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The Method of the Evaluation of the Efficiency of the Processes Carried Out at Traffic Maintenance Subsystem Posts

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Abstract

In the systems of the exploitation of means of transport in order to assure correct carrying out of the assigned transport tasks, it is necessary to maintain the required number of vehicles in the state of task availability. It is obtained as a result of carrying out of service and repair processes at the traffic maintenance subsystem posts. From the point of view of the effectiveness of the operation of the systems of the exploitation of means of transport, the damaged technological objects should be serviced in the shortest possible time. In the case of the system of the exploitation of means of transport the task of the traffic maintenance subsystem is the servicing of the required number of damaged means of transport over the assigned time interval defined by the transport task timetable. One of the methods of the evaluation of the degree of carrying out of the processes at the traffic maintenance subsystem posts is defining the efficiency of the posts of the subsystem. This paper presents the method of defining the efficiency of traffic maintenance subsystem in the system of the exploitation of means of transport measured by the probability value of servicing of the required number of technological objects over the assigned time interval. The resulting characteristics were presented on a general basis as well as in the form of exponential distribution and Erlang distribution. Moreover, for the utilization data obtained in tests carried out in an actual means of transport system the values of the analyzed characteristics were defined. The evaluation of the efficiency of the traffic maintenance subsystem posts may serve as the basis of decision making as to the change of the number and type of the used service and repair posts in the tested system as well as modification of the carried out exploitation strategy. The suggested method may be used for individual post, a group of posts of a given type as well as for a subsystem of traffic maintenance analyzed as a whole.

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1. Introduction

The basic goal for the existence of means of transport exploitation systems is carrying out of the assigned transport tasks via an executive subsystem consisting of elementary subsystems of the man – technological object (driver – mean of transport) type. While carrying out the transport tasks the utilized means of transport, due to the influence of various types of coercive factors undergo damage. Correct and effective carrying out of the assigned transport tasks is possible only when the required number of technological objects (means of transport) is available to perform the task. This is achieved as a result of the carrying out of the service and repair processes at the traffic maintenance posts.

In the case of the systems of means of transport exploitation, the task assigned to the traffic maintenance posts is defined by the number of technological objects (means of transport) which should be serviced over the assigned time interval. The task thus defined is carried out at the individual traffic maintenance posts by teams of specialists equipped with appropriate devices and tools. One of the methods of the evaluation of the degree of carrying out of the service and repair tasks assigned to the posts of the traffic maintenance subsystem in the evaluation of the efficiency of these posts, determined on the basis of the value of the efficiency index determined as the probability of the servicing of the required number of technological objects over the assigned time interval.

This paper presents the method of determining the value of the efficiency index of the traffic maintenance posts. The suggested method may be used for a single post, a group of a given type of posts, i.e. repair posts as well as the traffic maintenance subsystem as a whole. The presented characteristics were determined on general basis as well as in the form of exponential distribution and Erlang distribution. The use of the established models in the form of exponential distribution is limited to the sole instance of when the analyzed random variables have exponential distributions (the so called "no memory" models) [1, 6, 11]. On the other hand, the characteristics determined for Erlang distribution are implemented when the analyzed random variables are a sum of independent random variables in exponential distribution. Then, i.e. the random variable T with Erlang distribution may be presented as [2, 3, 9, 15]:

$$T = T_1 + T_2 + ... + T_m,$$

where $T_i, i = 1, 2, ..., m$ – independent random variables with exponential distribution with parameter λ. Such case may take place when, for example, the lengths of time of repairs of individual means of transport systems are analyzed separately.

2. The Efficiency of the Posts of the Traffic Maintenance System in the System of the Exploitation of Means of Transport

Let the random variable T be the length of time the technological object remains outside the traffic maintenance subsystem and $F(t)$ the distribution function of random variable T:

$$F(t) = P(T < t), \tag{1}$$

then the distribution of the number of technological objects directed to the traffic maintenance subsystem posts over the time interval τ is the relation:

$$P(L(\tau) = n) = F^{(n)}(\tau) - F^{(n+1)}(\tau), \; n = 0, 1, 2, ..., \tag{2}$$

where:

$L(\tau)$ is the number of technological directed to the traffic maintenance subsystem posts over the time interval τ and

$$F^{(n)}(\tau) = \int_0^t F^{(n-1)}(\tau - x)\, dF(x), \; n = 2, 3, ... \tag{3}$$

