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DEPENDENCE OF TRIP LENGTH FREQUENCY DISTRIBUTION ON CHARACTERISTICS OF ROUTE NETWORK ELEMENTS

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Abstract: *The up-to-date methods of the demand modelling for municipal public transport (MPT) services are not perfect and they require an objective estimation, enhancement and improvement. These refinements are to take into account the functions of population allocation in modern cities.*

The possibility of the definition of population allocation functions on the basis of characteristics of public transport stops allocation, the regularities of distances between adjacent stops and distances between a pair of stops are investigated.

Key words: *allocation function, distribution law, stop co-ordinate, distance between stops, trip matrix.*

1. Introduction

The modern theory of the travel demand forecasting uses, in general, only one characteristic of route as the criterion of choosing the Origin-Destination (O-D) pair by a person. At the same time, ignoring the other factors, which do not refer to the means of travel or its "impedance", makes the travel to be not the way to hit the target, but the purpose in itself.

Although, it is obvious that the purpose of majority of trips cannot be the movement process per se, all well-known methods of travel demand modelling are based only on the transport factors. The main arguments for this are the obvious desire of a person to minimize travel cost. Also the results of various investigations demonstrate a clear correlation between trip distance and probability of trip making [1, 5, 7, 11, 12, 18, 23, 24, 26, 27]. In transportation science the correlation is called a trip length frequency distribution function. The reason of its occurrence is the desire of people to live near the main places that satisfy their needs [23, 24, 26, 27].

Determination of existing regularities in allocation of Origins and Destinations of city population trips will clarify the possibilities of transport demand modelling on the basis of "impedance" ("deterrence") of a travel between Origin and Destination points and provide additional information to improve the transportation planning accuracy and efficiency.

2. Literature review

Transport demand models have a primary importance in transport planning because they define the accuracy of a transport model and efficiency of planning results. The transport demand is usually presented as trip matrices or OD matrices [3, 7, 9, 13, 18]. To make transport demand modelling most accurate the passenger trips in city transport systems are divided into groups by day period, purposes, socio-economic status of people, etc. The main group of city population trips is home-based work trips, which create the greatest load on a transport network and cause peak-periods. That's why their modelling is required during development of transport system models [1, 5, 7, 9, 11, 23, 24, 26, 27]. In addition, work trips are the basis of majority of spatial interaction models as they have strong influence on the choice of residences or workplaces [1, 5, 11, 14]. Meanwhile, all the authors [1, 5, 9, 11, 19, 23-27] consider that trip matrix and trip length frequency distribution is influenced by a choice of the «Home – Work» pair made by population. It makes home-based work trips to be the universally recognized base for determining the regularities in the spatial allocation of urban population.

Among existing approaches to model the distribution of population demand for travelling over the city territory the gravity models of interaction between Origins and Destinations are wide spread [1, 5, 7, 9, 11, 12]. The use of the gravity-type models is based on the assumption about dependence of spatial interaction between transport attractors on their size and travel cost between them. For example, a large city or an enterprise generates more traffic and attracts more labour than the small one. Concerning travel cost, any spatial interactions between objects such as work trips, exchange of product or information are realised over some distance. It is logical to consider that the greater the distance, the lower the interaction intensity and, hence, the probability of travel this distance [12].

Justification for using the transport factors in demand modelling can be illustrated by next examples. The land cost in a city or business centres is higher than in other areas. In most cases it can be explained by their high transport accessibility. These associated parameters can be explicitly or implicitly taken into account in the known land use models [3, 10, 11].

The other example of transport factors use is the modelling of traffic flows on the basis of Wardrop's principles: 1) transport network users independently choose the route that meets their minimum transportation cost; 2) transport network users choose the route to minimize general transportation cost in the whole network [4, 6, 15, 18]. These principles certainly have sense but they can be applied only in the case of travel between Origin and Destination which are already known.

