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INDIRECT OBSERVATION OF REROUTING PHENOMENA IN TRAFFIC NETWORKS – CASE STUDY OF WARSAW BRIDGES

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Abstract: *In this paper we propose estimation procedure in which traffic flows resulting from rerouting model are matched with traffic flows observed during unexpected events. We show practical value of observing a entire cut-set of the transportation network and propose theoretical closed-form formulation of estimation problem for the rerouting model. We apply proposed framework on field-data from Warsaw bridges to observe rerouting phenomena. Most importantly we observed that: a) around 20% of affected traffic flow reroutes, b) rerouting flows are increasing in time, c) drivers show strategic capabilities, d) and maximize their utility while rerouting. All of the which were hypothesized in Information Comply Model (Kucharski et. al., 2014) and are now supported with field observations.*

Key words: *dynamic traffic assignment, rerouting phenomena, information comply model, traffic events, traffic management, real-time traffic*

1. Introduction

Our aim is to model rerouting phenomena so that resulting traffic flows are matched with what is observed during unexpected events. By rerouting we mean changing the currently chosen path after either receiving some information or observing consequences of an unexpected traffic event. In this paper we model rerouting with the Information Comply Model (ICM – (Kucharski et. al., 2014)), which models the phenomena through calculating probability of rerouting for given place and time in the network subject to current situation and information. ICM is parameterized with: a) how information reaches drivers, b) how do they react and c) how do they choose their new routes, which all influence the resulting probability. Detailed representation of ICM model yields realistic results as it covers cognitive process of rerouting, unfortunately it is hard to estimate and validate. ICM models the probability of rerouting at each point in space and time which itself is latent and cannot be observed directly, yet the aggregated outcomes of those decisions can be observed by looking at traffic flows.

Previously proposed framework (Kucharski, et. al, 2014a) in which ICM model was estimated with direct observations of paths is here redefined to work with indirect observations of traffic flows. In the first part we propose theoretical framework on how traffic flows in case of events can be analysed. We come up with a well-founded framework for proposed estimation. In the second part we apply this framework to the field-data. We look at the traffic flows crossing Wisła river in Warsaw, Poland, to see how it changes in case when one of the bridges is blocked. Though the additional input needed for estimation (real-time dynamic traffic assignment model) was not available, we have managed to obtain rough estimates of the most important characteristics of the rerouting phenomena, namely: a) total volume of rerouting vehicles, b) time of rerouting, c) new route choice pattern. Which are a reasonable starting point for further estimation of ICM model.

2. Background

Both travel times and traffic flows can vary significantly due to unexpected events. An unexpected event affects both the supply side and the demand side. The supply side is affected not only at the place of event i.e. via reduced capacity, speed, closure, but it also propagates to upstream part of the network causing delays and spillback. Modelling effect on the supply side is relatively straightforward subject to fixed demand pattern (Corthout et al., 2009) and can provide valuable information on potential effects that event can cause.

The more complicated phenomena due to events take place at the demand side, when drivers react to the events. We call such reaction rerouting - changing the currently chosen path after either receiving some information or observing consequences of a traffic event (similar can be called: en-route rerouting (Snowdon et al., 2012), or adaptation (Gao et al., 2008)). Indeed, when the forecasted performance pattern of travel times and costs, known or only perceived, changes significantly, drivers may react by shifting their current route to a better one. The representation of such situations is particularly challenging if the information reaches a driver who is already travelling toward the destination. It is important to stress that we address rerouting not in terms of pre-trip route choices (referred as route swapping), but mainly en-route rerouting which takes place while travelling through the network. Recently we proposed ICM model to address the rerouting phenomena in Dynamic Traffic Assignment.

In this paper we follow classical definition of road network represented by means of an oriented graph $G(N, A)$, where N is the set of nodes and $A \subseteq N\times N$ is the set of arcs. Each arc $a\in A$ is described through a vector of characteristics $\delta_a(\tau)$ that allow to represent its performances. Such network is the input of DTA procedure providing state of the network represented with: a) time-varying traffic flows $q_a(\tau)$ on the network arcs and their cumulatives $Q_a(\tau)=\int_0^{\tau} q_a(\theta)d\theta$, b) time-varying travel times resulting from the traffic flows $t_a(\tau)$.

3. Observing rerouting phenomena

In this section we define the input of the procedure and derive desired characteristics. Traffic network G is observed through measuring the traffic flows $q_a(\tau)$ at some observed arcs $A_{obs}\subseteq A$. In practice, continuous observation of traffic flow is discretized and aggregated to a given time discretization as in (1), in this paper we work with traffic flows aggregated to hourly values.

$$q_a(H)=\int_{H-1}^{H} q_a(\tau)d\tau \tag{1}$$

Network is observed over several consecutive days and we further denote $\mathbf{q}_a^D$ as the daily traffic flow profile observed during day D and the long-term observation over set of observed days $\boldsymbol{D}_{obs}$ is represented through a set of daily profiles $\mathbf{q}_a=\left\{\mathbf{q}_a^D : D\in \mathbf{D}_{\mathbf{obs}}\right\}$. Within observed days $\mathbf{D}_{\mathbf{obs}}$ let's specify the subset of typical days $\mathbf{D}_{\mathbf{typical}}$, that is the days for which nothing atypical (events, excessive demand, severe weather, road closures etc.) took place. From observations of typical days $\mathbf{q}_a^{typical}=\left\{\mathbf{q}_a^D : D\in \mathbf{D}_{\mathbf{typical}}\right\}$ we can see if day-to-day fluctuations are significant. Although there is number of procedures in literature to obtain the mean traffic flow and its variance (for reference see Hranac, et. al 2012), here we apply simplified Student's t-statistics and define the confidence interval as in (2), where $\sigma\left(q_a^{typical}(\tau)\right)$ is the observed standard deviation of flows at arc a at time τ on set of typical days. t statistics are taken for $\left|D_{typical}\right|-1$ degrees of freedom, in field-data we obtained fit at α level of *90%*.

$$\varepsilon_a^{\max}(\tau)=t_{1-\alpha/2,\left|D_{typical}\right|-1}\frac{\sigma\left(q_a^{typical}(\tau)\right)}{\sqrt{\left|D_{typical}\right|-1}} \tag{2}$$

If observed values for the typical days are within the confidence interval $E(q_a^{typical}(\tau))\pm\varepsilon_a^{\max}(\tau)$ we can assume that there is a typical traffic flow profile $\hat{q}_a(\tau)=E(q_a^{typical}(\tau))$ at arc a during typical day.

