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Shock Processes on Railway Vehicles with One-Stage Spring Suspension

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Abstract

The influence of the shock processes on railway vehicles with one-stage spring suspension is analyzed in this paper. These processes are excited in normal operation. Kinetic energy is lost as a result of the shock loads. Expressions are derived to calculate the kinetic energy of the different units and of the railway vehicle as a whole. These expressions allow calculating the loss of the kinetic energy of the mechanical system. Another objective of this work is to solve optimization tasks. This optimization allows the parameters of the railway vehicle to be chosen so that the loss of kinetic energy to be minimal and the effective work of the transport vehicle to be guaranteed in normal operation.

1. Introduction

The periodic shock loads on railway vehicles with one-stage spring suspension are analyzed in this study. These loads are excited when the transport vehicle passes through the rails joints. Analogous loads are performed when the brake gear locks the wheelset. In this case the wheels are skimmed along the rails and flats are formed on the tyres. Periodic impacts also arise when the wheels are placed on the axle eccentrically or when the wheelset itself is dynamically unbalanced. Shock loads arise in a result of these processes [1, 2, 3, 4, 18, 19, 21, 23]. They load up the railway vehicles and change the kinematic components of motion – linear and angular velocities. The kinetic energy is lost as a result of the different kinematic components.

The objective of this paper is determination of the expressions to calculate the kinetic energy of the different units (wheelsets, car body and bogies) and of

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the mechanical system as a whole. These expressions allow the complete energy loss of the railway vehicle to be calculated. Another objective of this work is to solve optimization tasks [24, 25, 26, 27, 28]. This optimization allows to choose the parameters of the railway vehicle, i.e. the elastic constants of the spring suspension and the coefficients of the damping so that the loss of kinetic energy to be minimal depending on the mass of the wagon and the transport vehicle to work appropriately in normal operation.

2. Expose

2.1. Dynamic model

The dynamic model is shown in Fig. 1 in order to examine the influence of the periodic shock loads [11, 13].

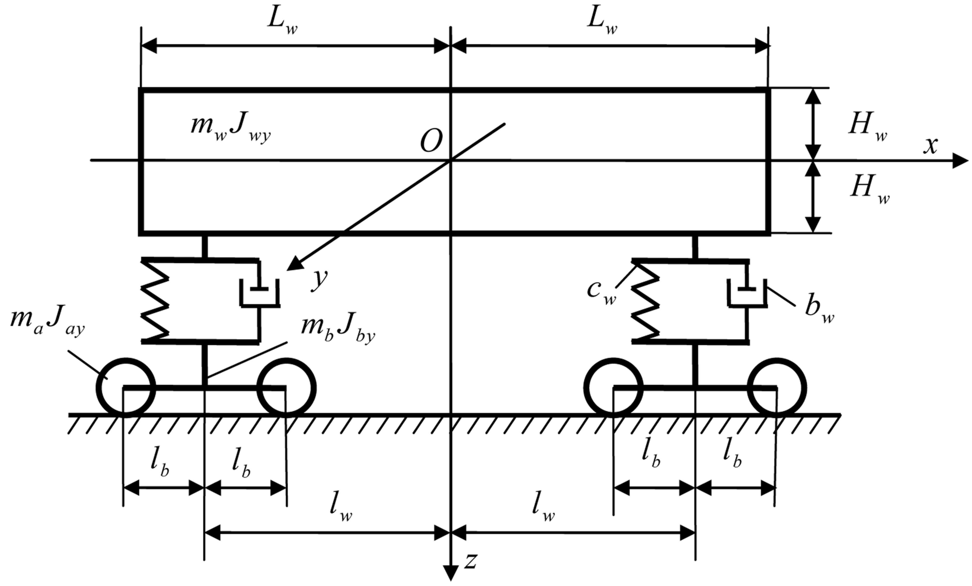


Fig. 1. Dynamic model of one-stage spring suspension railway vehicle

We define the following symbols:

J_{ay} , J_{by} , J_{wy} are respectively the mass moments of inertia of the wheelset, the frame of the bogie and the car body of the wagon,

m_a, m_b, m_w are respectively the masses of the wheelset, the frame of the bogie and the car body of the wagon,

$2l_b$ and $2l_w$ are the bases of the bogie and the car body of the wagon respectively,

$2L_w$ and $2H_w$ are the length and the height of the car body.