During the determination of Origin and Destination (transport zones), transportation cost should be paralleled with the positive part of a travel, that is, the purpose which a person realizes through travel. On the base of such paralleling in the work trips, the paper [19] shows that when a person chooses the workplace as continual transport attractor, transport factors are not decisive and the main role is played by other factors which are called social. This is true for individual behaviour, but it doesn't explain underlying conditions of collective, overall behaviour of people that is illustrated by dependence of travel probability on "impedance" of the connection between transport zones, i.e. the existence of population allocation functions.

The main role in the study of the functions of population allocation in cities and regions belongs to scientists from the former socialist countries. Theoretical evidence of Soviet scientists is mainly based on assumption of people's desire to choose the residence or workplace that is reasonably close to each other to minimize the time of regular trips between them. The result of such collective behaviour is the allocation function. It's general form in most empirical cases can be illustrated by gamma distribution with shape parameter that is more than 1 (Fig. 1) [23, 24, 26, 27].

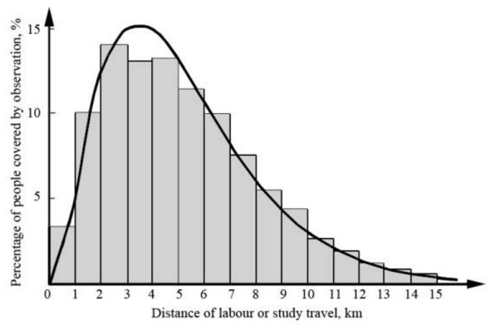

Fig. 1. Passenger distribution by travel distance in Prague, 1966, determined by Yan Tsybulka [26]

The form of this curve is typical for most cities investigated and differs only by the slope of a curve. As the modal value of distribution is shifted to zero, it leads to conclusion about the substantial dependence of the probability of travel on its "impedance" [23, 24, 26, 27].

The results of the formal testing of the hypothesis of coincidence between empirical data and a theoretical gamma-distribution curve were not found in literature but similarity of empirical and theoretical data may be considered as sufficient. The more important task is finding the reasons of existence of such regularities.

All the known population allocation curves for the cities of the former USSR are the same as in Fig. 1. They were built by approximation of a great number of empirical data [23, 24, 26, 27]. Such investigations require considerable resources and a strong administrative impact on enterprises in all economic sectors. It explains the greater effectiveness of socialist scientists in investigation of trip length distributions in cities.

It should be noted that the use of such popular theoretical models of the trip matrix calculation as gravity and entropy ones results in a matrices which give identical population allocation curves [19]. It would be an advantage of the methods, but matrices which are obtained in another way lead to the similar results: for example, randomly filled matrices with known transport zones' capacities; matrices of extreme travel cost or matrices that minimize and maximize passenger kilometres on trip realization [19]. The last two types of matrices are considered to be theoretical and they are practically impossible. On the basis of this information a preliminary conclusion about small dependence of population allocation functions on the method of receiving the trip matrix can be made. At the same time a large number of possible trip matrix variants with constant transport zones capacities indicate incomplete knowledge about population allocation in the city and a need for additional investigation of allocation functions.

In general the allocation curve is the result of proper processing of data that are contained in a trip matrix when its elements are grouped into segments depending on the correspond intervals of travel time or distance. Lack of real trip matrices for modern cities makes the receiving of empirical allocation functions very difficult in practice, and the use of theoretical matrices that are obtained by any of the known methods, for constructing the allocation functions is senseless because it will not characterize the factual population allocation completely.

More real researches are investigations of the allocation functions of the workers of separate enterprises, but they are selective in terms of the general allocation functions and may cause the significant deviations. Such a situation is a result of the forming of those functions under the influence of concrete enterprise characteristics, primarily according to their location, but not under the labour-market as a whole. An example of the mentioned allocation function is the illustration of results of the white collars survey in five office blocks in Krakow, Republic of Poland (Fig. 2) [2].

In contrast to the most common variant, such a curve seems to be shifted to the right from the modal value that causes doubts in the using of transport factors during travel demand modelling.

Investigations of H. Sheleikhovskyi can be treated as the first theoretical studies of people allocation in cities [27]. They are completely based on transport factors and their perception by person, which the author described as the psychophysical Weber-Fechner law that is generally used to describe a human reaction on a physical stimulus.