We propose to further decompose single day observed flow $q_a(\tau)$ of an arc for which there is a typical profile and define it as a mixture of typical $\hat{q}_a(\tau)$ and extra parts of flow $\varepsilon_a(\tau)$, as defined in (3). We can say that observation of arc a is typical as long as $\left|\varepsilon_a(\tau)\right|\le\varepsilon_a^{\max}(\tau)$ and it becomes atypical otherwise. We further denote atypical flow $\tilde{q}_a(\tau)$ and measure it with extra flow $\varepsilon_a(\tau)$. For atypical observation we define the time instant $\tau_a^{e^-}$ at which the flows starts falling out of confidence interval and $\tau_a^{e^+}$ at which it becomes typical again, we further define impact period τ_a^e with (4).

$$\varepsilon_a(\tau)=\tilde{q}_a(\tau)-\hat{q}_a(\tau) \tag{3}$$

$$\tau_a^e=\left\langle\tau_a^{e^-},\tau_a^{e^+}\right\rangle : \tau\in\tau_a^e \Rightarrow \left|\varepsilon_a(\tau)\right|>\varepsilon_a^{\max}(\tau) \tag{4}$$

$$E_a(\tau) = \int_{\tau_a^{e^-}}^{\tau} \varepsilon_a(\theta) d\theta \quad (5)$$

To measure the atypical flows we will cumulate the event impact through (5) and define the total impact with total extra flow $E_a(\tau_a^{e^+})$. In the remainder of the analysis we assume that traffic flow profile outside of impact period τ_a^e is typical $\hat{q}_a(\tau)$, thanks to this we can restrict analysis to τ_a^e and use $\varepsilon_a(\tau) = 0 : \tau \notin \tau_a^e$ to isolate impact from random traffic fluctuations not related to the event.

In this framework we propose to define an arc impacted by event as an arc for which flows are atypical for some impacted period τ_a^e. We can further extend this definition by looking at profile of $E_a(\tau)$ and distinguishing several cases for impacted arcs:

a) negatively influenced arc, for which $E_a(\tau_a^{e^+}) < 0$;
b) positively influenced arc, for which $E_a(\tau_a^{e^+}) > 0$;
c) neutrally impacted arc, for which $E_a(\tau_a^{e^+}) = 0$;

By definition we can say that for impacted arcs there is strict relation between *E* and *Q*, so that the total traffic flow of impacted arc over the impacted period is:

a) $\tilde{Q}_a(\tau_a^{e^+}) < \hat{Q}_a(\tau_a^{e^+})$ if $E_a(\tau_a^{e^+}) < 0$;
b) $\tilde{Q}_a(\tau_a^{e^+}) > \hat{Q}_a(\tau_a^{e^+})$ if $E_a(\tau_a^{e^+}) > 0$;
c) $\tilde{Q}_a(\tau_a^{e^+}) = \hat{Q}_a(\tau_a^{e^+})$ if $E_a(\tau_a^{e^+}) = 0$;

The negatively impacted arc can be seen as an arc for which the traffic conditions has been worsened due to event *e* so that cumulated flow was lower than typically as the traffic flows shifted to positively impacted arcs for which the cumulated flow was greater than typically. The special case arises for impacted arcs for which there is an impact, but the cumulated flow remains typical. In this case only the profile of flow $q_a(\tau)$ has changed, resulting in negative $\varepsilon_a(\tau)$ in the first, building, phase and positive $\varepsilon_a(\tau)$ in the second, unloading, phase. For this case we can define middle point at which a building-phase becomes an unloading-phase. If the *E* of building-phase equals to *E* of unloading phase it means that arc is neutrally impacted, and rerouting phenomena is not observed. Empirically it would correspond to the case when event causes some impact (i.e. slows down the traffic), but it doesn't alter the route-choice model, no-one waiting in queue decides to reroute.

Mind that τ_a^e is defined separately for each arc *a* and is not unique for the whole network, thanks to this we can observe the temporal dimension of the rerouting phenomena. For the negatively impacted arcs we assume that $\tau_a^{e^-}$ is the time instant at which backward wave (Lighthill, et al. 1955) propagated from the place of event *e* reaches arc *a*, i.e. a moment at which the queue caused by event *e* reaches arc *a*. So that $\tau_a^{e^-} = \tau^e + \Delta w(e,a)$ where τ^e is the time of the event and $\Delta w(e,a)$ is the wave propagation time from arc of the event *e* to arc *a*. $\Delta w(e,a)$ can be straightforwardly derived from the traffic flow model, i.e. (Gentile, 2010). The beginning of impact time for negatively impacted arcs results from the phenomena taking place at the supply side, mainly queue formation. On contrary the impact period at positively impacted arcs arises from rerouting phenomena, i.e. the drivers who decide to shift their routes from the negatively impacted arcs to positively impacted arcs.

4. Cut-set observation

Based on the above general considerations of traffic flows observed in atypical days, we focus our considerations on specific case of screenline observation which, as we will show later is valuable to estimate rerouting phenomena. We observe the subset of observed arcs $A_{obs} \subseteq A$ being cut-set of the graph dividing the network into two subgraphs, such that for each *od* pair with origin laying in one subgraph and destination in another there is no path connecting *o* with *d* which does not contain at least one arc of the cut-set. Practically such cut-set is obtained by looking at screenline of some linear barrier, i.e. railway line, motorway, or river; in our dataset we looked at the river and cut the network into left- and east-bank subgraphs. Thanks to this we are sure that we observe the total traffic crossing the river. Moreover, for clarity, we analysed the event which took place at the cut-set, so that we directly observe the event arc *e* for which the event took place. Time of the event is then directly observed and, as $\Delta w(e,a)=0$, it coincides with beginning of impact period at event arc *e* (which is, by the way, the only negatively impacted observed arc). If so, we can say that the total rerouting flow *R* coincides with the total extra flow of event arc, $-E_e(\tau_a^{e^+})$.

