THE STUDY OF TRACK GAUGE INFLUENCE ON LATERAL STABILITY OF 4-AXLE RAIL VEHICLE MODEL

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Abstract: *Analysis of lateral stability of rail vehicle model is the subject of present paper. The method used by the author is based on bifurcation diagrams creation and analysis. The continued study of stability of vehicle model in straight track and curved track and form of the results presentation are original features of the method. Results for the straight track and wide range of radii of the curved track are presented jointly on the combined bifurcation diagrams in this paper. Multibody dynamics software VI-Rail was used for numerical analysis. Passenger vehicle model and track models were created. Analysis of track gauge influence on vehicle model stability is main aim of this paper. But analysis of possibility to adopt the method worked out earlier to the newly used numerical code and model of 4-axle vehicle is the aim either.*

Key words: numerical simulation, rail vehicle non-linear stability, critical velocity, track gauge.

1. Introduction

Improvement of real vehicles dynamics by means of computational methods, being constantly developed, is conducted for a long time [1-6, 10, 11, 13, 17, 18, 20-29]. Creation of mathematical and numerical model of the vehicle and then analysis of results of the numerical simulation is the first stage of investigations, usually. Reduction of entire venture cost and research time shortened are main benefits from the theoretical analysis. This paper represents new results obtained by the author by means of numerical simulations. Well known bifurcation approach to stability analysis was applied [8, 34-38]. Essence of the method consists in creating and then analysis of bifurcation plots. Such plots enable to determine chosen parameter changes in so-called active parameter domain [37, 38]. Leading wheelset's lateral displacement y_p (of the 4-axle vehicle) is the chosen, observed and recorded parameter. It represents either stable or unstable solutions in the vehicle velocity v (bifurcation parameter) domain. Constant value of the observed parameter y_p may appear for particular v value. The solutions are called stable stationary in such cases (Fig. 4c). On the other hand, periodic solutions are possible too, (Fig. 4d). Lateral displacements y_p of the wheelset take a limit cycle character then. Maximum absolute value of lateral displacements $|y_p|$/max and peak-to-peak (p-t-p)y_p value of the lateral displacements are then recorded in the study. Both these values are obtained from

single simulation performed on individual route (of particular curve radius R) for constant value of velocity v. So, in order to present changes of the parameters $|y_p|$/max and (p-t-p)y_p in whole range of vehicle velocity and for particular R several dozen simulations have to be done. Example result of such sort is shown in Figures 4a and 4b. Next, other R values should be processed in the same way. Finally, results obtained for different routes (with different R value) can be presented on a pair of diagrams that take account of a whole considered range of the curve radii. In these researches the complete diagrams of such type represent results obtained for radii R that start from $R = 1200$ m and finish for a straight track, where $R = \infty$. Such a pair of the complete diagrams is named the „stability map" [34]. The stability maps were adopted as a form of the results presentation in Figures 5 to 10.

The problem of rail vehicle lateral dynamics and its stability is to many people connected with running along a straight track. Motion along a curved track is supposed to be of a quasi-static (stable stationary) nature at the same time. Despite strong conviction of many that it is true, recently more researchers have studied and discussed the problem of railway vehicle stability in a curved track. This article confirms legitimacy and usefulness of rail vehicle lateral stability analysis in a curved track.

The method and idea of research was widely presented by authors' team in [8, 34-38]. Discrete two-axle freight car model as described in [17, 33]

was utilized in the mentioned works. The answers about influence on vehicle stability of parameters in suspension, wheels and rails wear, rail inclination and many more were the aims in the above cited works. In present investigations the idea was kept, but object of the research was changed. Four-axle passenger car model was build up with engineering software code VI-Rail. Vehicle model is supplemented with discrete models of vertically and laterally flexible tracks. The VI-Rail preprocessing interface used by the author in present study makes the process of numerical model building significantly easier and faster than traditional deriving equations of motion and next their implementation into the numerical (simulation) software. Here, when vehicle and track models are completed, simulation process begins. VI-Rail generates equations of motion for the adopted model structure according to Legrange'a I order formula (formalism). Next the equations of motion are solved. Well known and verified method, specialised for ordinary differential equations (ODE) of „stiff" type was chosen to solve the equations. This method is based on Gear's algorithm, widely applied in multi-body system (MBS) simulation tools.

The method of mathematical description (ODE), numerical procedures to calculate the contact forces (FASTSIM program), numerical method to take account of the non-linear wheel-rail contact geometry (RSGEO program) and equation of motion solver (Gear's procedure) correspond to these applied for the two-axle vehicle model mentioned earlier. So hopefully, despite significant differences in construction (structure) between two- and four-axle vehicles, results can be compared.

It is well known problem how to generalize simulation results that concern single or countable number of the cases. On the other hand simulation became the most popular method of the studies for large multidimensional systems. In some sense there is no choice. We are fated to use simulation as no real alternative exists. Nevertheless, one always has to remember when generalization is formulated, that just limited number of the cases was examined and generalization is for sure at least limited or conditional. Despite this, very many commercial software packages for MBS became available on the market nowadays. They are used to solve problems both in the engineering and

research. In case of the research a serious formal problem appears, however. Because codes of the commercial software are not available to their users, so in fact they do not know how their MBS is modelled. Consequently the user cannot conclude in a formal scientific manner basing on the results obtained with such software. The issue is called the black box problem in the literature. In order to reduce this problem the benchmarking is used. It does not solve the problem formally, however it is a kind of the software verification and makes the software more credible. In case of the MBS codes used in railway vehicle dynamics the specific benchmark test were developed, e.g. [9, 12, 13]. All highly ranked commercial codes, including VI-Rail, are more or less subject to these tests [13]. In order to reduce the black box problem the author is going to go even further in his study. He is going to compare results presented in the current article with the results from the ULYSSES program for MBS. Code of the uncommercial program ULYSSES [7] (many information also in [39]) is at the author's disposal. Use of the ULYSSES program will be the next stage in the author's research on the influence of track gauge on railway vehicle stability properties.