The elastic constant of the spring suspension and the damping coefficient are marked with c_w and β_w.

2.2. Kinematic components of motion

The kinematics components of motion are the angular velocities of the rigid bodies, building the railway vehicle and the linear velocities of their centres of masses. These kinematics components change as a result of the shock impacts. We use a given computational scheme in order to determine them as well as equations corresponding to the type of motion of the bodies at the end of the shock process. The shock impulses on each body are determined as well. We must take into account the condition of the mechanical system before the shock impact on the wheelset.

2.2.1. Wheelset motion kinematic components

The wheelset performs plane motion. We use Fig. 2 in order to examine the influence of the periodic shock loads on the wheelsets in one-stage spring suspension wagons [12].

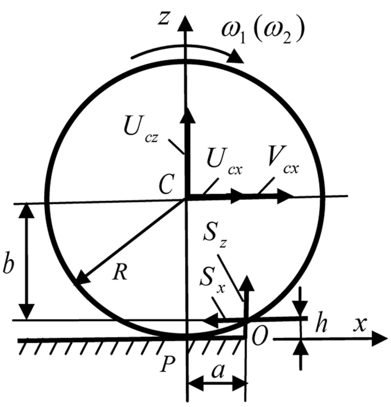


Fig. 2. Wheelset motion kinematic components

The kinematic components of motion before the shock impact are the velocities V_{Cx} and V_{Cz} of the centre of mass C and the angular velocity ω_1 of the wheelset. They are calculated by using the expressions written below:

$$V_{Cx} = V_{res}\,, \quad V_{Cz} = 0\,, \quad \omega_1 = V_{res}/R, \tag{1}$$

where V_{res} is such velocity at which the car body of the wagon will perform vertical vibrations with maximum amplitude as a result of the influence of the vertical impulses. The following dependence for the resonant velocity is received in the work [11], using the formula between t_0 and T_ε:

$$V_{res} = \frac{l_0 \omega_\varepsilon}{2\pi n_1}, \tag{2}$$

where l_0 is the length of each railway section.

The phenomenon “shock resonance” arises if the instantaneous impulse period t_0 contains an integer number of free vibration periods T_ε, i.e. $t_0 = n_1 T_\varepsilon$, (n_1 is an integer. We may assume $n_1 = 2$). We determine the free vibrations period T_ε using the dependence $T_\varepsilon = 2\pi/\omega_\varepsilon$. ω_ε is the smallest natural frequency of the mechanical system and it can determine by using the following expression [10, 12]:

$$\omega_\varepsilon = \sqrt{\omega_0^2 - \varepsilon^2}, \tag{3}$$

where $\varepsilon = \beta_\Sigma / 2m_w$ and $\omega_0 = \sqrt{c_\Sigma / m_w}$ is the wagon’s natural frequency disregarding damping.

We denote the summary elastic constant of the spring suspension and the summary damping coefficient in the upper expressions by c_Σ and β_Σ. They are shown in Fig. 3 [11, 12] and are determined for every concrete case.

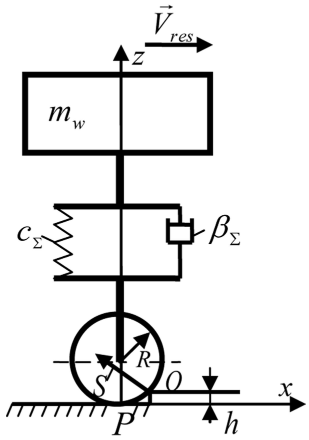


Fig. 3. Simplified dynamic model

The kinematic components of motion after the shock impacts are the velocities U_{Cx} and U_{Cz} of the centre of mass C and the angular velocity ω_2 of the wheelset. They are calculated by the expressions written below [14]:

$$U_{Cx} = \frac{2hm_{wi}\omega_\varepsilon b}{m_a a} + kV_{res}, \quad U_{Cz} = \frac{2hm_{wi}\omega_\varepsilon}{m_a}, \quad \omega_2 = \frac{V_{res}}{R} - \frac{2hm_{wi}\omega_\varepsilon (b-a)}{J_{ay}}, \tag{4}$$

where m_{wi} is the part of the mass of the car body on the first wheelset.