The quantity of rerouting phenomena defined through *R* results from sum of all rerouting decisions made by drivers throughout the space (network arcs $a \in A$) and time (τ) represented with $r_a(\tau)$ – which is the central variable of rerouting model. We can satisfactorily define the rerouting phenomena through determining $r_a(\tau)$, which , unfortunately, is not observed directly and can only be approximated, as we show below. First approximation of *R* can be derived from *E* of negatively impacted arc as in (6). But we shall check whether flow conservation (7) holds true. If so it means that total flow crossing the cut-set is conserved, i.e. total extra flow on all positively impacted observed arcs is equal to extra flow on negatively impacted observed arcs. Which means that the whole impacted flow has shifted to observed arcs, so that *R* can be appropriately calculated with (6). Otherwise it means that some part of impacted flow has resigned from the trip and didn't cross the cut-set at all. In such case it is better to define the total rerouting *R* through (8) which sums ε only for positively impacted arcs.

$$R \equiv \sum_{a \in A} \int_{\tau_a^{e-}}^{\tau_a^{e+}} r_a(\tau) = -E_e(\tau_a^{e^+}) \tag{6}$$

$$-E_e(\tau_e^{e^+}) = \sum_{a \in A_{obs} \setminus \{e\}} E_a(\tau_a^{e^+}) \tag{7}$$

$$R \equiv \sum_{a \in A} \int_{\tau_a^{e-}}^{\tau_a^{e+}} r_a(\tau) = \sum_{a \in A_{obs} \setminus \{e\}} E_a(\tau_a^{e^+}) \tag{8}$$

$$r_a(\tau) = \sum_{o \in A_{obs}} r_a^o(\tau) \tag{9}$$

$$\varepsilon_o(\tau) = \sum_{a \in A} r_a^o\left(\tau - \Delta t(a,o)\right) \tag{10}$$

We can further say that unobserved rerouting flows $r_a(\tau)$ can be indirectly observed at $o \in A_{obs}$ through $\varepsilon_o(\tau)$ as in equation (10). In this formula we shift the decisions made by individuals through $r_a(\tau)$ back in time when they were made, $\Delta t(a,o)$ before observing the flow at observed arc. For this end we use departure time from arc *a* to arrive at observed arc *o* at time τ which equals $\tau - \Delta t(a,o)$, and is direct result of traffic flow model. We also use $r_a^o(\tau)$ being the decomposition of rerouting flow $r_a(\tau)$ per observed arcs $o \subseteq A_{obs}$, as in (9), i.e. part of rerouting flow at choosing to cross cut-set at alternative arc *o*

5. Estimating rerouting phenomena

Using the above framework we can define estimation problem of rerouting phenomena. The problem is solved through searching rerouting flows $r_a(\tau)$ in space *a* and time τ with (10). This problem can be paraphrased as a problem of finding unknown rerouting flows $r_a^o(\tau)$ which will sum up to observed extra flows at the observed cut-set, or more technically: find $r_a^o(\tau)$ so that for each observed arc $\varepsilon_o(\tau) = \sum_{a \in A} r_a^o\left(\tau - \Delta t(a,o)\right)$. Which is a very underdetermined problem and should be further constrained. The constrains should include: a) at the supply side, the traffic flow theory to determine travel times Δt; b) characteristics of rerouting phenomena (reaction of drivers to unexpected events). Therefore in the following section $r_a(\tau)$ is defined through the ICM model. We also provide background on how travel times $\Delta t(a,o)$ and traffic flows $q_a(\tau)$ are obtained and linked through Dynamic Traffic Assignment.

6. Dynamic Traffic Assignment

Before we can introduce the ICM model we need to provide a brief introduction to DTA. In general, DTA determines the traffic flows on the network satisfying the demand (Cascetta, 2009). It is done through an assignment methods, typically following the 'user-equilibrium' concept of balancing travel costs of all drivers (Wardrop, 1952). In dynamic context 'user-equilibrium' becomes a dynamic user equilibrium (DUE), defined as a traffic pattern at which no driver finds it convenient to (unilaterally) change his/her route and departure time (see i.e. Friesz et. al, 2000). DUE is obtained through an iterative process where every iteration route choices are adjusted based on outcomes of decisions made in previous iterations. The process is converging to a fixed-point, where demand and supply are stabilized (Banach, 1922).

The results of DTA are the network performances (i.e. temporal profile of travel costs and times) and the demand pattern. Demand pattern of DTA is either explicit set of *od* paths defined with specific temporal profile of flows, or, alternatively, (Meschini et al., 1999) set of implicit, local routing decisions (arc conditional probabilities) denoted as $p_a^d(\tau)$, defined for each node which coupled with origin demand becomes equivalent to explicit paths. Implicit representation through local decisions is way more convenient for modeling rerouting phenomena. In particular we introduce two demand patterns (obtained through a route choice model (RCM)); one calculated with typical travel costs and times: $\hat{p}_a^d(\tau) = RCM(\hat{c},\hat{t})$ and one calculated using actual (atypical) costs and times: $\tilde{p}_a^d(\tau) = RCM(\tilde{c},\tilde{t})$. By typical costs (denoted throughout the paper with superscript ^) we mean conditions observed during a typical day (when no unexpected events are present), which coincide with costs and times of dynamic user equilibrium and are, at the same time, the conditions expected by individuals to occur when making route choices. Actual (atypical) conditions (denoted throughout the paper with superscript ~) in turn are those observed as a consequence of unexpected event. Actual (atypical) travel times and costs are used by individual to choose a new path to avoid consequences of unexpected event. The outcomes of DTA are travel costs and flows of the network obtained through equilibrium for typical case.

7. ICM model

ICM rerouting model provides the same outcomes as DTA, yet for atypical situation of unexpected event, i.e. it provides atypical flows $\tilde{q}_a(\tau)$ including extra flow $\varepsilon_a(\tau)$. This section briefly summarizes information comply model (ICM) proposed in (Kucharski et al. 2014a). Here we provide only essential information about ICM model needed to understand how rerouting phenomena is modelled and what is the meaning of parameters.

ICM models probability of individual to reroute $\alpha_i^d(\tau)$ for each point in time τ and space i based on current situation and destination d. It is designed to resemble reasoning process made by each individual who is represented through three sub models:

– information model $\iota_i^d(\tau)$, telling if individual has received information about the event,

– observation model $o_i^d(\tau)$, telling if individual has observed atypical situation and linked it with the event, and:

– compliance model $\kappa_i^d(\tau)$, telling if individual reroutes to avoid negative impact of the event,

each parameterized to fit to actual observed behavior.