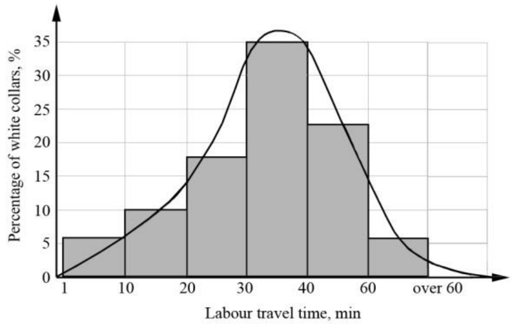

Fig. 2. Distribution of white collars by the work trip time

The effect of this law is valid at the average stimulus impact on senses and significantly degrades at the large or boundary ones [8]. Regarding to the distance or time of travel between residence and workplace it is not acceptable. H. Sheleikhovskyi had been accepted the general form of trip length frequency distribution (Fig. 1), but on the basis of his own theory he argued that any allocation would follow up to the «normal» according to the city evolution, Fig. 3 [27].

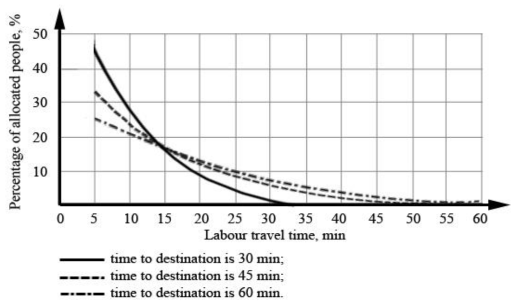

Fig. 3. The scale of «normal» allocation by H. Sheleikhovskyi

Such a theoretical allocation function looks like the exponential distribution law, which is a special case of gamma distribution. This fact provides the visual conformity between Sheleikhovskyi's theory and empirical data in an extreme case.

This theory and some other allocation theories imply that if the residential real estate and job vacancies are in excess, any person would seek to change the residence in order to live nearby the workplace and vice versa – change the workplace for the same reason [23, 24, 27]. Such assumptions are necessary to explain the results of city population collective behaviour and to justify the using of travel "impedance" between transport zones as the only factor of formation the travel demand model. However, these assumptions are not theoretically justified and contradict the real state of both labour and real-estate markets as well as behaviour of people in situations of collective decision making, e.g. family decision making. Therefore, the reasons of origin of allocation function shown in Fig. 1 that describes empirical data quite well [23, 24, 26, 27] cannot be considered theoretically grounded that causes a need to search and test other hypotheses.

One of these hypotheses is contained in work [3]. The authors of [3] consider transport and route networks as self-organized hierarchical structures which formation is influenced by interests of government, large business owners, network users and other subjects and, therefore, can be considered in many aspects as random despite of some plans [3, 10]. Attempts to explain the results of such interaction were made on the basis of analogy between a process of transportation network development and microorganisms' behaviour during the searching of nutrients [3] but fundamental progress in explaining the existing population allocation regularities using this analogy was not achieved by authors cited.

Drawing a line under publication review, it can be considered that there are two main views on the origin of the trip length frequency distribution function: 1) it is the result of people deliberate behaviour to locate as close as possible to their workplaces or 2) it is a natural process of transport demand distribution over the city that is caused by the city growth under influence of many factors. The first view is already developed in the existing allocation theory; the second one needs to be tested theoretically and experimentally.

3. Problem statement

At present it is possible to consider that general plot of the trip length distribution, which is shown in Fig. 1, is known. At the same time, there are no considerable differences when grouping by a criterion of travel distance or time, as the travel distance covers more than 95% of a total travel time variation [19]. It should be pointed out that these data concern only public transport and they are more than relevant for Ukrainian cities as 70-80% of city dwellers use public transport services during work trip making.

The publication review has shown that the route network (RN) of municipal public transport (MPT) and its characteristics are not directly considered in the known population allocation theory. Upon that, MPT is seen only as a factor which defines travel "impedance" on its basic directions according to own level of development, which can be described by velocities, vehicles comfort level, network branching degree etc. [23, 27].

