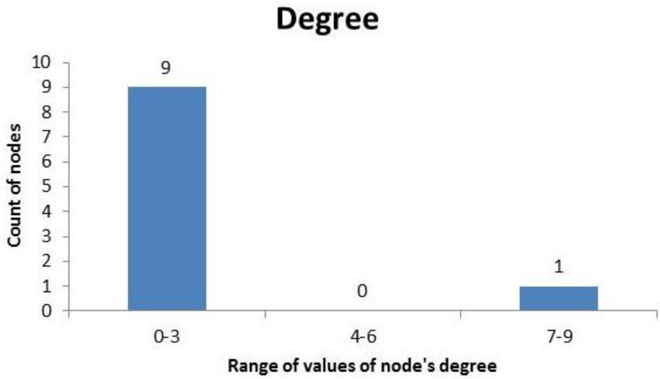

Fig. 4. Node degree distribution in the S_1 network from Fig. 3

In the Table 2 a comparison of characteristics for network from Fig.3 with analogical networks in the world have been given (a little different values of parameters from the Table 2 for USA are given in Cheung and Gunes (2012) and for China in Wang et al. (2011)). The network in Poland has small average distance between any pair of nodes (L=1.73; it means that about 0.73 changes is needed in average case to achieve any node in the network from another node) and it is the smallest value among other networks in the Table 2, but simultaneously it has small value of clusterization C (the smaller value of this coefficient has network in Italy only). Taking into account interpretation of this measure, we can say that in Poland "probability" that the first neighbours of any node are their first neighbours is equal C=0.36.

Table 2. Characteristics of passenger air transportation networks in the world (Zanin and Lillo, 2013) and in Poland

Network	*N*	*M*	*L*	*C*	γ
World	3883	27051	4.4	0.62	1.0
USA	272	6566	1.9	0.73	2.63
China	128	1165	2.07	0.73	4.16
India	79	442	2.26	0.66	2.2
Italy	42	310	1.97	0.10	1.7
Spain	35	123	1.84	0.78	-
Poland	**10**	**12**	**1.73**	**0.36**	-

Network S_2 from Fig.5 (nodes – EC/IC railway stations, edges – EC/IC railway connection; node characteristics are presented in the Table 3) consists of 13 nodes and 14 edges (on the edges the type of connection is described). Node degree distribution in this network (Fig.6) is similar to power distribution: a few central nodes (Warszawa, Poznań, Katowice) exist, which has high degree and majority of nodes has low degree. This network has small average distance between any pair of nodes (L=2.72; it means that about 1.72 changes is needed in average case to achieve any node in the network from another node) but simultaneously it has small value of clusterization C=0.1.

Fig. 5. Network S_2 of express railroad connections in Poland, 2007 year
Source: own work on the basis of http://www.intercity.pl

Table 3. Centrality measures of nodes in the network S_2 from Fig.5

Wierzchołek *i*	*Degree* k_i	*Normalized degree* dc_i	*Eccentricity* ec_i	*Radius* rc_i	*Closeness* cc_i	*Betweenness* bc_i	*Clusterization* gc_i
Gdańsk	2	0.17	4	0.25	2.42	20	0
Gdynia	2	0.17	5	0.20	3.17	11	0
Gliwice	2	0.17	5	0.20	2.50	5	0
Katowice	4	0.33	4	0.25	2.17	17	0.17
Kraków	2	0.17	4	0.25	2.50	0	1
Łeba	1	0.08	6	0.17	4.08	0	0
Poznań	4	0.33	4	0.25	2.00	30	0
Rzepin	1	0.08	5	0.20	2.92	0	0
Szczecin	1	0.08	5	0.20	2.92	0	0
Warszawa	4	0.33	3	0.33	1.83	37	0.17
Wrocław	3	0.25	5	0.20	2.42	14	0
Zebrzydowice	1	0.08	5	0.20	3.08	0	0
Żary	1	0.08	6	0.17	3.33	0	0