For each node i in the network we can compute result of ICM model using input from DTA, most importantly typical and actual travel times and costs. ICM computes $\alpha_i^d(\tau)$ as shown in formula (11) which links three submodels of ICM defined through formulas (12) to (14):

$$\alpha_i^d(\tau) = \left(1-\iota_i^d(\tau)\right)\left(1-o_i^d(\tau)\right)k_i^d(\tau) \tag{11}$$

$$\iota_i^d(\tau) = I_0 \cdot e^{-\frac{a_1}{M(\tau)}}\left(1-e^{-\frac{\tau^2}{2(a_2/M(\tau))^2}}\right) \tag{12}$$

$$o_i^d(\tau) = a_3(\Delta t_i)^{a_4} \tag{13}$$

$$\kappa_i^d(\tau) = \frac{1}{1+e^{-\left(\beta_{\Delta p_{id}}\Delta p_{id}(\tau)+\beta_{\Delta w_{id}}\Delta w_{id}(\tau)\right)/\eta}} \tag{14}$$

Submodels of ICM model ((12) to (14)) are defined using following terms:

M(τ) – global impact of the event calculated as total network delay at time τ:

$$M(\tau) = \sum_{a\in A}\int_0^\tau \tilde{t}_a(\theta)d\theta \Big/ \sum_{a\in A}\int_0^\tau \hat{t}_a(\theta)d\theta \tag{15}$$

$\Delta t_i(\tau)$ – cumulated delay at node i:

$$\Delta t_i^d(\tau) = \sum_{a\le i}\left(\tilde{t}_a(\tau)-\hat{t}_a(\tau)\right) \tag{16}$$

$\Delta p_i^d(\tau)$ – cosine similarity between $\hat{p}_i^d(\tau)$ and $\tilde{p}_i^d(\tau)$ vectors, showing how the demand pattern differs:

$$\Delta p_i^d(\tau) = \frac{\sum_{a\in i^{+d}} \tilde{p}_a^d(\tau)\hat{p}_a^d(\tau)}{\sqrt{\sum_{a\in i^{+d}} \tilde{p}_a^d(\tau)^2}\times\sqrt{\sum_{a\in i^{+d}} \hat{p}_a^d(\tau)^2}} \tag{17}$$

$\Delta w_i^d(\tau)$ – relative gap between actual and typical node satisfaction *w* (expected cost to get to destination – Dial, 1971):

$$\Delta w_i^d(\tau) = \frac{\hat{w}_i^d(\tau)}{\tilde{w}_i^d(\tau)} = \frac{\sum_{a \in i^{+d}} \hat{p}_a^d(\tau)\tilde{w}_{a^+}^d(\tau)}{\sum_{a \in i^{+d}} \tilde{p}_a^d(\tau)\tilde{w}_{a^+}^d(\tau)} \quad (18)$$

ICM is parameterized through set of parameters $\mathbf{a}=\{a_1,\dots a_4\}$ plus parameters of logit model $\beta_{\Delta p_{id}}, \beta_{\Delta w_{id}}, \eta$ with following meaning:

a_1,a_2 sensitivity of information spread to total severity of the event, a_1 alters the total probability of receiving information and a_2 the pace at which information is spread.

a_3,a_4 probability of *guessing* the event from the total delay.

$\beta_{\Delta p_{id}}, \beta_{\Delta w_{id}}, \eta$ relative weight of $\Delta p_{id}(\tau)$ and $\Delta w_{id}(\tau)$ in the rerouting utility used in binomial logit model (14) with logit parameter η

8. Estimating ICM model

The rerouting problem that we proposed based on equation (10) was underdetermined, though it can be now redefined using ICM model through (11) as shown in (19). We substitute $r_a^o(\tau)$ of (10) with formula (19) where the traffic flows at arcs q_i^d (decomposed per destination) are multiplied with rerouting probability α_i^d and with $\tilde{p}_{a\to o}^d(\tau)$ which tells from DTA route choice model which share of flow q_i^d at arc a reaches observed arc o at time τ (it can be seen as equivalent of $r_a^o(\tau)$ resulting from the DTA model).

$$r_a^o(\tau) = \sum_{d\in D} q_a^d(\tau)\cdot\alpha_a^d(\tau)\cdot\tilde{p}_{a\to o}^d(\tau) \quad (19)$$

$$\tilde{\varepsilon}_o(\tau) = \sum_{a\in A} r_a^o(\tau - \Delta t(a,o)) \quad (20)$$

Thanks to this elaboration (10) is no longer underdetermined as we can use $r_a^o(\tau)$ of (19) where the flows q_a^d and share $\tilde{p}_{a\to o}^d(\tau)$ result from DTA model, while α is modelled with ICM. This way degrees of freedom are limited to the set of ICM parameters $\{\mathbf{a}, \beta_{\Delta p_{id}}, \beta_{\Delta w_{id}}, \eta\}$.

With this extension (10) is no longer a simple linear equation, but becomes an estimation problem where objective is to parameterize ICM so that it will produce the observed extra flows ε through (20). First let's distinguish theoretical extra flow $\tilde{\varepsilon}_o(\tau)$ at arc o defined through (21) and define the objective function through some distance measure $\|\varepsilon_o(\tau) - \tilde{\varepsilon}_o(\tau)\|$. This way we can propose the estimation problem (22) to estimate ICM model parameters.

$$\tilde{\varepsilon}_o(\tau) = \sum_{a\in A}\sum_{d\in D} q_a^d(\tau - \Delta t(a,o))\cdot\alpha_a^d(\tau - \Delta t(a,o))\cdot\tilde{p}_{a\to o}^d(\tau) \quad (21)$$

$$(\mathbf{a}, \beta_{\Delta p_{id}}, \beta_{\Delta w_{id}}, \eta) = \underset{\mathbf{a},\beta_{\Delta p_{id}},\beta_{\Delta w_{id}},\eta}{\arg\min}\left\{\sum_{o\in A_{obs}}\sum_{\tau\in\tau_a^e}\|\varepsilon_o(\tau) - \tilde{\varepsilon}_o(\tau)\|\right\} \quad (22)$$

9. Field-data observations

Above procedure is run with the input from temporal profiles of flows $q_a(\tau)$ for each bridge crossing Wisła river in Warsaw (fig. 1). Flows were collected over 11 consecutive days (including day of the event), 6 of which were identified as typical.

On the last observed day there was a severe event impacting the flows in the city. The event took place at Siekierkowski bridge (further denoted as the event arc e) on 9th April 2014, at 9:00 in eastbound direction. As a result two out of three lanes were blocked until 11:30 and capacity was reduced to ca. 1700 vehicles per hour and causing severe delays. During the day of the event drivers had access to number of information sources. Majority of the drivers were equipped with smartphones and some part of them uses traffic information services. Several companies provide traffic forecasts based on historical data coupled with actual state (mainly from FCD data), radio broadcasted information about the event with approximately 20 minute delay.

The event arc e is the southernmost bridge in Warsaw and the arterial leading to it has only few junctions. The topology of the network points the only one logical 'escape junction' at which drivers could reroute to avoid impact of the event (marked with a cross at fig. 1). Such arbitral topological assumption reduces the search space of $r_a(\tau)$ to a single junction. Rerouting drivers escaping at the escape-junction drivers switch to Wisłostrada – arterial parallel to the river leading to all the alternative bridges.