The formation of the route network of city public transport is performed under the influence of a large number of factors. Their interaction causes some randomness of RN development [3, 10]. The most influential factor is passenger demand for transportation because its satisfaction is the main purpose of a route network existence. Therefore, the location of the objects of route network infrastructure, that represents the transport supply, is determined by transport demand. At a considerable part of passenger transportation by public transport, which is typical for Ukrainian cities, the RN of MPT is a very good tool to study the process of formation of trip length frequency distribution. It is stipulated by such route network elements as stops because they are the places of urban trips concentration. All other characteristics of the RN of MPT including the Distances Between Adjacent Stops (DBAS) and the Distances Between a Pair of Stops (DBPS) are derived from the stops spatial location. The first parameter, which is determined by the stops location, is the DBAS regardless of their belonging to different MPT types – bus, trolleybus or tram.

In terms of walking time to and from a stop it can be assumed that this time can be neglected when studying the population allocation regularities.

Therefore, distribution of work trips by public transport can be taken as the basis to determine the reasons for trip length frequency distribution by example of Ukrainian cities of the 21st century.

The forming of the trip length distribution function can be presented as a process of transforming the square matrix of DBPSes in a linear trip distance set that can be described by corresponding distribution law. The gist of transformation process is that each distance l_{ij} between stops i and j is repeated in a linear trip distance set as many as h_{ij} times if h_{ij} is a quantity of trips between stops i and

j. Unfortunately, absence of a real trip matrix for any Ukrainian city makes this transformation impossible.

So, to achieve the goal it is necessary to investigate and explain the reasons for DBASes distribution and then compare last one with the known trip length distributions. In the case of similarity of these distributions it can be considered that the trip matrix has a negligible influence on the allocation function (trip length distribution) and the main reason of the origin of population allocation regularities is the natural processes of enterprises and population allocation in the vicinity of the city centre. Otherwise, it is necessary to determine conditions of transforming DBASes into allocation function, i.e., those trip matrix variants that will provide such a transformation. Analysis of these matrices will provide opportunities to put forward the hypotheses about the way of trip distribution and to check the likelihood of the existing models of trip matrix calculation.

To investigate regularities in DBASes distribution it is necessary to connect DBASes and stops location on the city territory that will describe existing dependence between spatial characteristics of a RN infrastructure.

This investigation is based on the hypothesis of randomness of stop plane co-ordinates in the city which are influenced by many factors including surroundings, individual and collective activity of population and enterprises, history, minerals etc. Thus, the applicate coordinates can be neglected.

4. Theoretical background

To achieve the goal it is necessary to theoretically determine the type of distribution law of distances between stops i and j for every possible pair of stops, number of which is N ($\forall i, j \in [1, N]$). Such distances are formed as a sequence of DBASes that are on the route from stop i to stop j:

$$l_{ij} = \sum_{k=1}^{n_{ij}} l_k \qquad (1)$$

where:

l_{ij} – distance between stops i and j (DBPS), km;

n_{ij} – number of DBASes between stops i and j;

l_k – factual DBAS k, km.

DBAS l_k is a distance on a route network. That's why determining the l_k properties should be based on the investigation of regularities of stops distribution over the city. It is this distribution that is the starting point for the formation of required regularities in DBPSes. According to the hypothesis in Problem Statement, distribution of stops over the city can be characterized by the bivariate distribution of random coordinates (X;Y). Taking into account a great number of factors that determine stops location coordinates, it is reasonable to assume that a hypothetical law of their distribution is an asymptotically bivariate normal law.

Since the stops coordinates are a random variable then the distance between public transport stops will also be a random variable and in majority of cases can be presented as the shortest distance between adjacent stops. Individual cases of unstraight DBASes cannot significantly affect the random variable l_k distribution so they can be neglected in the theoretical part of investigation.