The coefficient of restitution k lies in the interval $0 \leq k \leq 1$. We assume $k = 0.55$. The difference between two neighboring rails is marked with h. The geometric sizes, shown in Fig. 2, can be calculated by the expressions: $a \approx \sqrt{2Rh}$, $b = R - h$.

The expressions above can be given in the following form, regarding four-axle wagon:

$$U_{Cx} = \frac{hm_w\omega_\varepsilon b}{2m_a a} + kV_{res}, \quad U_{Cz} = \frac{hm_w\omega_\varepsilon}{2m_a}, \quad \omega_2 = \frac{V_{res}}{R} - \frac{hm_w\omega_\varepsilon(b-a)}{2J_{ay}}. \tag{5}$$

2.2.2. Car body motion kinematic components

The car body of the wagon performs translational motion before the shock impact. The horizontal velocity is V_{Ox} and it is calculated through the dependence $V_{Ox} = V_{res}$. This velocity is car body motion kinematic component before the shock.

The car body of the wagon performs plane motion provoked by shock loads. Motion kinematic components are the angular velocity $\dot{\phi}_w$ and the velocities U_{Ox} and $\dot{z}_w$ of the centre of mass O. These kinematic components are shown in Fig. 4.

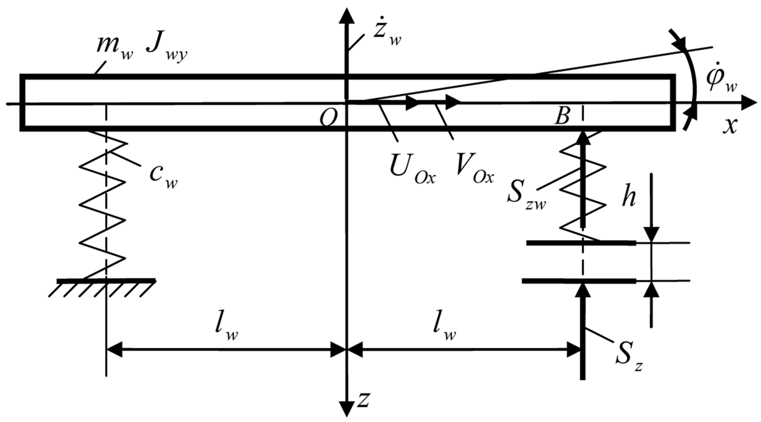


Fig. 4. Shock loads on the wagon car body

They are calculated by the expressions written below [14]:

$$U_{Ox} = \frac{2hm_{wi}\omega_\varepsilon b}{m_a a} + kV_{res}, \quad \dot{z}_w = h\omega_\varepsilon, \quad \dot{\phi}_w = \frac{hm_w l_w}{J_{wy}}\omega_\varepsilon \tag{6}$$

In case of four-axle wagon, we receive the following expressions:

$$U_{Ox} = \frac{hm_w\omega_\varepsilon b}{2m_a a} + kV_{res}, \quad \dot{z}_w = h\omega_\varepsilon, \quad \dot{\phi}_w = \frac{hm_w l_w}{J_{wy}}\omega_\varepsilon \tag{7}$$

2.2.3. Leading bogie motion kinematic components

One-stage spring suspension bogies are used mainly in boxcar. The spring suspension can be either central spring suspension or axle box spring suspension. Figure 5 shows an axle box spring suspension [6, 14, 22]. The spring pairs are situated between the bogie frame and the wheelsets. G_{wj} is the part of the weight of the car body, which load up the bogie.

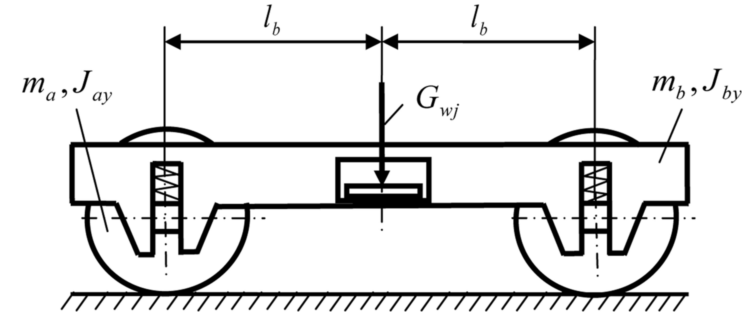


Fig. 5. An axle box spring suspension

The bogie frame performs translational motion before the shock impact and it is moving at resonant velocity $V_{b1x} = V_{res}$. It performs plane motion after the shock impact. The scheme of the leading bogie is shown in Fig. 6, presented below:

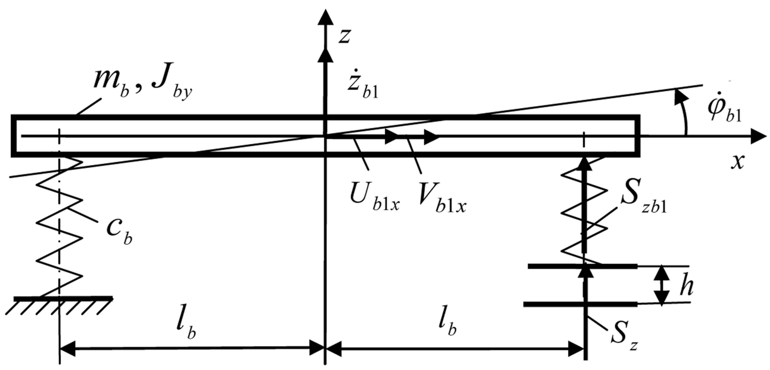


Fig. 6. Scheme of the leading bogie

Kinematic components of motion after the shock are the velocities U_{b1x} and $\dot{z}_{b1}$ of the centre of mass and the angular velocity $\dot{\phi}_{b1}$. We assume that vertical velocities are equal, i.e. $\dot{z}_{b1} = \dot{z}_w$, because the joint between the car body and the bogie is a rigid one. We determine the angular velocity $\dot{\phi}_{b1}$ from the expression, given below:

$$J_{by}(\dot{\phi}_{b1} - \omega_{b1}) = S_{zb1} l_b \tag{8}$$

where $\omega_{b1} = 0$ is the bogie angular velocity before the shock impact. The vertical impulse S_z is shown in Figs. 2 and 6. S_{zb1} is the impulse that loads the bogie [10, 15, 16]. We assume that $S_{zb1} = S_z = hm_{wj}\omega_\varepsilon$ because the rigidity of the spring suspension is big. The angular velocity after the shock is calculated from the following expression:

$$J_{by}\dot{\phi}_{b1} = hm_{wj}\omega_\varepsilon l_b\,. \tag{9}$$

where m_{wj} is the part of the mass of the car body that loads the bogie.

Finally, we get the following expression for the kinematic components $\dot{z}_{b1}$ and $\dot{\phi}_{b1}$:

$$\dot{z}_{b1} = h\omega_\varepsilon\,, \quad \dot{\phi}_{b1} = \frac{hm_{wj}l_b\omega_\varepsilon}{J_{by}}\,. \tag{10}$$

We calculate the horizontal velocities before and after the shock using expressions (2) and (7), i.e. $V_{b1x} = V_{Ox}, U_{b1x} = U_{Ox}$. The expressions above can be given in the following form, regarding four-axle wagon:

$$\begin{aligned} &V_{b1x} = V_{res} = \frac{l_0\omega_\varepsilon}{2\pi n_1}, \quad U_{b1x} = \frac{hm_w\omega_\varepsilon b}{2m_a a} + kV_{res}\,, \\ &\dot{z}_{b1} = h\omega_\varepsilon\,, \quad \dot{\phi}_{b1} = \frac{hm_w l_b\omega_\varepsilon}{2J_{by}}\,. \end{aligned} \tag{11}$$

2.2.4. Trailer bogie motion kinematic components

The trailer bogie performs translational motion before the shock load and plane motion after the shock impact. The scheme of the trailer bogie is shown in Fig. 7.