The probability that the number of directed to the traffic maintenance subsystem posts over the time interval τ is lower than n is defined by the relation:

$$U(n) = P(L(\tau) < n) = 1 - F^{(n)}(\tau). \tag{4}$$

Let the random variable Θ be the length of time the technological object remains at the traffic subsystem post and $G(t)$ the distribution function of the random variable distribution function Θ:

$$G(t) = P(\Theta < t), \tag{5}$$

then the distribution of the number of technological objects serviced at the traffic maintenance posts over the time interval τ is defined by the relation:

$$P(N(\tau) = n) = G^{(n)}(\tau) - G^{(n+1)}(\tau), \; n = 0, 1, 2, ..., \tag{6}$$

where:

$N(\tau)$ is the number of technological objects directed to the traffic maintenance subsystem posts over the time interval τ

$$G^{(n)}(\tau) = \int_0^t G^{(n-1)}(\tau - x)\, dG(x), \; n = 2, 3, ... \tag{7}$$

The probability of the number of technological objects serviced at the traffic maintenance subsystem posts overt the time interval τ is lower than n is determined by the relation:

$$V(n) = P(N(\tau) < n) = 1 - G^{(n)}(\tau). \tag{8}$$

Bearing the above in mind, the distribution function of the number $M(\tau) = L(\tau) - N(\tau)$ of technological objects not serviced at the traffic maintenance subsystem posts over the time interval τ is defined as follows:

$$\begin{aligned} Z^{(k)} &= P(M(\tau) < n) = P(L(\tau) - N(\tau) < n) = \\ &= \sum_{b=0}^{\infty} P(N(\tau) = b, L(\tau) < n + b) = \\ &= \sum_{b=0}^{\infty} P(N(\tau) = b) \cdot P(L(\tau) < n + b) = \\ &= \sum_{b=0}^{\infty} \left[G^{(b)}(\tau) - G^{(b+1)}(\tau)\right] \cdot U(n + b) = \\ &= \sum_{b=0}^{\infty} \left[G^{(b)}(\tau) - G^{(b+1)}(\tau)\right] \cdot \left[1 - F^{(n+b)}(\tau)\right], \; n = 1, 2, ... \end{aligned} \tag{9}$$

The distribution function $Z^{(k)}$ expresses the probability that the number of objects not serviced at the traffic maintenance subsystem posts over the time interval τ is lower than $n = L(\tau) - k$, i.e. that the number of technological objects serviced at the traffic maintenance subsystem posts over the time interval τ equals at least k. Then the relation $Z^{(k)}$ thus determined is the characteristic defining the efficiency of the traffic maintenance subsystem posts.

The random variables T and Θ being the length of time the technological object remains, respectively, outside the traffic maintenance posts or at the traffic maintenance posts, may be defined by distributions of various types. In the analysis of the technological objects service systems, we often use the exponential and Erlang distribution to talk about time distribution between consecutive reports and the time of service. Making an assumption about the exponential distribution of time is, first of all, comfortable from the mathematical point of view, and, second of all, results from the fact that implementing exponential distribution makes it possible to perform a precise enough analysis of the operation of any service system if the analyzed system is a "no memory" one. The probability of completing the servicing of any technological object at any consecutive time interval is, then, independent from the length of time interval planned for its service [1, 2, 8, 17, 18]. Implementing Erlang distribution makes it possible to analyze a wider array of technological objects service systems for which the times between consecutive reports and times of service may be defined using exponential distribution, gamma distribution as well as in the case of the analysis of periodical and regular report times, when the length of the interval between consecutive reports is constant [2, 3, 6, 11]. The following formulas determine the value of efficiency index of the traffic maintenance subsystem posts efficiency established for exponential and Erlang distribution.

Exponential distribution

If random variable T is the length of time of the technological object remaining outside the subsystem of traffic maintenance and has exponential distribution with distribution function

$$F(t) = P(T < t) = 1 - e^{-\gamma \cdot t}, \ t \geq 0, \tag{10}$$

then the number $L(\tau)$ of technological objects directed to the traffic maintenance subsystem posts over the time interval τ has Poisson distribution with parameter $\gamma \cdot \tau$, i.e.:

$$P(L(\tau) = n) = \frac{(\gamma \cdot \tau)^n}{n!} e^{-\gamma \cdot \tau}, \ n = 0, 1, 2, ..., \tag{11}$$

whence:

$$F^{(n)}(\tau) = 1 - \sum_{b=1}^{n-1} \frac{(\gamma \cdot \tau)^b}{b!} e^{-\gamma \cdot \tau}, \ n = 1, 2, ... \tag{12}$$