If each stop is the beginning of coordinates and measure existing distances to adjacent stops on RN avoiding repeated measurements of the same distances, the overlaying of all possible initial stops at the beginning of coordinates will result in the distribution of final stops in neighbourhood of a certain radius which equals the maximum DBAS in the city.

If the distribution of (X;Y) is bivariate normal circular with mean values of both co-ordinates $m_x = m_y = 0$ and standard deviations of co-ordinates $\sigma_x = \sigma_y$, DBAS can be presented as $l_k = \sqrt{X^2 + Y^2}$ and described by Rayleigh distribution [17]:

$$F(l_k) = 1 - e^{-\frac{l_k^2}{2\sigma^2}} \quad (l_k > 0) \qquad (2)$$

where:

σ – Rayleigh distribution parameter – standard deviation of l_k [17], km^2.

The first condition for the bivariate normal circular distribution of DBASes is satisfied automatically for distances between stops due to sequential investigation of stops of the DBASes for MPT as the beginning of coordinates and their imaginary overlaying at one point. As for the second condition which defines the scale and shape of distribution, we could not theoretically determine it.

Despite of lack of the proof for the second condition, the hypothesis is that DBASes correspond to Rayleigh distribution. However, due to the specific formation of this random variable, when the initial and final stops of DBASes are close to each other, the requirement for bivariate normality of the initial co-ordinates distribution is not so rigid. It is important that this condition should be met after imaginary overlaying of initial stops of DBASes at the beginning of coordinates. In this case, the proximity of transformed coordinates to bivariate normal distribution can be confirmed by coincidence between empirical DBASes distribution and theoretical Rayleigh distribution.

However, in our case the Rayleigh distribution is not further suitable as analytical expression for the DBPS l_{ij} cannot be obtained by using it because the known analytical transformation of this distribution has not been found. The use of Rayleigh distribution as the basis for further calculations is sophisticated by the fact that the DBAS l_k enters the expression of l_{ij} in a linear form and in the relative Rayleigh distribution function they are squared. That's why it is reasonable to converse to another distribution with linear DBAS as an argument. The conversion of squared DBAS into linear ones can be performed using the Taylor expansion [17]:

$$l_k^2 = f(l_k) = f(a) + f'(a)\cdot(l_k - a) \quad (3)$$

where:
a – point of expansion;
$f'(a)$ – the first-order derivative value in point a, i.e $f'(l_k) = (l_k^2)' = 2l_k$.

To make the expansion, the point-value of 0.5 km should be chosen. It will result in the shift of factual DBAS by 0.25 km. and the value of $f'(a)$ is 1 after Taylor expansion:

$$l_k^2 = l_k - 0,25 = l'_k \quad (4)$$

If $\lambda = 1/(2\sigma^2)$ Rayleigh distribution will converse into exponential:

$$F(l'_k) = 1 - e^{-\lambda(l_k - 0,25)} = 1 - e^{-\lambda l'_k} \quad (5)$$

where:

$$l'_k = l_k - 0,25 > 0 \quad (6)$$

It is important for the linearization of squared DBAS to have the constant in equation (6). This constant determines the shift of the random DBAS to the right relating to the beginning of coordinates. Taking into account the constant mentioned, equation (1) looks like

$$l_{ij} = \sum_{k=1}^{n_{ij}} l'_k + n_{ij}\cdot q \quad (7)$$

where:
$q = \text{const}$ – the shift parameter of the exponential distribution of the continuous random variable l'_k, km.

The first part of the equation (7) is convolution of the exponential distribution of the random part of DBAS, which is Erlang of n_{ij} degree distribution

$$G_{(n_{ij})}(l') = \frac{\lambda(\lambda l'_k)^{n_{ij}-1} e^{-\lambda l'_k}}{(n_{ij}-1)!} \quad (l'_k > 0,\ n_{ij} = 1, 2, \ldots) \quad (8)$$

where:
λ – parameter of exponential distribution of the random part of DBAS [17].