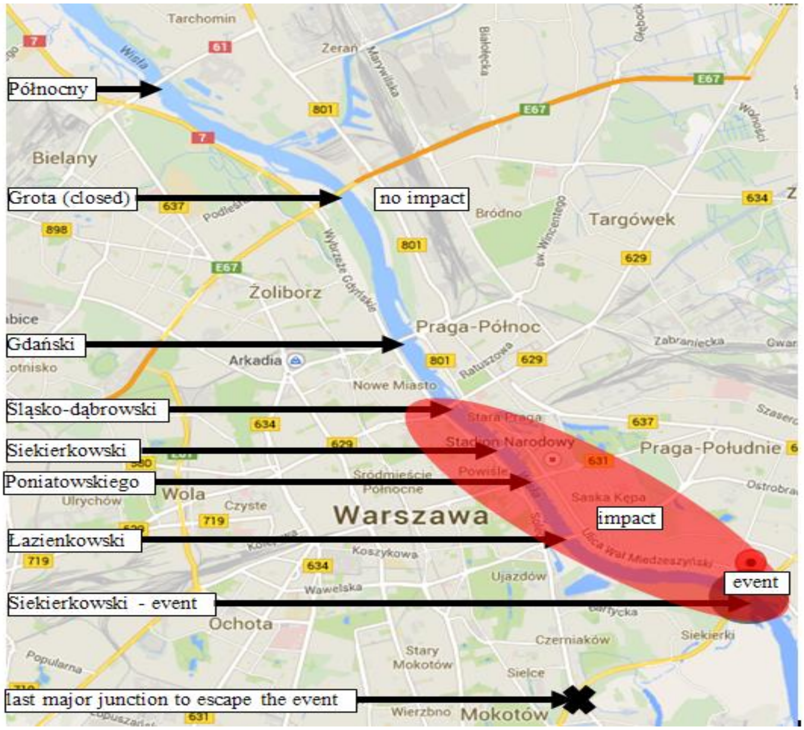


Fig. 1. Warsaw bridges, southernmost is affected

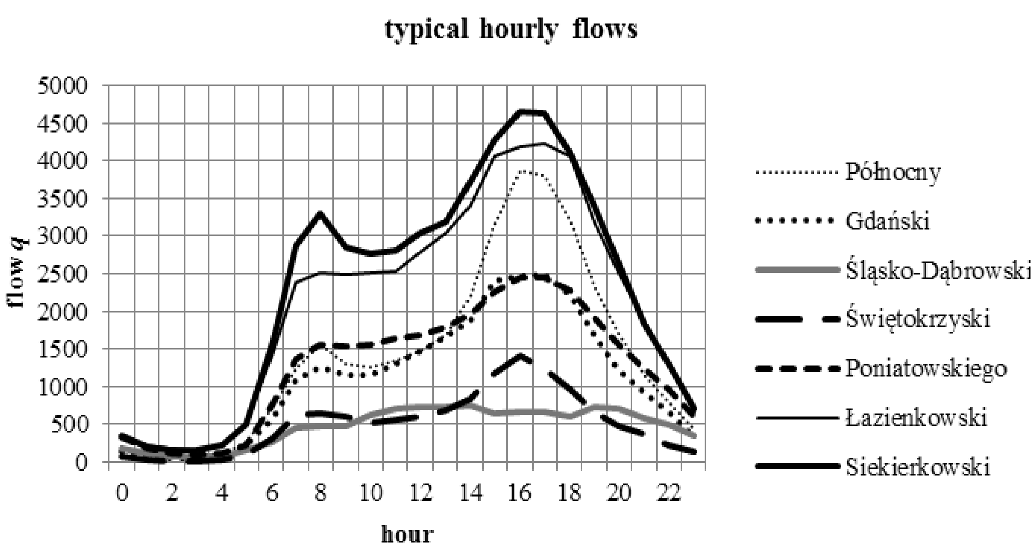


Fig. 2. Typical observed temporal profile of traffic flow $\hat{q}_a(\tau)$ at the cut-set