If random variable Θ is the length of time the technological object remains at the posts of the traffic maintenance subsystem and has exponential distribution with distribution function

$$G(t) = P(\Theta < t) = 1 - e^{-\beta \cdot t}, \ t \geq 0, \tag{13}$$

then the number $N(\tau)$ of technological objects serviced at the traffic maintenance subsystem posts over the time interval τ has Poisson distribution with parameter $\beta \cdot \tau$, i.e.:

$$P(N(\tau) = n) = \frac{(\beta \cdot \tau)^n}{n!} e^{-\beta \cdot \tau}, \ n = 0, 1, 2, ..., \tag{14}$$

whence:

$$G^{(n)}(\tau) = 1 - \sum_{b=1}^{n-1} \frac{(\beta \cdot \tau)^b}{b!} e^{-\beta \cdot \tau}, \ n = 1, 2, ... \tag{15}$$

Then the relation (9) is as follows:

$$\begin{aligned} Z^{(k)} &= P(M(\tau) < n) = P(L(\tau) - N(\tau) < n) = \\ &= \sum_{b=0}^{\infty} \left[\frac{(\beta \cdot \tau)^b}{b!} e^{-\beta \cdot \tau} \cdot \sum_{c=0}^{n+b-1} \frac{(\gamma \cdot \tau)^c}{c!} e^{-\gamma \cdot \tau} \right] \end{aligned} \tag{16}$$

Erlang distribution

If random variable T is the length of time the technological object remains outside the traffic maintenance subsystem and has Erlang distribution rank s with distribution function

$$F(t) = P(T < t) = \int_0^{\gamma \cdot x} \frac{u^{s-1}}{(s-1)!} e^{-u} du, \; x \geq 0, \; \gamma > 0, \tag{17}$$

then the number $L(\tau)$ of technological objects directed to the traffic maintenance subsystem posts over the time interval τ has the following distribution:

$$P(L(\tau) = n) = \sum_{k=n \cdot s}^{n \cdot s + s - 1} \frac{(\gamma \cdot \tau)^k}{k!} e^{-\gamma \cdot \tau}, \quad n = 0, 1, 2, ... \tag{18}$$

If random variable Θ is the length of time the technological object remains at the posts of the traffic maintenance subsystem has Erlang distribution rank r with the following distribution function

$$G(t) = P(\Theta < t) = \int_0^{\beta \cdot x} \frac{u^{r-1}}{(r-1)!} e^{-u} du, \quad x \geq 0, \quad \beta > 0, \tag{19}$$

then the number $N(\tau)$ of technological objects serviced at the traffic maintenance subsystem posts over the time interval τ has the following distribution:

$$P(N(\tau) = n) = \sum_{k=n \cdot r}^{n \cdot r + r - 1} \frac{(\beta \cdot \tau)^k}{k!} e^{-\beta \cdot \tau}, \quad n = 0, 1, 2, ... \tag{20}$$

Then the relation (9) is as follows

$$\begin{aligned} Z^{(k)} &= P(M(\tau) < n) = P(L(\tau) - N(\tau) < n) = \\ &= \sum_{b=0}^{\infty} \left[\sum_{k=b \cdot r}^{b \cdot r + r - 1} \frac{(\beta \cdot \tau)^k}{k!} e^{-\beta \cdot \tau} \cdot \sum_{l=(n+b) \cdot (s-1)}^{(n+b) \cdot s - 1} \frac{(\gamma \cdot \tau)^l}{l!} e^{-\gamma \cdot \tau} \right] \end{aligned} \tag{21}$$

3. Examples of Test Results

Following are examples of efficiency index calculation results determined for repair post at the traffic maintenance subsystem. The values of repair efficiency index were determined for both exponential distribution and Erlang distribution characteristics, with the assumption that the expected values of the length of time the technological object (means of transport) remains at the repair posts and length of time the technological object (means of transport) remains outside the repair posts are of the same value for both types.

Table 1 presents the values of basic parameters defining the process of carrying out repairs at the traffic maintenance subsystem. The values of the parameters necessary for determining the efficiency index were established based on the exploitation data obtained from exploitation database used at the tested means of transport exploitation system.

Table 1

Values of basic parameters defining the repair process carried out at the means of transport exploitation system

Time outside post	Average value	$\bar{T}$	91.53 [h]
	Standard error	$\sigma(T)$	22.88 [h]
Time at post	Average value	$\bar{\Theta}$	3.24 [h]
	Standard error	$\sigma(\Theta)$	1.62 [h]
Erlang distribution shape parameter for random variable T		s	4
Erlang distribution shape parameter for random variable Θ		r	2
Average 24 hour number of repairs at post		N	2.9825

Following are charts of the efficiency index for repair post at the traffic maintenance subsystem determined for the values of parameters presented in Table 1 and various values of numbers of repaired technological objects (means of transport).