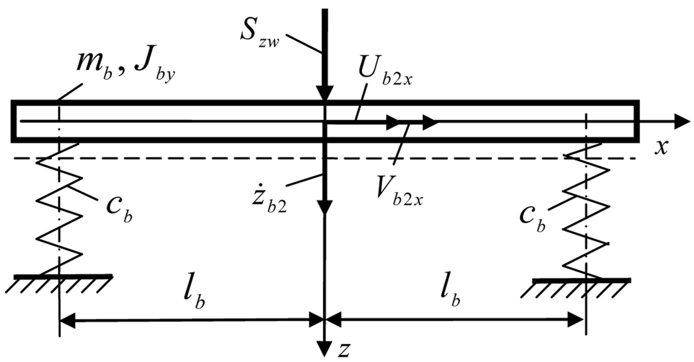


Fig. 7. Scheme of the trailer bogie

The bogie is moving at velocity $V_{b2x} = V_{res}$ before the shock. It is moving with horizontal and vertical velocities U_{b2x} and $\dot{z}_{b2}$ after the shock. Horizontal velocities of trailer and leading bogies after the shock impact are equal, i.e. $U_{b2x} = U_{b1x}$ and can be determined by using the dependence (11).

We use the following dependence in order to determine the vertical velocity $\dot{z}_{b2}$ of the centre of mass after the shock:

$$\dot{z}_{b2} = \dot{\phi}_w l_w - \dot{z}_w \,. \tag{12}$$

We derive the final dependence for the velocity $\dot{z}_{b2}$ substituting $\dot{\phi}_w$ and $\dot{z}_w$ with their equal values:

$$\dot{z}_{b2} = \frac{h}{J_{wy}} \left(m_w l_w^2 - J_{wy} \right) \omega_\varepsilon. \tag{13}$$

The final expressions for the motion kinematic components in case of four-axle wagon are presented in Table 1.

Table 1

Motion Kinematic Components before and after the shock process

Before the shock process			
Wheelset	Car body	Leading bogie	Trailer bogie
$V_{Cx} = V_{res} = l_0\omega_\varepsilon/2\pi n_1$	$V_{Ox} = V_{res}$	$V_{b1x} = V_{res}$	$V_{b2x} = V_{res}$
$V_{Cz} = 0$	$V_{Oz} = 0$	$V_{b1z} = 0$	$V_{b2z} = 0$
$\omega_1 = V_{res}/R$	0	0	0
After the shock process			
$U_{Cx} = \dfrac{hm_w\omega_\varepsilon b}{2m_a a} + kV_{res}$	$U_{Ox} = \dfrac{hm_w\omega_\varepsilon b}{2m_a a} + kV_{res}$	$U_{b1x} = \dfrac{hm_w\omega_\varepsilon b}{2m_a a} + kV_{res}$	$U_{b2x} = U_{b1x}$
$U_{Cz} = \dfrac{hm_w\omega_\varepsilon}{2m_a}$	$\dot{z}_w = h\omega_\varepsilon$	$\dot{z}_{b1} = h\omega_\varepsilon$	$\dot{z}_{b2} = \dfrac{h\left(m_w l_w^2 - J_{wy}\right)\omega_\varepsilon}{J_{wy}}$
$\omega_2 = \dfrac{V_{res}}{R} - \dfrac{hm_w\omega_\varepsilon(b-a)}{2J_{ay}}$	$\dot{\phi}_w = \dfrac{hm_w l_w}{J_{wy}}\omega_\varepsilon$	$\dot{\phi}_{b1} = \dfrac{hm_w l_b\omega_\varepsilon}{2J_{by}}\,.$	0

2.3. Mechanical system kinetic energy

We use expressions for the kinematic components of motion (the angular velocities of the wheelsets, the car body and the bogies and the velocities of their centres of masses) presented in Table 1 to calculate the kinetic energy of the mechanical system.

2.3.1. Wheelset kinetic energy

We use well-known expressions in the literature [17] in order to determine its kinetic energy before and after the shock process.

We determine the kinetic energy before the shock using the following expression:

$$T_{a1} = \frac{1}{2}m_a V_{res}^2 + \frac{1}{2}J_{ay}\omega_1^2 = \frac{1}{2}\left(m_a + \frac{J_{ay}}{R^2}\right)\left(\frac{l_0}{2\pi n_1}\right)^2 \omega_\varepsilon^2. \tag{14}$$

The kinetic energy after the shock can be determined using the following formula:

$$T_{a2} = \frac{1}{2}m_a\left(U_{Cx}^2 + U_{Cz}^2\right) + \frac{1}{2}J_{ay}\omega_2^2, \tag{15}$$