Track gauge has a great influence on the performance of rail vehicle. It is especially significant in case of the lateral dynamics analysis. Smaller track gauges in comparison to the nominal value may possibly reduce the self-exciting vibrations appearance and extend range of velocity for which stable stationary solutions exists only. To confirm or deny this supposition is the main detailed aim of this article.

2. The Model

The MBS was build up with the engineering software VI-Rail (ADAMS/Rail formerly). This is the environment which enable users to create any rail vehicle – track model by assembling typical parts (wheelsets, axleboxes, frames, springs, dampers and any other) and putting typical constrains on each of kinematical pairs. Exemption of users from deriving the equations of motion by themselves is the main advantage of this software. This and many other advantages of the software reduce the time devoted to build the model significantly. The simulation model being tested in the paper consists of vehicle and track. Complete system has 82 kinematic degrees of freedom.

2.1. Track model

Discrete, two level, vertically and laterally flexible track models were assumed (Fig. 1). But models of track flexibility are simplified. For low frequency analysis (less than 50 Hz) simplified track model is accepted when dynamics of vehicle motion is considered. It means also that no Bernoulli or Timoshenko beams were applied. Rails and sleepers are treated as a lumped masses (m_r, m_s) of the corresponding rigid bodies. No track irregularities are taken into account. Periodic support of the rails in real track is neglected in the model too. So, the non-inertial type of the moving load is adopted here. Linear elastic springs and dampers connect the track parts (rigid bodies) to each other (see Appendix). Similar approach is used in many works in vehicle dynamics where just low frequency deformations of the track are of the interest, e.g. [4, 5, 13, 15, 28].

The track has got nominal UIC60 rails with a rail inclination 1:40. Each wheelset is supported by a separate track section consisting of two rail parts and sleepers that correspond to 1m length of typical ballasted real track. Every wheelset – track subsystem has homogenous properties and is independent from one another. Each route of

curved track model is composed of short section of straight track, transition curve and regular arc. Constant value of superelevation depending on curve radius value is applied for each curved track route (Table 1).

2.2. Vehicle model

Typical 4-axle passenger vehicle model is employed in the simulations (Fig. 2). Vehicle model corresponds to the 127A passenger car of Polish rolling stock. Bogies of the vehicle have 25AN designation in Polish terminology. The model consists of fifteen rigid bodies representing: carbody, two bogies with two solid wheelsets and eight axleboxes. Each wheelset is attached to axleboxes by joint attachment of a revolute type. So rotation of the wheelsets around the lateral axis with respect to axleboxes is possible only. Arm of each axlebox is attached to bogie frame by pin joint (bush type element). They are laterally, longitudinally and rotary flexible elements. The linear and non-linear characteristics of the primary and secondary suspension are included in the model (see Appendix). They represent metal (screw) springs and hydraulic dampers of primary and secondary suspension.

Table 1. Curve Radii Tested and Track Superelevations Corresponding to Them

Curve radius R [m]	1200	2000	3000	4000	6000
Superelevation h [m]	0.16	0.155	0.110	0.077	0.051
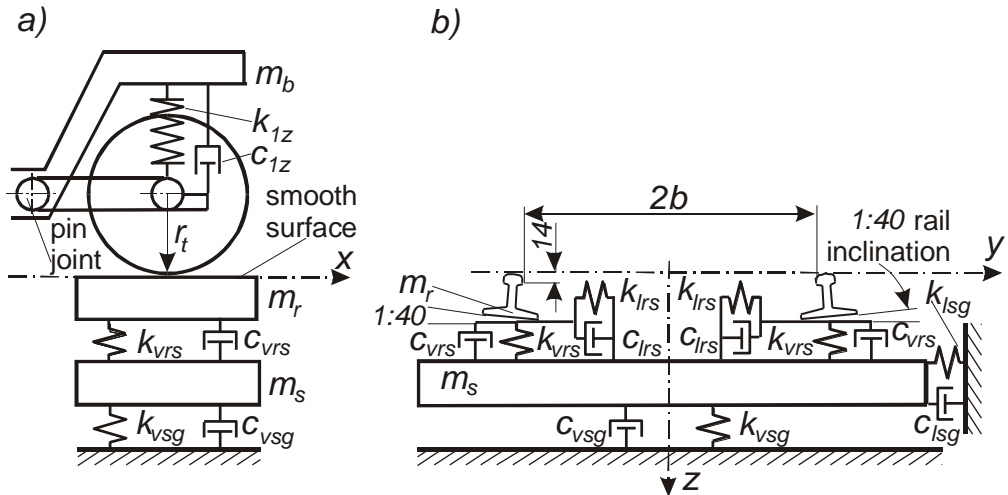
Fig. 1. Track Nominal Model Structure: a) side view, b) cross section view