The Erlang distribution is a special case of gamma distribution when shape parameter, in our case n_{ij}, is an integer. But the quantity of DBASes is constant only when considering one distance between stops l_{ij}. When considering all possible l_{ij} for the city, the quantity of DBASes that came on the route from stop i to stop j will be a random variable. Random variable l_{ij} is the probabilistic mixture of exponential distributions [16]. In general, summation of variables that have Erlang distribution and $n_{ij}\cdot q$, according to formula (7), will result in the loss of shape parameter integrity and conversion of DBPSes distribution into gamma distribution with scale parameter b and shape parameter c[22]:

$$F(l) = \frac{1}{\Gamma(c)} \int_0^{bl} t^{c-1} e^{-t} dt \quad (9)$$

$$f(l)=\frac{b^c l^{c-1}e^{-bl}}{\Gamma(c)} \quad (l>0,\ b>0,\ c>0) \qquad (10)$$

It indicates the known population allocation regularities at the stage of stops location in the city. It is necessary that the discrete part in expression (7) $n_{ij} \cdot q$ has not influenced the shape of distribution l_{ij}. The influence of this part of (7) upon the form of the DBPSes l_{ij} distribution curve can be different and even may cause several peaks in the resulting distribution. Such an effect is possible if DBASes exponential distribution shift parameter q is considerable. If this parameter is small, i.e. several times less than the mean value of l'_k, the discrete part will not cause the change of the distribution l_{ij}.

To test this statement it is necessary to select the appropriate value of the l'_k shift parameter and check the possibility of the description of the DBASes variation using exponential distribution. In this case it is logical to select the minimum DBAS in the city as a shift parameter:

$$q=\min(\forall l_k, k\in\{1,M\})=l_{\min} \qquad (11)$$

where:
M – total quantity of DBAS in the city;
$l_{\min}$ – minimum DBAS in the city.

If the empirical data of DBASes having such a shift parameter are described by exponential distribution, the curve of gamma distribution of DBPSes l_{ij} will be the same as shown in Fig. 1. Thus, if all above theoretical background is practically proved, it will be proved that the common type of the function of population allocation in the cities is determined by distribution of MPT stops over the city and trip matrix doesn’t influence significantly on the allocation function.

5. Experimental test of hypotheses

To receive empirical data the models of RN of MPT in the cities of Sumy, Kharkiv, Kyiv and KryvyiRih of Dnipropetrovsk region that were designed using PTV Vision® VISUM software are used. These models allow getting the stop co-ordinates, DBASes and matrices of distances between pairs of stops [13].

The first stage of experimental investigations is the testing of the correspondence between DBASes empirical distribution and theoretical Rayleigh distribution, which illustrated in Fig. 4. [20]. Results of such correspondence testing for other cities mentioned are analogous.

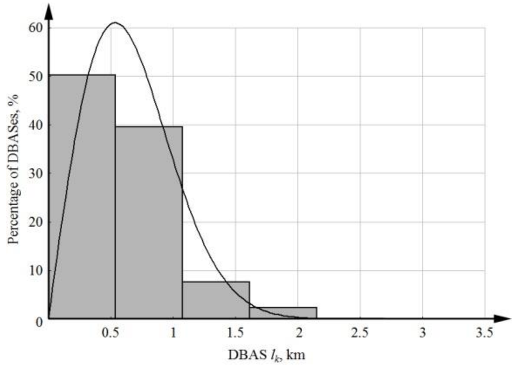

Fig. 4. Rayleigh distribution of DBASes in Kharkiv MPT

The hypothesis of correspondence between Rayleigh and DBASes distributions was correct for all the cities what confirms both the randomness of stops distribution over the city area and proper theoretical background. The parameters of Rayleigh distribution of DBASes in cities investigated are shown in Table 1.

Table 1. Characteristics of Rayleigh distribution of DBAS in Ukrainian cities

Distribution parameter	City			
	Sumy	Kryvyi Rih	Kharkiv	Kyiv
Standard deviation, km	0.42	0.57	0.54	0.41
Mathematical expectation, km	0.554	0.746	0.682	0.557
Probability of Pearson criterion	0.37	0.11	0.18	0.12

The next stage of experimental investigations is the testing of the usefulness of exponential distribution with a shift parameter which equals the minimum DBAS in each city (q = l_{min}) for the description of the DBASes set. Such a shift unlike the shift of

0.25 km preserves the factual DBAS set in equation (6).
The example of such a distribution chart, which is analogous for all the cities investigated, is presented in Fig. 5; the distribution parameters are given in Table 2.