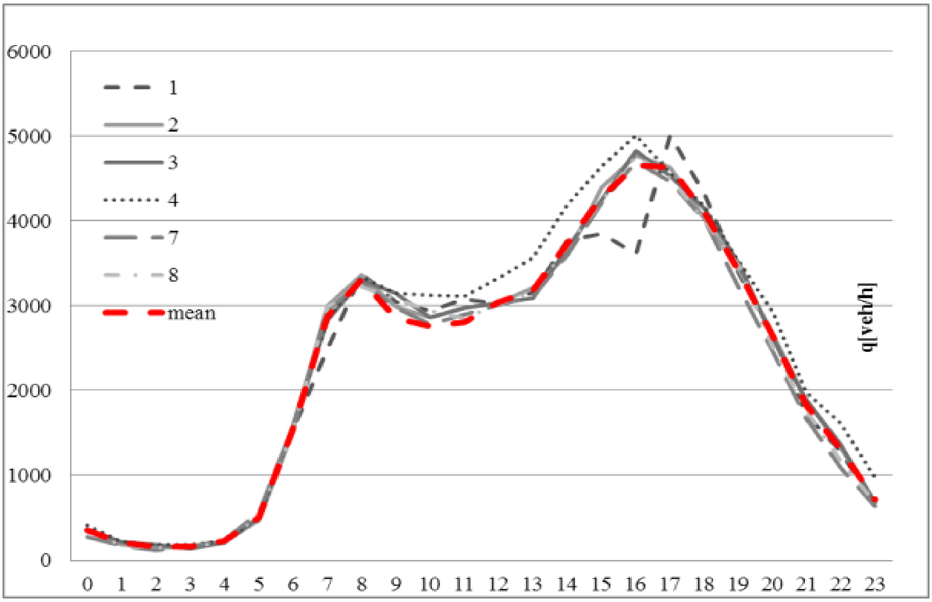


Fig. 3. Flow observed over typical days at the affected bridge *e*

Table 1 Typical hourly traffic flows and their standard deviations

bridge	Północny		Gdański		Śląsko-Dąbrowski		Świętokrzyski		Poniatowskiego		Łazienkowski		Siekierkowski	
hour	**mean**	**std**	**mean**	**std**	**mean**	**std**	**mean**	**std**	**mean**	**std**	**mean**	**std**	**mean**	**std**
0	**200**	*24*	**132**	*22*	**185**	*16*	**68**	*5*	**335**	*51*	**325**	*47*	**353**	*44*
1	**105**	*11*	**78**	*15*	**110**	*22*	**36**	*7*	**179**	*32*	**228**	*48*	**200**	*29*
2	**75**	*10*	**45**	*11*	**81**	*27*	**20**	*8*	**119**	*33*	**188**	*56*	**156**	*23*
3	**69**	*9*	**44**	*14*	**69**	*12*	**20**	*5*	**106**	*17*	**149**	*30*	**160**	*14*
4	**103**	*9*	**61**	*10*	**77**	*10*	**26**	*5*	**107**	*10*	**224**	*57*	**223**	*15*
5	**261**	*10*	**220**	*34*	**156**	*14*	**87**	*3*	**224**	*14*	**503**	*87*	**504**	*33*
6	**714**	*20*	**555**	*83*	**266**	*8*	**315**	*21*	**751**	*25*	**1408**	*95*	**1542**	*29*
7	**1240**	*47*	**1095**	*163*	**463**	*13*	**618**	*23*	**1375**	*18*	**2388**	*108*	**2871**	*184*
8	**1550**	*26*	**1259**	*218*	**471**	*21*	**640**	*10*	**1553**	*36*	**2517**	*86*	**3304**	*58*
9	**1295**	*49*	**1149**	*175*	**475**	*29*	**600**	*22*	**1536**	*42*	**2500**	*111*	**2852**	*564*
10	**1267**	*66*	**1164**	*139*	**630**	*27*	**523**	*32*	**1551**	*64*	**2504**	*134*	**2758**	*442*
11	**1349**	*37*	**1309**	*135*	**700**	*15*	**556**	*20*	**1643**	*67*	**2527**	*122*	**2804**	*485*
12	**1478**	*71*	**1467**	*116*	**726**	*30*	**610**	*50*	**1694**	*70*	**2790**	*175*	**3042**	*140*
13	**1684**	*196*	**1665**	*120*	**724**	*39*	**685**	*49*	**1797**	*65*	**3040**	*218*	**3184**	*182*
14	**2188**	*272*	**1886**	*72*	**745**	*52*	**846**	*39*	**1971**	*34*	**3402**	*205*	**3731**	*216*
15	**3143**	*200*	**2414**	*93*	**647**	*16*	**1172**	*71*	**2261**	*29*	**4050**	*183*	**4277**	*240*
16	**3873**	*84*	**2475**	*29*	**669**	*30*	**1417**	*34*	**2453**	*92*	**4184**	*235*	**4656**	*473*
17	**3806**	*101*	**2484**	*109*	**666**	*16*	**1250**	*97*	**2439**	*59*	**4236**	*293*	**4629**	*182*
18	**3206**	*108*	**2206**	*346*	**602**	*28*	**957**	*10*	**2276**	*90*	**4058**	*225*	**4097**	*116*
19	**2338**	*277*	**1684**	*220*	**739**	*31*	**665**	*26*	**1909**	*108*	**3182**	*234*	**3409**	*104*
20	**1704**	*220*	**1217**	*109*	**703**	*16*	**467**	*24*	**1560**	*91*	**2507**	*276*	**2630**	*150*
21	**1165**	*96*	**919**	*72*	**579**	*68*	**369**	*27*	**1230**	*54*	**1876**	*208*	**1831**	*114*
22	**803**	*113*	**678**	*48*	**504**	*57*	**214**	*24*	**962**	*90*	**1278**	*118*	**1307**	*165*
23	**426**	*103*	**336**	*48*	**348**	*81*	**134**	*34*	**594**	*62*	**673**	*152*	**717**	*119*

10. Analysis

In this and following sections field data is analysed to obtain the characteristics of the rerouting phenomena that can be derived from the observations.

In the first step, representative subset of typical days $\mathbf{D}_{\mathbf{typical}}$ is selected by excluding atypical observations (i.e. excessive demand on Friday). This way typical flow $\hat{q}_a(\tau)$ can be determined and observed extra flow $\varepsilon_a(\tau)$ in most cases falls within the confidence interval computed with (2). As depicted on fig. 3, day-to-day traffic flow fluctuations of the event arc *e* are insignificant and typical flow well represents the expected/mean flow profile. The same holds true for most of the bridges, typical traffic flow profiles $\hat{q}_a(\tau)$ and their standard deviations over all bridges of a cut-set are shown in table 1. and at fig. 2.

The event arc *e* has the highest total cumulative flow *Q*. Bridges in eastbound direction are more congested in the afternoon peak than in the morning. At the time of the event at alternative bridges there is still some capacity in eastbound direction available i.e. for rerouting vehicles. Centrally located Śląsko-Dąbrowski bridge is an exception as it works at capacity (ca. 700 veh/h) roughly throughout the day.

Typical flows $\hat{q}_a(\tau)$ can be compared with atypical flows observed during the day of the event $\tilde{q}_a(\tau)$) to derive extra part $\varepsilon_a(\tau)$. At the event arc *e* negative extra flow $\varepsilon_e(\tau)$ is evident (see fig. 4, table. 2), both at the level of flows and their cumulatives. In total during the day 3705 vehicles less has passed the bridge than typically $(E_e(\tau) = -3705)$ which stems for around 6% of daily flow and 25% of flow during impact period. Impacted period has a strict beginning at 09:00 with clearly identifiable loading-phase $(09:00 - 13:00)$. Interestingly the queue dissipation after the road is cleared is not observed. What we expected is that the queue which formed upstream of the event will dissipate at the level of capacity until it reaches back the typical values when queue diminishes. While what we observed is the flow recovering to typical levels (13:00), reaching capacity much later, during afternoon peak (16:00). Traffic flows are slightly above the typical values only for a short period $(14:00 - 16:00)$. Therefore the limitation of impacted period τ_e^e is not obvious. Here we assume that impact period coincides with capacity reduction time (loading-phase) and the flows above typical in the unloading phase are neglected. Nevertheless the observation at event arc *e* fits into definition of negatively impacted arc proposed in former section. For the remainder of calculations $\tau_e^e = (09:00 - 13:00)$ is used. Having estimated the impact period τ_e^e , *R* according to (6) can be estimated. In total during the impact period *R*=–*3 705* vehicles less has travelled through the bridge. What shall, however, be tested is the conservation rule (7), for this end typical and atypical flows at the remainder of cut-set are compared and $E_e(\tau)$ for each bridge is obtained (depicted at fig. 11).