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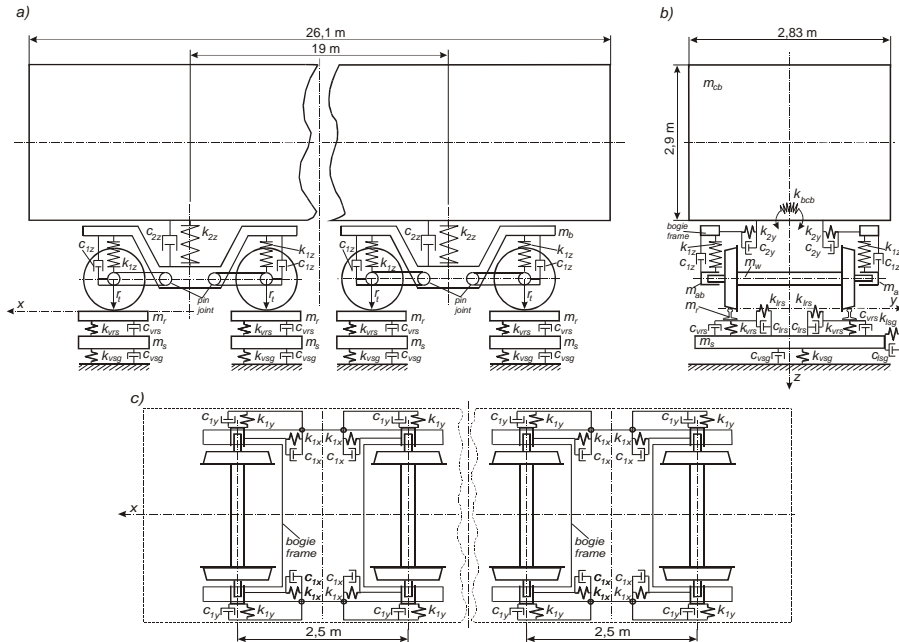

Fig. 2. Vehicle – track Nominal Model Structure: a) side view, b) front view, c) top view

In addition torsion springs (k_{bcb}) are mounted between car body and bogie frames. To restrict car body – bogie frame lateral displacements, bumpstops with 0.03 m clearance were applied (not visible in Fig. 2). A new S1002 wheel and UIC60 rail pairs of profiles are considered [30, 31]. Non-linear geometry of wheel - rail contact description is assumed. Contact area and other contact parameters are calculated with use of RSGEO subprogram (implemented into VI-Rail). To calculate wheel-rail contact forces, results obtained from RSGEO are utilized. In order to calculate tangential contact forces between wheel and rail, so called non-linear simplified theory of the rolling contact by J.J. Kalker is applied. It is implemented in the computer code FASTSIM [14, 22, 23] used worldwide.

2.3. Parameters arrangement in simulations

The VI-Rail software enables users to arrange and adjust many of the computational parameters. The simulation specification, selected method of mathematical description of real elements, and equation solver procedure choice have significant influence on the final results. List of the parameters applied in each simulation is presented below.

− Simulation time – 15 s;
− Number of Steps – 2500;
− Contact Configuration File – mdi_contact_tab.ccf;
− Track Type – flexible;
− Wheel – rail coefficient of friction – 0.4;
− Young Modulus – 2.1E+11;
− Poisson's Ratio – 0.27;
− Cant Mode – Low Rail;
− Solver Selection – F77;
− Solver Dynamics Setting: Integrator – GSTIFF, Formulation – I3

The VI-Rail program enable to choose the integration procedure. Among a few possible schemes the Gstiff procedure was applied. It utilizes the Gear's procedure, applied for the Adams backward-difference method [19]. The method executes the predictor-corrector algorithm of calculations. Because of wide region of absolute stability the method is dedicated for numerical integration of large non-linear systems of ordinary differential equations „stiff type" (typical for vehicle-track models).

3. The method of stability analysis

Basically, the method used by the author in the present study is based on the bifurcation approach to the analysis of rail vehicle lateral stability. This approach is widely used in the rail vehicle lateral dynamics, e.g. [8, 10, 11, 17, 25, 26, 28, 29 and 34-39]. This method takes account of the stability theory, however is less formal than the theory but more practical instead. In another word, it also makes use of some assumptions and expectations from the system being studied, which are based on the already known general knowledge about the rail vehicle systems. In accordance with that, building the bifurcation plot is the main objective here but formal check if the solutions on this plot are formally stable is not such an objective. That is why one does not adopt some solution as the reference one in this approach and then does not introduce some perturbation into the system to check if the newly obtained solution stays within some narrow vicinity of the reference solution, what definition of the stability (theory) would require. The approach assumes that any solution typical for railway vehicle systems (stationary or periodic) is stable. Such assumption could be accepted based on the understanding within the railway vehicle dynamics that periodic solutions are the self-exciting vibrations that are governed by the tangential forces in wheel-rail contact. Thanks to it, the self-exciting vibrations theory can be used to expect (predict) typical behavior of the system. Only when serious doubts about stability appear, formal check for the stability (with the initial conditions variation to introduce the perturbation) is then performed.

On the other hand, another very important reason exists to vary the initial conditions. This is the need to get all the solutions in order to build the complete bifurcation diagram. Varying the initial conditions carefully, widely and knowingly enables to obtain all multiple solutions for the particular velocity v value. It is the case for both the stable and unstable solutions as well as stationary and periodic solutions known in the railway vehicle dynamics. Repeating the procedure for all velocity range makes it possible to get the bifurcation plots, as e.g. Figs. 4a and 4b. As it is seen on these figures the bifurcation plots represent stability properties of the system as they show precisely areas of the stable and unstable solutions, both the

stationary and periodic ones. The crucial elements on the plots are saddle-node bifurcation and subcritical Hopf's bifurcation that correspond to the stable solutions lines and velocities v_n and v_c, respectively (see Figs. 4a and 4b). The v_n and v_c are well known in the rail vehicle stability analysis non-linear and linear critical velocities, respectively.

It is worth adding that bifurcation approach, focused on building the bifurcation diagrams, is also suitable to represent less typical behaviors of the railway vehicle systems, as chaotic ones. Then more formal activities are necessary, however.