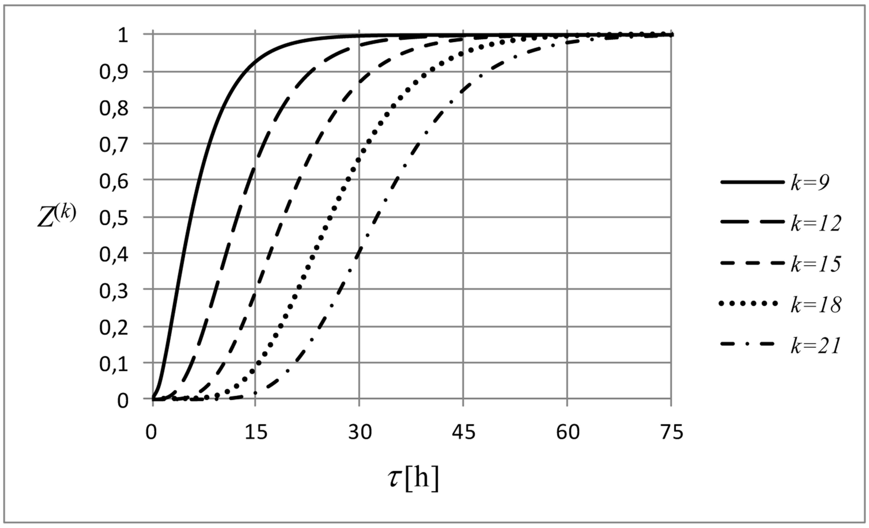

Fig. 1. Values of the repair post efficiency index in relation to time of repair τ as well as number of repaired objects k – exponential distribution

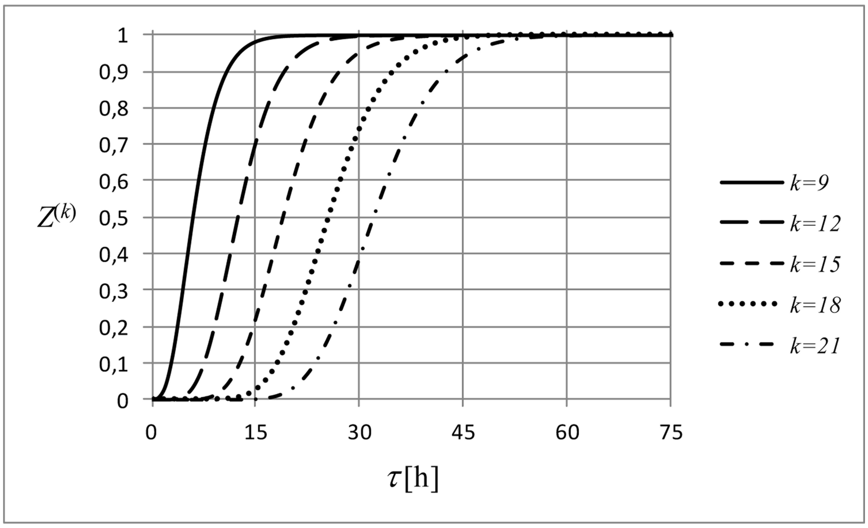

Fig. 2. Values of the repair post efficiency index in relation to repair time τ as well as number of repaired objects k – Erlang distribution

4. Summary

Based on the traffic maintenance subsystem repair post efficiency index test results presented (in Figures 1 and 2), one notices the following:

1. as servicing time τ increases, for the given number of repaired technological objects k the value of the traffic maintenance subsystem repair post efficiency index increases,
2. the minimum repair time $\tau_{min.}$ for the given number of repaired technological objects k increases as number k increases,
3. when random variables T and Θ are defined by Erlang distribution for the given number of the repaired technological objects k, the repair efficiency index achieves higher values over a shorter time interval than in the case of exponential distribution.

In the case of tests of the processes carried out at the traffic maintenance subsystem, the values of the subsystem posts efficiency index determined based on the presented models may be used for the evaluation of the degree of carrying out of the assigned service and repair tasks, for both individual posts as well as the whole subsystem. The evaluation of the efficiency of the traffic maintenance subsystem posts thus obtained forms a piece of information the use of which during the process of exploitation control carried out in the tested system will facilitate rational decision making in terms of adjusting the traffic maintenance subsystem to the current needs of the means of transport exploitation system, e.g. as a result of:

- modernization of the posts,
- replacing the posts with higher efficiency posts,

- changing the number of the individual types of posts,
- implementing multi-purpose posts where, if needs be, servicing, diagnostics as well as means of transport repair may be carried out,
- changes in organization and conditions of carrying out of the processes at the analyzed system posts.

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