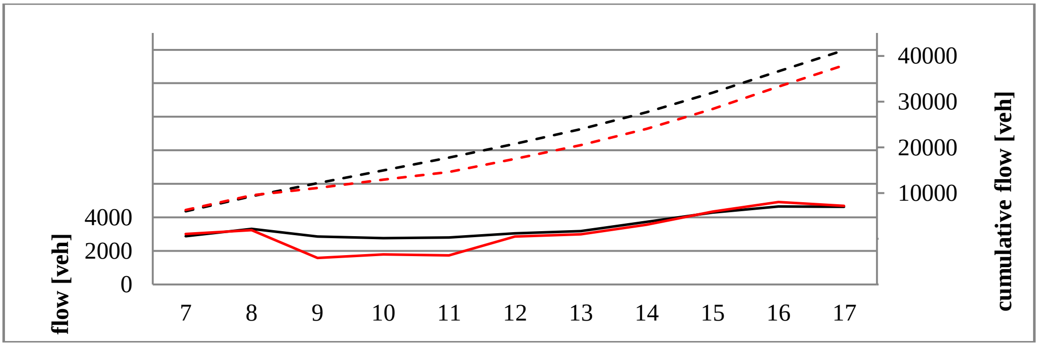


Fig. 4. Flow and cumulative flow (dashed) of affected bridge, typical and atypical (red)

What is interesting to see is the flow profile crossing the whole cut-set during the impacted time (typical and atypical) as in fig. 5. Traffic crossing the river drops significantly at time of the event (09:00) which coincides with the drop at the event arc *e* at 09:00, yet at 10:00 it is recovers approaching to typical and at 11:00 it goes above, going back to typical values at 14:00. Base on the above the total impact time of the whole cut-set is identified as $(09:00 - 14:00)$.

Having rough overview of the entire cut-set the situation at single alternative bridges can be looked at. From the charts showing typical and atypical flows, extra flows ε of each bridge can be identified. At the neighbouring bridge (Łazienkowski, fig.6) positive ε appears at 10:00 and reaches back the typical level at 13:00, analogous behaviour can be seen at three consecutive bridges (fig. 7-9). While on the two northernmost bridges: Most Północny (fig.10), and Most Gdański the observed flows are typical throughout the whole day–they are not impacted by the event.

Table 2. Flow *q* and cumulative flow *Q* of affected bridge, typical and atypical, impact period in red

	traffic flows *q*		cumulatives *Q*			traffic flows *q*		cumulatives *Q*	
hour	typical	atypical	typical	atypical	hour	typical	atypical	typical	atypical
0	353	*391*	353	*391*	12	3042	*2850*	20768	*17461*
1	200	*245*	553	*636*	13	3184	*2994*	23952	*20455*
2	156	*167*	708	*803*	14	3731	*3566*	27683	*24021*
3	160	*174*	868	*977*	15	4277	*4348*	31960	*28369*
4	223	*240*	1091	*1217*	16	4656	*4911*	36615	*33280*
5	504	*543*	1595	*1760*	17	4629	*4693*	41245	*37973*
6	1542	*1505*	3137	*3265*	18	4097	*4034*	45342	*42007*
7	2871	*3017*	6009	*6282*	19	3409	*3354*	48751	*45361*
8	3304	*3238*	9312	*9520*	20	2630	*2632*	51382	*47993*
9	2852	*1583*	12164	*11103*	21	1831	*1919*	53213	*49912*
10	2758	*1784*	14922	*12887*	22	1307	*1294*	54520	*51206*
11	2804	*1724*	17725	*14611*	23	717	*729*	55237	*51935*

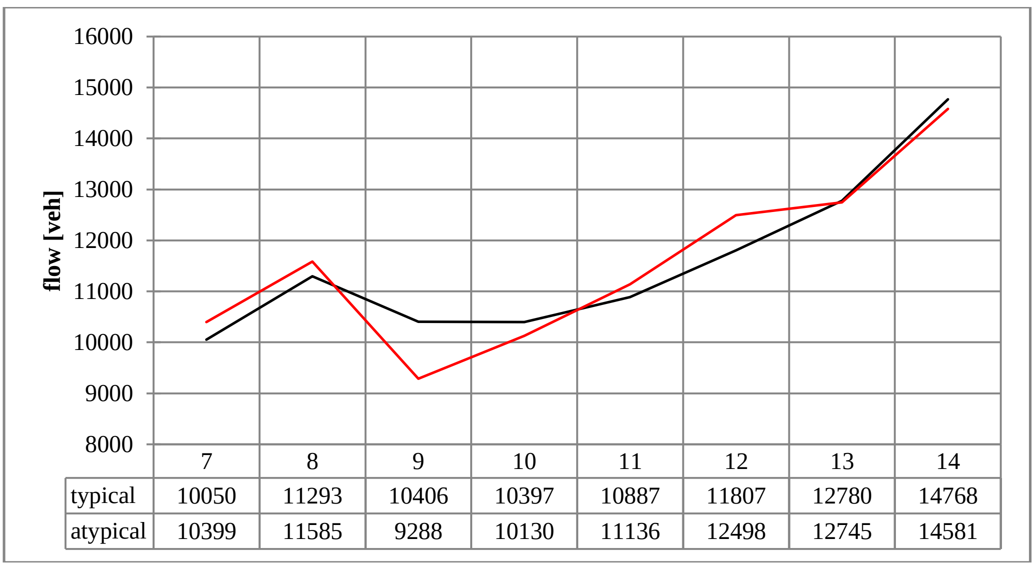
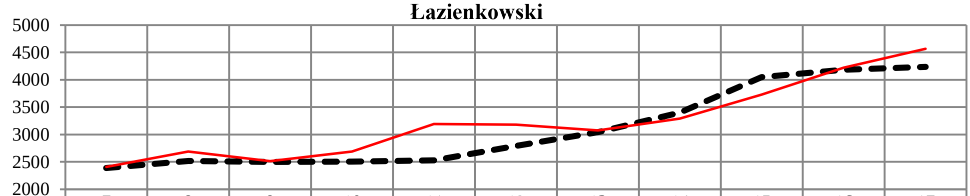


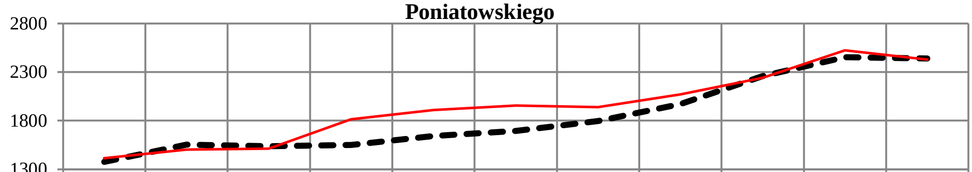
Fig. 5. Flow of whole screenline typical and atypical (red)