Interpretation and extension to the above statements, including physical aspects, can be found in [37, 38 and 39] where thorough considerations are presented, which enable dipper understanding of the rail vehicle lateral stability analysis. The method presented in [38] is more formal than that in [37], while both methods are discussed in [39]. In [39] different methods to determine non-linear critical velocity v_n are also discussed.

According with the above the information is given below, referring to the considered objects, on how the bifurcation plots for the needs of the present paper were built. The first part refers to the straight track case, while the second one to the circular curve case.

In the method used, the first bogie's leading wheelset lateral displacements y_p are observed and recorded in time domain (as in Figs. 3, 4c and 4d). The stable stationary solutions can appear (Figs. 3a and 4c) in the system. They are typical for vehicle velocity less than the critical value v_n. Sometimes in a curved track for velocity higher than the critical one they appear as well. In case of the stable stationary solutions zero lateral displacements and peak-to-peak values in straight track are observed (Fig. 3a). In addition, the tested vehicle model is example of hard excitation system, e.g. [37]. It means that some minimum value of initial conditions have to be imposed to initiate periodic solutions (self-exciting vibrations in real system). Alternatively, some other perturbation in the system has to be introduced. Thus, in order to initiate vibrations the straight track test section has got singular lateral irregularity situated 200 m from the track beginning. The irregularity has half of sine function shape. Its amplitude equals 0.006 m

and wave length 20 m. So all wheelsets are shifted in lateral direction in straight track during the irregularity negotiation (about 2.5 s from the route beginning in Fig. 3).
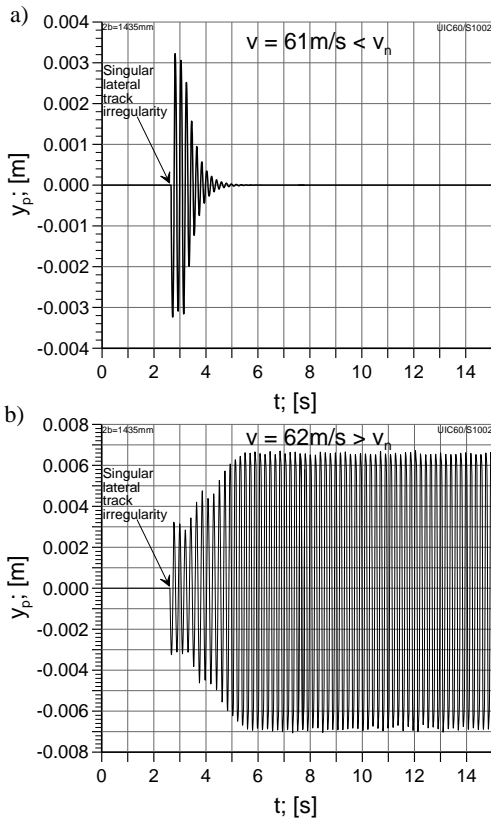
a)

b)

Fig. 3. Leading Wheelset Lateral Displacement in Time Domain for Velocity: a) Lower, b) Bigger Than the Critical Value v_n. Straight Track Motion

Afterwards the wheelsets tend to central position (for velocity lower than the critical value v_n, Fig. 3a) or lateral displacements increase and may change periodically (for velocity equal or bigger then the critical value v_n, Fig. 3b). The smallest motion velocity for which stable periodic solutions (limit cycles) appear is accepted as a critical value v_n. The step of velocity changes equal 0.1 m/s was applied in particular simulations processes. Hence, the accuracy of critical velocity value determination is equal to 0.1 m/s, too. Existence of periodic solutions (self-exciting vibrations in real

object) means also energy dissipation in the system. Two conditions have to be met to initiate the periodic solutions. The first is some minimum value of energy delivered to the system (minimum velocity value in the tested system). The second is application of some minimum value of initial excitation (e.g. track irregularity of sufficient amplitude). The periodic solutions (limit cycles) are generally not desirable in real objects because vibration is always worse than stationary behaviour. On the other hand limited (and constant) value of the amplitude enables safe vehicle motion. Consequently, such type of solutions can be accepted as being the stable one. Amplitude as well as other limit cycle parameters can constitute some indicators of the system state. The maximum of wheelset lateral displacements (y_p max) and their peak-to-peak values (p-t-p y_p) are utilized in the method.

Non-zero lateral displacements (vibrations) appear in the initial part of curved track, usually (as for R = 2000 m in Fig. 4c). It is caused by the lack of balance between lateral (with respect to track plane) forces acting on the vehicle in curve. Another word, the lateral components of centrifugal and gravity forces do not neutralize each other. Stationary value of wheelset lateral displacement become established after enough long time (12 seconds in Fig. 4c). So, stable stationary non-zero solutions exist in curved track for velocity lower than v_n, usually. Exceeding the critical velocity value v_n means self-exciting vibrations appearance. It causes for vehicle model transfer (bifurcation) of solutions from the stable stationary to the stable periodic ones (Figs. 3b and 4d). The wheelsets move periodically along lateral axis y and rotate round their vertical axis z. It is the form of energy dissipation, typical in wheelset-track system. Similarly to the straight track case, two conditions should be fulfilled to initiate the self-exciting vibrations in circular curve too. The first one is some minimum velocity value of wheelset (vehicle). The second one is sufficiently big initial excitation of the wheelset. For the analysis of stability in curved track sections it is not sure if the initial excitation at the beginning of straight track section can play its role sufficiently. On the other hand transition curve negotiation appeared to be quite enough excitation to initiate periodic solutions in the regular curve (if vehicle velocity is

equal or exceeds the critical value v_n). That is why the lateral irregularity in straight track is not applied in curved track cases.