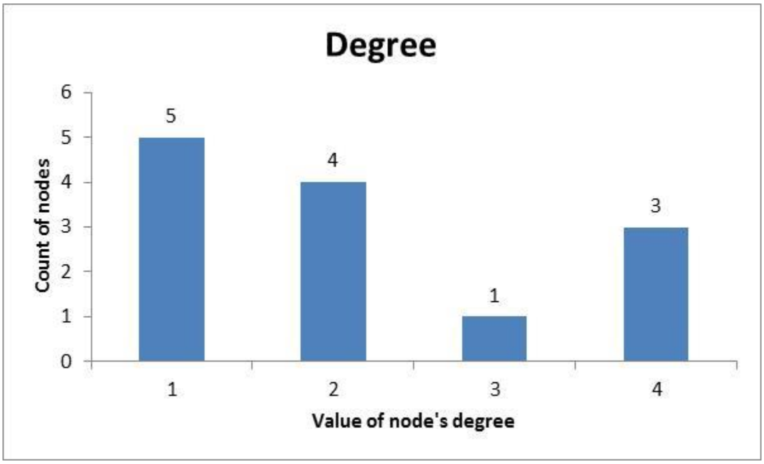

Fig. 6. Node degree distribution in the S_2 network from Fig.5

Network S_3 from Fig.7 (nodes – main expressway/highway nodes, edges – part of expressway/highway; node characteristics are presented in the Table 4) consists of 57 nodes and 80 edges (on the edges name description of expressway/highway is given). Node degree distribution in this network (Fig.8) is similar to power distribution: a few central nodes (Warszawa, Łódź, Szczecin) exist, which has high degree and majority of nodes has low degree. This network has relatively small average distance between any pair of nodes (L=4.77; it means that about 3.77 changes is needed in average case to achieve any node in the network from another node) but simultaneously it has small value of clusterization C=0.1. Moreover, node Warszawa has the highest value of betweenness centrality; it means that travelling by expressways/highways only between two different nodes in this network, in majority cases, need Warszawa node for change.

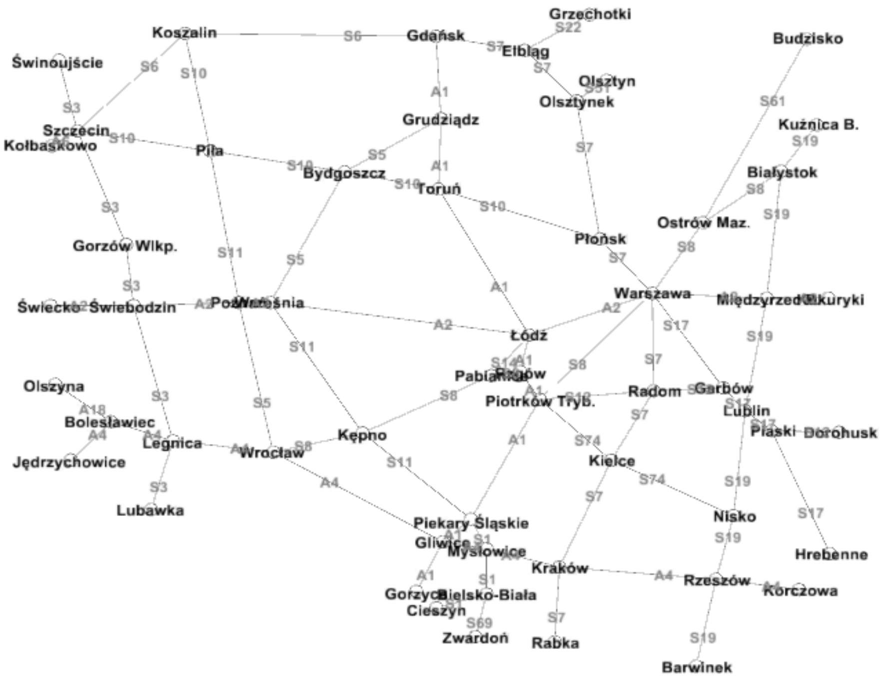

Fig. 7. The target structure of the network S_3 of expressways and highways in Poland
Source: own work on the basis of Polish GDDKiA.

Table 4. Centrality measures of nodes in the network S_3 from Fig.7

Node i	*Degree* k_i	*Normalized degree* dc_i	*Eccentricity* ec_i	*Radius* rc_i	*Closeness* cc_i	*Betweenness* bc_i	*Clusterization* gc_i
Barwinek	1	0,018	9	0,111	5,679	0	0
Białystok	3	0,054	9	0,111	4,893	60,924	0
Bielsko-Biała	3	0,054	8	0,125	4,768	109	0
Bolesławiec	3	0,054	10	0,100	5,5	109	0
Budzisko	1	0,018	9	0,111	5,286	0	0
Bydgoszcz	4	0,071	7	0,143	4,054	127,455	0,167
Cieszyn	1	0,018	9	0,111	5,750	0	0
Dorohusk	1	0,018	11	0,091	6,411	0	0
Elbląg	3	0,054	8	0,125	4,946	81,955	0
Garbów	3	0,054	8	0,125	4,054	81,722	0,333
Gdańsk	3	0,054	9	0,111	4,946	69,694	0
Gliwice	4	0,071	7	0,143	3,804	236,429	0,167
Gorzów Wlkp.	2	0,036	9	0,111	5	32,889	0
Gorzyce	1	0,018	8	0,125	4,786	0	0
Grudziądz	3	0,054	8	0,125	4,5	39,380	0,333
Grzechotki	1	0,018	9	0,111	5,929	0	0
Hrebenne	1	0,018	11	0,091	6,411	0	0
Jędrzychowice	1	0,018	11	0,091	6,482	0	0
Kępno	4	0,071	7	0,143	3,714	145,508	0
Kielce	4	0,071	8	0,125	4,107	126,694	0,167