Łazienkowski



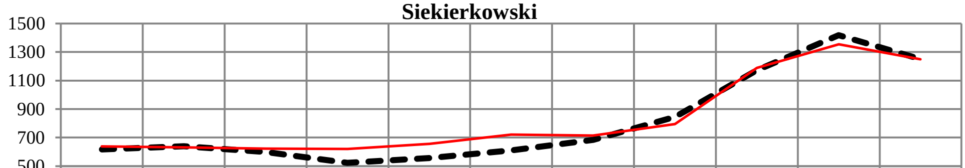
	7	8	9	10	11	12	13	14	15	16	17
typical	2388	2517	2500	2504	2527	2790	3040	3402	4050	4184	4236
atypical	2411	2692	2512	2691	3194	3180	3078	3290	3729	4218	4567

Poniatowskiego



	7	8	9	10	11	12	13	14	15	16	17
typical	1375	1553	1536	1551	1643	1694	1797	1971	2261	2453	2439
atypical	1415	1501	1511	1815	1911	1957	1940	2070	2242	2525	2430

Siekierkowski

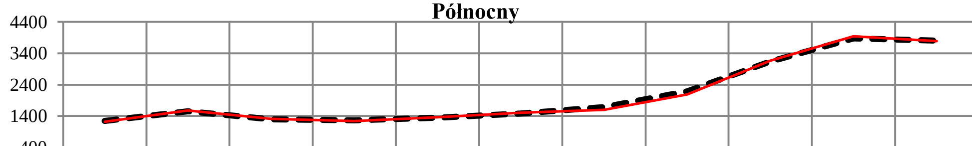


	7	8	9	10	11	12	13	14	15	16	17
typical	618	640	600	523	556	610	685	846	1172	1417	1250
atypical	638	631	623	621	657	721	715	794	1188	1353	1248

Śląsko-dąbrowski

	7	8	9	10	11	12	13	14	15	16	17
typical	463	471	475	630	700	726	724	745	647	669	666
atypical	458	501	505	715	813	753	708	762	651	735	714

Północny



	7	8	9	10	11	12	13	14	15	16	17
typical	1240	1550	1295	1267	1349	1478	1684	2188	3143	3873	3806
atypical	1218	1580	1307	1239	1360	1506	1601	2087	3161	3940	3789

Figures 6–10. Typical and atypical (red) flow at alternative bridges

11. Summary of observations

Table 3 shows extra flows at impacted, alternative bridges, based on this the total rerouting flow computed with (8) is $R = -2809$ which confronted with $R = -3705$ computed with (6) shows that flow conservation did not hold true and ca. 24% of flow did not cross the river at all. Therefore the corrected estimate of total rerouting flow computed with (8) is used as more appropriate, which yields lower total rerouting share of $2809/14640 = 19,2\%$.

The field data discretized every hour did not allow to precisely estimate the time dimension of rerouting flows $r_a(\tau)$. However we can say that there is a delay time between the event and time when rerouting flows are observed at alternative bridges. This delay is visible at fig. 11 when ε is negative for event arc at 09:00 and becomes positive for alternatives at 10:00. Which, taking into account the approximated travel time from 'escape-junction' to closest alternative (Most Łazienkowski) of 13 minutes, leads to the conclusion that rerouting starts less then hour past the event but not immediately. The above is all that can be stated this data discretization about timing of rerouting.

The extra flows $\varepsilon_a(\tau)$ are increasing in time reaching the top level ca. 2 hours past the event. Which is coherent with the ICM assumption that rerouting flows are positively correlated with time past the event (reflected in information spread model of ICM - (12)) and the delay caused by the event (reflected in all three ICM submodels – through M, Δt, Δw, Δp (11)).

To conclude the new route choice pattern being impact of the event can be approximated as shown in table 4. We see that the closest alternative takes biggest share of rerouting flows, and the share decreases as the distance (cost to reach the alternative) increases, which fits the assumption that users make rational choices in rerouting case (they minimize the travel costs), as well as the assumption that users take into account the choices made by others (not only single optimal alternative is used). Those observations support the ICM assumptions that a) rerouting is driven by cost minimizing formula, i.e. drivers reroute according to $\tilde{p}_a^d(\tau)$ b) more than one iteration of DTA is needed to reproduce rerouting route-choice pattern, with $\tilde{p}_a^d(\tau)$ being updated every iteration.

Table 3. extra flow ε at alternative bridges

hour	Północny	Śląsko-Dąbrowski	Świętokrzyski	Poniatowskiego	Łazienkowski	Siekierkowski
9	12	30	24	-25	12	-1269
10	28	85	98	264	187	-974
11	-11	113	101	268	667	-1080
12	-29	27	112	263	390	-192
13	83	-16	30	143	38	-190
sum	**83**	**239**	**364**	**912**	**1294**	**-3705**

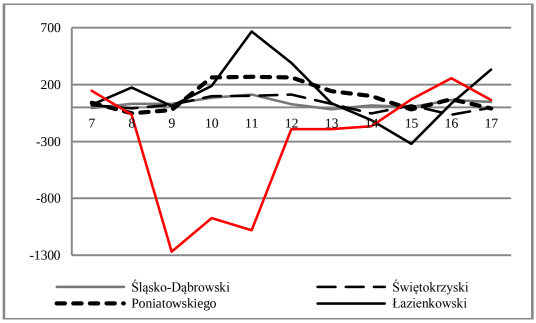


Fig. 11. extra flow ε at alternative bridges

Table 4. route-choice between bridges of a cut-set

share	Północny	Śląsko-Dąbrowski	Świętokrzyski	Poniatowskiego	Łazienkowski	Siekierkowski
typical	12.6%	5.8%	5.3%	14.6%	23.7%	26.0%
atypical	12.7%	6.3%	6.0%	16.5%	26.5%	19.8%
extra	3%	9%	13%	32%	46%	-

12. Conclusions

In this article we found evidence of existence of rerouting phenomena, which was observed through the field-data. We have seen an extra flow at alternative bridges when major bridge was blocked. We managed to approximate most important

characteristics of rerouting: share of impacted flow which reroutes ~20%, time of rerouting ~1hour after the event. The emerging route-choice pattern for rerouting flows show that not only optimal alternative is chosen when rerouting, but also others which evidences the strategic capabilities of (at least Polish) drivers. What is more we observed relation between additional travel cost of rerouting through alternative and its share among rerouting flows. Distant alternatives are not used and share increases while the cost to reach it decreases.
Furthermore we provide theoretical framework for rerouting model estimation using observation of traffic flows crossing the cut-set of the network (in our case the river). Full estimation requires both the observed data and established, real-time dynamic traffic assignment model which is yet not available.

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