In practice, to obtain the results for curved track, compound routes had to be applied. It is the consequence of VI-Rail software feature. It cannot start calculations in a curved track directly. Therefore simulations which finish in a curved track have to begin in straight track section (first 3 seconds in Fig. 4c and 4d). Then they pass through transition curve and finally regular the curved track section (R = const.) begins. If the wheelset's lateral motion takes form of limit cycle and exists until end of the test time (15 s usually), the state represents and is called the stable periodic solution (Fig. 4d).

Constant value of velocity is taken in each simulation. Two parameters – maximum of leading wheelset lateral displacement absolute value ($|y_p|$max) and peak-to-peak value of y_p (p-t-p y_p) are determined. Diagrams of these parameters in velocity domain (Fig. 4a and 4b) are created. Both graphs include the lines matching circular track

sections of radii from R = 1200 m to $R = \infty$ (straight track). So a few lines are presented in the complete diagrams usually. Each line is created following a series of single simulations for different v and the same route. The range of v starts at low velocities, passes critical value v_n and terminates in velocities v_d, called sometimes the derailment velocity.

The value v_d does not mean the real derailment, however. This is the lowest value of velocity for which results of simulations take no limit cycle shape and no quasi-static shape either. But the vehicle motion is possible often. In addition, if wheelset lateral displacements take large values the climb of wheels on rail head could happen. In curved track, outer wheel may be lift up and can loss contact at velocity v_d sometimes. It is effect of acting centrifugal force and it is treated as a derailment too. The pairs of diagrams like those visible in Figure 4a and 4b, which include results for all tested curve radii, are called a „stability maps" and selected as a form of results presentation.
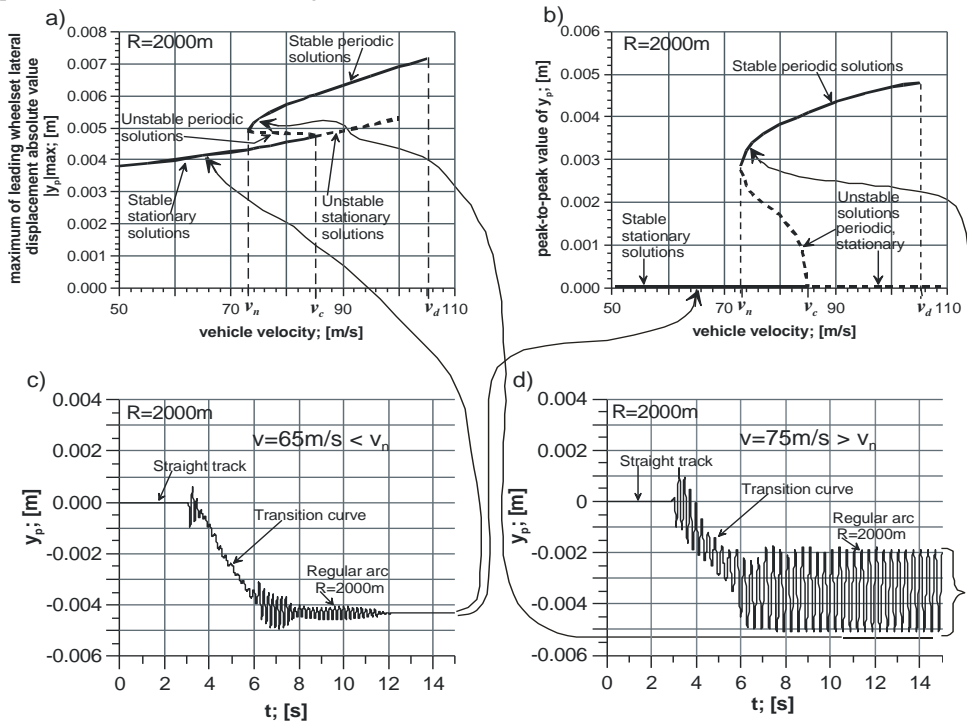

Fig. 4. Scheme of Creating the Pair of Bifurcation Plots Useful in The Curved Track Analysis

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The meaning of stable motion of vehicle in the current research should be expressed here, now. Just stable stationary solutions (constant value of wheelset lateral displacement y_p) or stable periodic solutions (limit cycle of y_p) are assumed to describe stable vehicle motion. Any other solutions are assumed to be the unstable ones. The periodic motion of wheelset corresponding to its limit cycle is not desirable in real vehicle exploitation of course. On the other hand, limit cycle in the stability analysis means constant peak-to-peak value and frequency of the wheelset lateral displacements. Consequently, if the maximum of wheelset lateral displacement value does not exceed the permissible value, vehicle motion is possible and to some extent safe

Great practical significance has the non-linear critical velocity v_n. It is a good idea to take it at least a bit higher than velocity permissible for real object (maximum service speed of the vehicle). Stable solutions exist in range of velocity smaller and bigger than the critical value v_n. But distance between the critical value v_n and the derailment value v_d can be significantly different in individual tests. This distance depends on the vehicle – track system parameters (see results). From practical point of view the critical velocity should be high and distance between critical velocity and derailment velocity possibly long.

4. The results

Considering the analysis of track gauge influence on the stability, the European nominal track gauge 1435 mm was applied at the beginning. In this case (Fig. 5) critical velocity in straight track appears at 61.7 m/s. Stable stationary solutions in straight track exist for velocity lower than 61.7 m/s, only. Above this value and straight track periodic solutions exist until velocity 130 m/s. The wheelset's maximum lateral displacements y_p in these conditions reach 0.0068 m at critical velocity and increase to 0.0095 m with increase of velocity. At the same time peak-to-peak value reaches 0.013 m at critical velocity, and further increases until about 0.0182 m.

On the route of the largest curve radius $R = 6000$ m, self-exciting vibrations appear at velocity 63 m/s (Fig. 5). So, this value of velocity should be accepted as a critical one, here. Stable stationary solutions exist for velocity lower than 63 m/s.