Kołbaskowo	1	0,018	10	0,100	5,875	0	0
Korczowa	1	0,018	9	0,111	5,679	0	0
Koszalin	3	0,054	9	0,111	4,964	62,925	0,333
Kraków	4	0,071	7	0,143	4,125	192,860	0
Kukuryki	1	0,018	9	0,111	5	0	0
Kuźnica B.	1	0,018	10	0,100	5,875	0	0
Legnica	4	0,071	9	0,111	4,589	234,564	0
Lubawka	1	0,018	10	0,100	5,571	0	0
Lublin	4	0,071	9	0,111	4,518	205,824	0
Łódź	5	0,089	6	0,167	3,589	288,163	0,1
Międzyrzec P.	4	0,071	8	0,125	4,018	189,624	0
Mysłowice	4	0,071	7	0,143	3,857	282,439	0,167
Nisko	3	0,054	9	0,111	4,679	102,673	0
Olsztyn	1	0,018	10	0,100	5,643	0	0
Olsztynek	3	0,054	9	0,111	4,661	125,555	0
Olszyna	1	0,018	11	0,091	6,482	0	0
Ostrów Maz.	3	0,054	8	0,125	4,304	96,8	0
Pabianice	3	0,054	6	0,167	3,911	30,727	0,333
Piaski	3	0,054	10	0,100	5,429	109	0
Piekary Śląskie	4	0,071	6	0,167	3,571	221,199	0,167
Piła	4	0,071	8	0,125	4,250	195,705	0,167
Piotrków Tryb.	5	0,089	7	0,143	3,589	244,038	0,2
Płońsk	3	0,054	8	0,125	3,929	204,627	0
Poznań	4	0,071	7	0,143	3,750	271,675	0
Rabka	1	0,018	8	0,125	5,107	0	0
Radom	4	0,071	7	0,143	3,786	54,720	0,5
Rzeszów	4	0,071	8	0,125	4,696	131,243	0
Rzgów	3	0,054	6	0,167	3,875	20,993	0,333
Szczecin	5	0,089	9	0,111	4,893	131,889	0,1
Świebodzin	4	0,071	8	0,125	4,429	150,264	0
Świecko	1	0,018	9	0,111	5,411	0	0
Świnoujście	1	0,018	10	0,100	5,875	0	0
Toruń	4	0,071	7	0,143	4,089	133,431	0,167
Warszawa	7	0,125	7	0,143	3,429	556,246	0,095
Wrocław	4	0,071	8	0,125	3,911	319,605	0
Września	4	0,071	6	0,167	3,554	233,560	0
Zwardoń	1	0,018	9	0,111	5,750	0	0

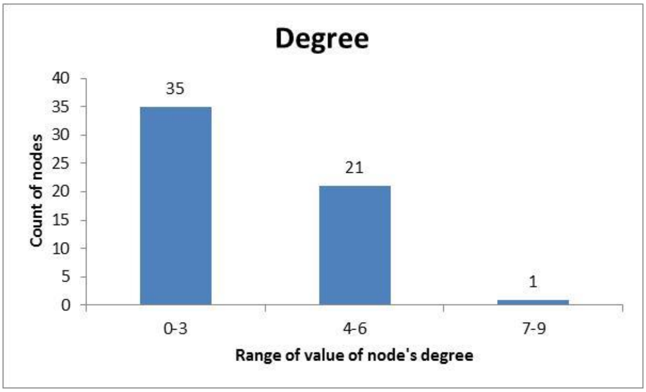

Fig. 8. Node degree distribution in the S_3 network from Fig.7

In the Table 6 we present comparison between network characteristics of passenger air links (S_1), express railroad IC and EC (S_2) and expressways/highways (S_3) in Poland.