Table 2.Characteristics of exponential distribution of DBAS in Ukrainian cities when a shift parameter is $q = l_{\min}$

Index	City			
	Sumy	Kryvyi Rih	Kharkiv	Kyiv
Minimum DBAS $l_{\min}$, km	0.043	0.079	0.074	0.070
Parameterλ	2.16	1.63	1.96	2.18
Mathematical expectation, km	0.511	0.667	0.608	0.487
Dispersion, km^2	0.540	0.334	0.408	0.263
Probability of Pearson criterion	0.77	0.44	0.25	0.31

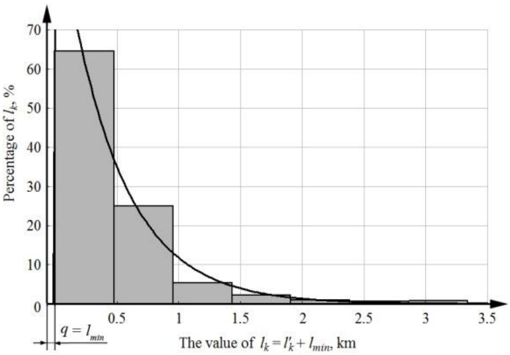

Fig. 5. Exponential distribution of DBASes in Kyiv when shift parameter is $q = l_{min}$

Theoretically such a result is evidence of gamma distribution of DBPSes but it requires experimental testing on the basis of a proper matrix. The example of the density of DBPSes distribution for Sumy is shown in Fig. 6.
The test of a possibility to describe DBPSes when using gamma distribution was made in other cities where the density charts of distribution are analogous to Fig. 6 [22]. The parameters of the distribution received are given in Table 3.
Thus, the experimental investigations do not contradict the theoretical background of population allocation regularities. In this case DBPSes in cities investigated are gamma-distributed that meets generally accepted trip length frequency distribution function. It means that trip matrix does not significantly influence on population allocation function, i.e. the use of transportation factors to calculate trip is not sufficiently grounded.

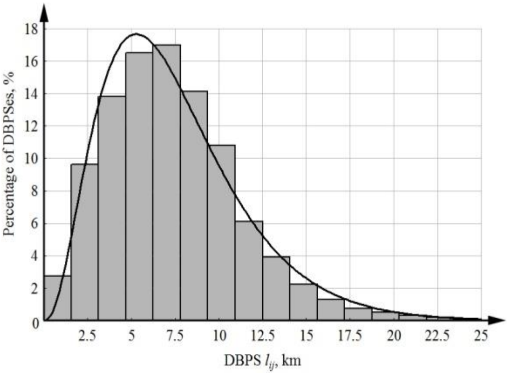

Fig. 6. Gamma distribution of DBPSes in Sumy

Table 3.Parameters of gamma distribution of DBPSes

Index	City			
	Sumy	Kryvyi Rih	Kharkiv	Kyiv
Scale parameter λ	2.20	7.43	3.59	4.10
Shape parameterα	3.40	2.53	3.25	3.40
Mathematical expectation, km	7.331	18.415	11.317	13.586
Dispersion, km^2	14.466	122.860	32.611	44.816
Probability of Pearson criterion	0.47	0.53	0.28	0.14

6. Conclusion

The investigations have theoretically and experimentally confirmed that the characteristics of transport supply of MPT, i.e. distribution law of DBPSes, are the basis to determine the regularities of population allocation in Ukrainian cities. It indicates that these regularities are the result of spatial allocation of city RN elements rather than the influence of transportation factors on the formation of Origin-Destination pair for population. When modelling the demand for MPT services, it is necessary to determine such states of trip matrix that do not converse gamma distribution of DBPSes into any other distribution and use the interval estimation of transport demand.

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