Wheelset's maximum lateral displacements slightly decrease from about 0.0015 m to 0.0012 m.
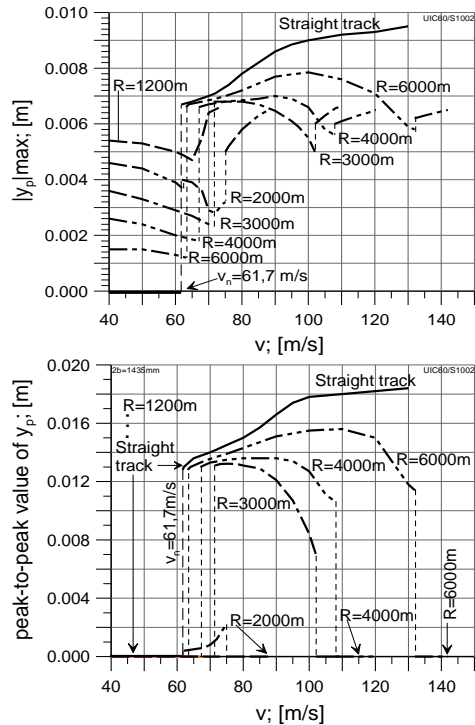

Fig. 5. Maximum of Absolute Value of Leading Wheelset Lateral Displacements and Peak-to-Peak Value of Leading Wheelset Lateral Displacements Versus Velocity for the Nominal Track Gauge 1435mm

Next they increase to about 0.0068 m, but stable periodic solutions appear, however. The maximum value 0.0079 m appears for velocity 100 m/s, and next decreases to 0.0058 m for velocity 132 m/s. Self-exciting vibrations disappear at this value of velocity and stable stationary solutions exists until velocity 142 m/s. Small increase of lateral displacements is observed from about 0.0058 m to 0.0062 m at the same time.

The second route of large curve radius $R = 4000$ m was tested next. Stable stationary solutions exist until velocity 67 m/s. Periodic solutions appear at velocity bigger than 67 m/s, so this value should be accepted as a critical value on this route. Wheelset lateral displacement achieves 0.0066 m and slightly

increases until 0.007 m at velocity 92 m/s. Then it decreases to 0.0056 m at velocity 108 m/s, while solutions return to stable stationary type. Wheelset lateral displacement increases to 0.0065 m at derailment velocity 120 m/s. Peak-to-peak value of wheelset's lateral displacements achieves about 0.013 m at critical velocity and decreases to 0.011 m at velocity 108 m/s.

In case of the smaller curve radius $R = 3000$ m the wheelset's lateral displacement decreases from about 0.0036 m to 0.0024 m, in the stable stationary solutions range. Self-exciting vibrations appear at velocity 72 m/s (critical velocity on this route). Lateral displacements increase to about 0.0068 m and than decrease to 0.005 m for velocity 102 m/s. Then they increase to 0.006 m and achieve 0.0066 m for velocity 109 m/s. Stable stationary solutions exist for velocity bigger than 102 m/s, however. Peak-to-peak values decrease from about 0.013 m at critical velocity to 0.007 m at 102 m/s.

Next, vehicle model motion on the route of even smaller curve radius $R = 2000$ m was tested. The wheelset's maximum lateral displacements decrease from about 0.0046 m to 0.0038 m for velocity lower than 62 m/s. Above this value stable periodic solutions appear and last until 75 m/s, only. Next stable stationary solutions exist until velocity 88 m/s and increase in the lateral displacements is noticed at the same time. The peak-to-peak values rise from about 0.0005 m to 0.002 m in the narrow range of velocity corresponding to the periodic solutions.

In case of the routes of curve radius 1200 m and less just stable stationary solutions exist. So just the maximum lateral displacements can effectively be observed (p-t-p $y_p = 0$, here).

The same range of simulations has been adopted for track gauge 1432 mm (3 mm less than before). The results are represented in Fig. 6. Critical velocity appears in straight track at $v_n = 61$ m/s. The wheelset lateral displacements y_p increase from about 0.005 m at critical velocity to almost 0.007 m at derailment velocity (the diagrams do not cover whole velocity range, however).

Peak-to-peak values in the straight track increase from about 0.01 m to 0.014 m in the same range of velocity. Critical velocities for all curved track routes have almost the same value, $v_n = 65$ m/s for $R = 2000$ m and 4000 m and 67 m/s for $R = 3000$ m

and 6000 m. But range of velocity in which periodic solutions exists increased in case of curve radius $R = 2000$ m and decreased in case of $R = 3000$ m, compared to the results obtained for the nominal gauge of track.
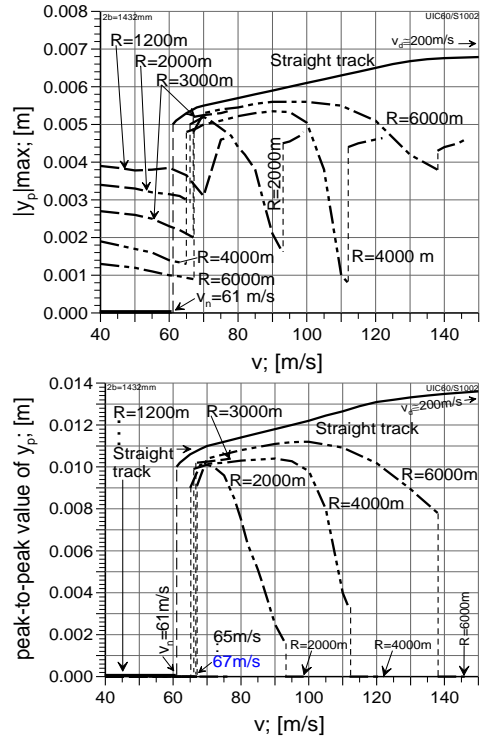