Table 6. Characteristics of passenger air links (S_1), express railroad IC and EC (S_2) and expressways/highways (S_3) in Poland

Network	N	M	D	R	L	C	$\overline{k}$	γ
S_1	10	12	2	1	1.73	0.36	2.60	-
S_2	13	14	6	3	2.72	0.10	2.15	0.71
S_3	57	80	11	6	4.77	0.10	2.81	2.97

5. Conclusions

The results obtained in the paper allow us to conclude that transportation networks in Poland have features of complex networks. Indeed, low value of clusterization coefficient in the networks S_2 and S_3 force us to consider whether these networks have *Small World* features (S_2 has very small size). Otherwise, we know that selected transportation networks (e.g. street network in a city) have specific feature (the closer a city centre, the better visible): node degree has the same value equals 4 (perpendicular crossroads), so they are local regular networks (see Fig.1b). However, many results of research (e.g. in Berche et al. (2012), Von Ferber et al. (2009)) show: the higher scale of the network (from micro-network to macro-network) the better visible *Scale Free* and *Small World* features of transportation networks. Note that in S_1 network we can see one of the disadvantages of *Scale Free* networks: if we conduct target attack/block on a node with highest value of degree (Warszawa), then we degrade this network after the first attack/block. It means that majority of nodes would have no possibilities to achieve majority of nodes by air transport (because Warszawa is change airport for travelling).

References

[1] BARABÁSI, A. and ALBERT, R., 1999. Emergence of Scaling in Random Networks. *Science*, 286(5439), pp. 509-512.

[2] BARTOSIAK, C., KASPRZYK, R. and TARAPATA, Z., 2011. Application of Graphs and Networks Similarity Measures for Analyzing Complex Networks. *Biuletyn Instytutu Systemów Informatycznych,* 7, pp. 1-7.

[3] BERCHE, B., FERBER, C.V., HOLOVATCH, T. and HOLOVATCH, Y., 2012. Transportation Network Stability: A Case Study Of City Transit. *Advances in Complex Systems*, 15(supp01), pp. 1-18.

[4] BRANDES, U., 2001. A faster algorithm for betweenness centrality. *Journal of Mathematical Sociology*, 25(2), pp. 163-177.

[5] CHEUNG, D.P. and GUNES, M.H., 2012. A Complex Network Analysis of the United States Air Transportation, *Proceedings of the 2012 IEEE/ACM International Conference on Advances in Social Networks Analysis and Mining, 26-29 August, 2012, Kadir Has University, Istanbul, Turkey* 2012, pp. 699-701.

[6] ERDŐS, P. and RÉNYI, A., 1959. On random graphs I. *Publ.Math.Debrecen*, 6, pp. 290-297.

[7] GEPHI webpage, The Open Graph Viz Platform. Available: http://gephi.org/.

[8] HAGE, P. and HARARY, F., 1995. Eccentricity and centrality in networks. *Social networks*, 17(1), pp. 57-63.

[9] LATAPY, M., 2008. Main-memory triangle computations for very large (sparse (power-law)) graphs. *Theoretical Computer Science*, 407(1–3), pp. 458-473.

[10] LIMA, M., 2004-last update, Transportation Routes. Available: http://www.mslima.com/mfadt/thesis/2004/08/transportation-routes.html.

[11] TARAPATA, Z. and KASPRZYK, R., 2009. An application of multicriteria weighted graph similarity method to social networks analyzing, *Proceedings of the 2009 International Conference on Advances in Social Network Analysis and Mining*, 20-22.07.2009 2009, IEEE Computer Society, pp. 366-368.

[12] TARAPATA, Z. and KASPRZYK, R., 2010. Graph-based optimization method for information diffusion and attack durability in networks, *Rough Sets and Current Trends in Computing, Lecture Notes in Artificial Intelligence, 6086 (2010)* 2010, Springer-Verlag Berlin Heidelberg, pp. 698-709.

[13] TRAVENÇOLO, B.A.N. and DA F. COSTA, L., 2008. Accessibility in complex networks. *Physics Letters A*, 373(1), pp. 89-95.

[14] VON FERBER, C., HOLOVATCH, T., HOLOVATCH, Y. and PALCHYKOV, V., 2009. Public transport networks: empirical analysis and modeling. *The European Physical Journal B,* 68(2), pp. 261-275.

[15] WANG, J., MO, H., WANG, F. and JIN, F., 2011. Exploring the network structure and nodal centrality of China's air transport network: A complex network approach. *Journal of Transport Geography,* 19(4), pp. 712-721.

[16] WATTS, D.J. and STROGATZ, S.H., 1998. Collective dynamics of 'small-world' networks. *Nature*, 393(6684), pp. 440-442.

[17] XIE, F. and LEVINSON, D., 2009. Topological evolution of surface transportation networks. *Computers, Environment and Urban Systems,* 33(3), pp. 211-223.

[18] ZANIN, M. and LILLO, F., 2013. Modelling the air transport with complex networks: A short review. *The European Physical Journal Special Topics,* 215(1), pp. 5-21.