Fig. 6. Maximum of Absolute Value of Leading Wheelset Lateral Displacements and Peak-to-Peak Value of Leading Wheelset Lateral Displacements Versus Velocity for Track Gauge 1432 mm

In the next stage, the track was narrowed down to 1429 mm gauge (Fig. 7). Critical velocity in straight track is a little bit bigger than for previously tested track gauges, i.e. equals $v_n = 63.5$ m/s. Wheelset's maximum lateral displacements increase from about 0.0038 m to 0.005 m in the range of periodic solutions existence. The peak-to-peak values of the displacements increase from about 0.0074 m to 0.01 m in this case. Critical velocity in the curved track routes increase to 67÷74 m/s, in comparison with previously analysed case. But ranges of the velocity in which periodic solutions exist slightly decreased. Generally

significant decrease of the wheelset lateral displacements and peak-to-peak values compared to the nominal track gauge is observed.
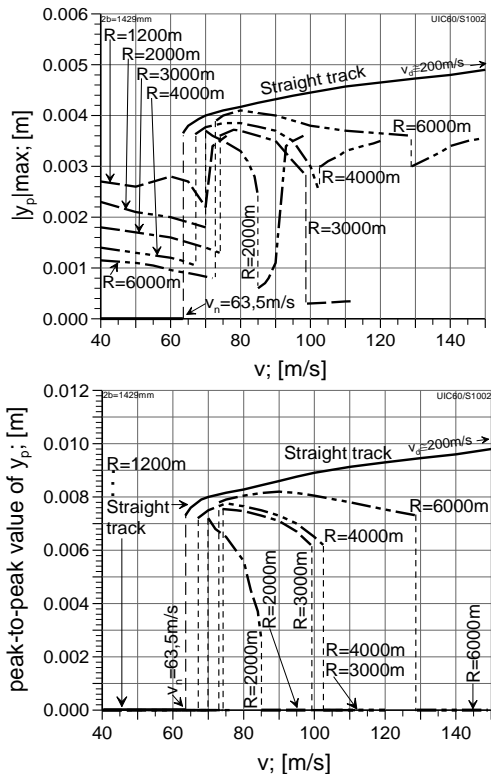

Fig. 7. Maximum of Absolute Value of Leading Wheelset Lateral Displacements and Peak-to-Peak Value of Leading Wheelset Lateral Displacements Versus Velocity for Track Gauge 1429 mm

At last the track was narrowed down to 1426 mm gauge (Fig. 8). Critical velocity in straight track increased to 71 m/s (in comparison with previous case). Wheelset latreal displacements increase from 0.0021 m at critical velocity to 0.0026 m at derailment velocity. Peak-to-peak values increase from 0.004 m to 0.0052 m in the same range of velocity. Just stable stationary solutions for low range of velocity are observed in curve track cases. For some value of velocity (depending on curve radius) loss of stability appear. Some example of unstable solution is shown in Fig. 9. The solution is neither stationary nor periodic.
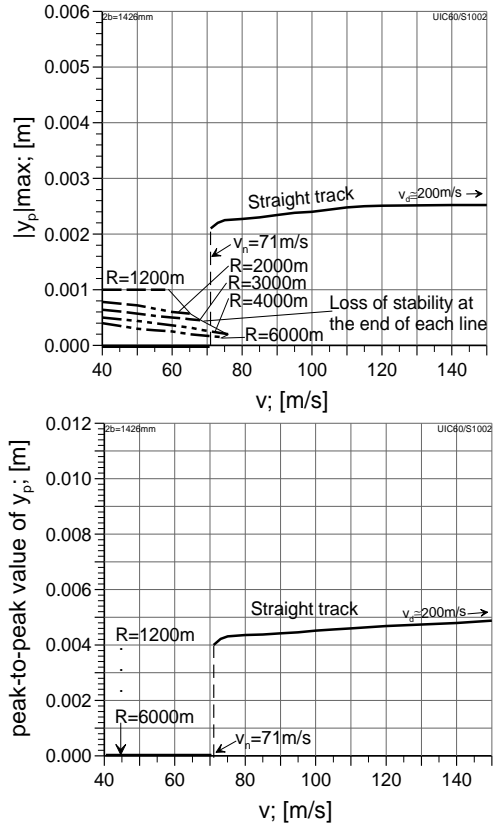

Fig. 8. Maximum of Absolute Value of Leading Wheelset Lateral Displacements and Peak-to-Peak Value of Leading Wheelset Lateral Displacements Versus Velocity for Track Gauge 1426 mm

Additionally, significant changes of solutions are observed, although all model parameters are constant. This can mean that chaotic behavior is observed here.

In order to complete the outlook of track gauge influence on vehicle model stability, similar simulations for the widened track have been done. The track gauge 1438 mm (3 mm bigger than the nominal value) was applied (Fig. 10). Critical velocity in straight track increased to $v_n = 99$ m/s (in comparison with the nominal track gauge case). The wheelset lateral displacements y_p increased from about 0.0108 m at critical velocity to 0.012 m at the end of periodic solutions range of existence (not shown on the diagrams).
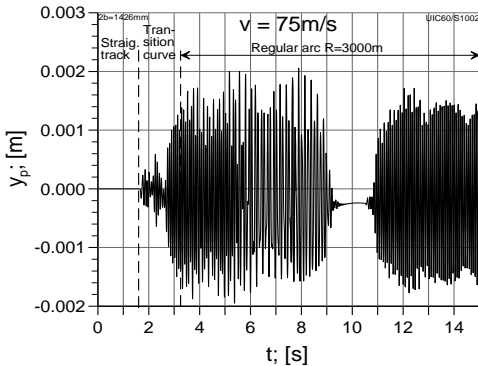
Fig. 9. Leading Wheelset Lateral Displacements in time domain for Track Gauge 1426 mm. Curve radius R = 3000 m, at Velocity 75 m/s. An Example of Unstable Solutions
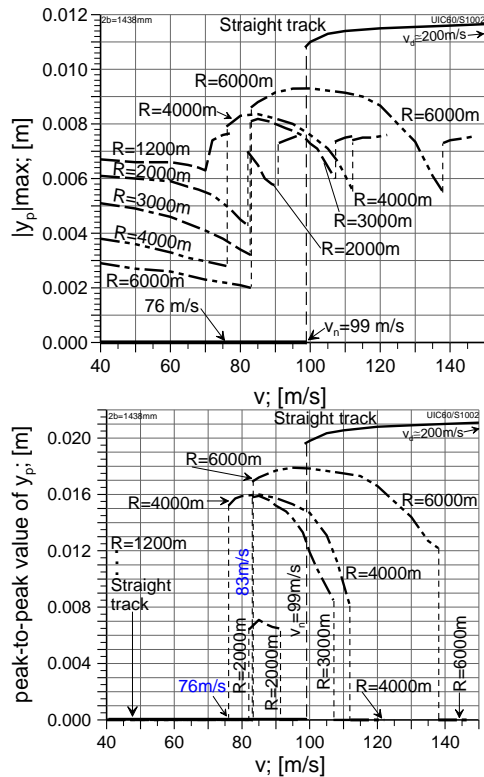

Fig. 10. Maximum of Absolute Value of Leading Wheelset Lateral Displacements and Peak-to-Peak Value of Leading Wheelset Lateral Displacements Versus Velocity for Track Gauge 1438 mm

The peak-to-peak values possess big values too, and reach 0.020 m. Critical velocity in curved track routes increased to $76 \div 83$ m/s, in comparison with the previously analysed cases. The wheelset's lateral displacements and peak-to-peak values are also higher than for the nominal track gauge case, but character of their changes is kept. Stable stationary solutions exist as the only solution for curve radius $R = 1200$ m and below this value.

5. Conclusion

It exists possibility to study lateral stability properties of 4-axle railway vehicle model by use of the presented method in the light of the performed research. High value of lateral stiffness k_{1y} between axle box and bogie frame, characterize this bogie construction. So the wheelset's lateral displacements are transmitted from axle boxes to bogie frame. Consequently, entire bogie rotate around pin joint of bolster (it is observed in the VI-Rail „Animation Controls"). Stable periodic solutions as an effect of self-exciting vibrations existence appear in straight track analysis and the curved track analysis as well. The critical velocity value in straight track is sligthly different to this in the curved track cases. It is similar as compared to 2-axle vehicle model [34-38], where usually critical velocity value in straight track is equal to this in the curved track. But great difference between these two models are values of leading wheelset lateral displacements in the curved track cases. They are always smaller than in the straight track case. They were usually bigger in case of the 2-axle vehicle model. In the real railway system the track gauge increases in comparison to the nominal gauge value as an effect of a few dozen years of the operation, usually. The results show that effect is not completely negative. Although wheelset lateral displacements and p-t-p y_p values increase (as compared to the results in Fig. 5 and Fig. 10), the critical velocity significantly increased in curved track and in straight track as well. It exists theoretical possibility to decrease the real track gauge during the exploitation time period. But smaller track gauges do not reduce risk of periodic solutions (self-exciting vibrations) appearance. The results show, that although the wheelset's lateral displacements and p-t-p y_p values decrease according to track gauge decrease, the critical velocity may increase a few m/s as well as decrease (Fig. 5, 6, 7 and 8). For track gauges 1426 mm just

only straight track motion is possible in high range of velocity value (above the critical value). In curve track cases, stable stationary solutions exists only and loss of stability appear prior the periodic solutions appear. It may be a dangerous situation although the wheelset's lateral displacements are very small (less than 0.001 m). For track gauges smaller than 1426 mm, smaller wheelset's lateral displacements should be expected. But it means wheel flange − rail contact in real object. It is unacceptable state due to big value of the lateral wheel − rail contact forces and risk of derailment. On the routs of curve radius R = 1200 m and less, stable stationary solutions are observed only. It does not mean that phenomena of self-exciting vibrations cannot exist on these routes [34-39]. Great values of centrifugal forces are acting on the vehicle in the tested range of velocity on such routes. This is a reason of unsymmetrical wheel − rail contact forces distribution between inner and outer wheel of each wheelest. The VI-Rail „Animation Controls" tool enable to observe the vectors of contact forces. Decrease of inner wheel − rail contact forces to zero for some velocity value can be observed on these routes. In addition, if we think about wheelset radial position then rolling radii of both wheels have to be different. So the creepages for the inner and outer wheels can differ significantly and consequently the tangential contact forces are different, too. These facts may influence on lack of the self-exciting vibration appearance. It should be also noticed that maximum value of velocity for which stable stationary solution exists on the route of curve radius R = 1200 m, just a few m/s exceeds the critical velocities on the routes of the bigger curve radii. Sometimes it is less than critical velocity values on other curves (Fig. 10). Development and improvement of the presented method of stability analysis is worth putting an effort into. Especially in the context of its application to other vehicle models like locomotive or motor units. As already stated and in accordance with opinions expressed in other works [11], multiple stable solutions on the diagrams may sometimes exist. As also stated, they can be detected by variation of initial conditions of the model. Since the study presented was not focused on such solutions, so it would be interesting to check carefully if it is the case for the model studied in this paper.

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