The expression above describes the wheelset kinetic energy after the shock impact and can be written in the following form:

$$T_{a2} = \frac{1}{2}m_a\left[\left(\frac{2hm_{wi}b}{m_a a} + \frac{kl_0}{2\pi n_1}\right)^2 + \left(\frac{2hm_{wi}}{m_a}\right)^2\right]\omega_\varepsilon^2 + \frac{1}{2}J_{ay}\left[\frac{l_0}{2\pi n_1 R} - \frac{2hm_{wi}(b-a)}{J_{ay}}\right]^2\omega_\varepsilon^2, \tag{16}$$

In case of four-axle wagon we receive the following expressions:

$$T_{a2} = \frac{1}{2}m_a\left[\left(\frac{hm_w b}{2m_a a} + \frac{kl_0}{2\pi n_1}\right)^2 + \left(\frac{hm_w}{2m_a}\right)^2\right]\omega_\varepsilon^2 + \frac{1}{2}J_{ay}\left[\frac{l_0}{2\pi n_1 R} - \frac{hm_w(b-a)}{2J_{ay}}\right]^2\omega_\varepsilon^2. \tag{17}$$

2.3.2. Wheelset kinetic energy loss

We create the following formula in order to determine the kinetic energy loss of the wagon wheelset [17]:

$$_\Delta T_a = T_{a2} - T_{a1}, \tag{18}$$

where T_{a2} and T_{a1} are replaced by their equivalents from expressions (14) and (16). As the result we receive:

$$_\Delta T_a = \frac{1}{2}\left(M_{red} + J_{red2} - J_{red1}\right)\omega_\varepsilon^2. \tag{19}$$

We perform the following replacements in the expression given above:

$$J_{red1} = \left(m_a + \frac{J_{ay}}{R^2}\right)\left(\frac{l_0}{2\pi n_1}\right)^2, \quad J_{red2} = J_{ay}\left[\frac{l_0}{2\pi n_1 R} - \frac{2hm_{wi}(b-a)}{J_{ay}}\right]^2,$$

$$M_{red} = m_a\left[\left(\frac{2hm_{wi}b}{m_a a} + \frac{kl_0}{2\pi n_1}\right)^2 + \left(\frac{2hm_{wi}}{m_a}\right)^2\right].$$

We can calculate the kinetic energy loss for the four-axle wagon, using the expressions (14) and (17) given above by ignoring the vertical velocity of the first wheelset after the shock impact. In this case, the expressions are the same for all four wheelsets:

$$_\Delta T_a = \frac{1}{2}m_a\left(A_a m_w^2 + B_a m_w + C_a\right)\omega_\varepsilon^2, \tag{20}$$

where the constants A_a, B_a, C_a are determined from the dependences:

$$A_a = \frac{h^2}{4m_a}\left[\frac{b^2}{m_a a^2} + \frac{(b-a)^2}{J_{ay}}\right], \quad B_a = \frac{h}{2m_a}\left[\frac{kl_0 b}{\pi n_1 a} - \frac{l_0(b-a)}{\pi n_1 R}\right], \quad C_a = \frac{l_0^2(k^2-1)}{(2\pi n_1)^2}.$$

2.3.3. Car body kinetic energy

The car body kinetic energy before the shock impact is calculated using the expression, shown below:

$$T_{w1} = \frac{1}{2} m_w V_{res}^2. \tag{21}$$

The kinetic energy after the shock can be determined using the following formula:

$$T_{w2} = \frac{1}{2} m_w \left(U_{Ox}^2 + \dot{z}_w^2\right) + \frac{1}{2} J_{wy} \dot{\phi}_w^2, \tag{22}$$

We receive the expressions for calculation of the car body kinetic energy before and after the shock, using formulae (21) and (22). These expressions are presented in the following expressions.

$$\begin{aligned} T_{w1} &= \frac{1}{2} m_w \left(\frac{l_0}{2\pi n_1}\right)^2 \omega_\varepsilon^2, \\ T_{w2} &= \frac{1}{2} m_w \left[\left(\frac{2hm_{wi}b}{m_a a} + \frac{kl_0}{2\pi n_1}\right)^2 + h^2\left(1 + \frac{m_w l_w^2}{J_{wy}}\right)\right] \omega_\varepsilon^2. \end{aligned} \tag{23}$$

We transform the expressions (23) in case of four-axle wagon.

$$T_{w2} = \frac{1}{2} m_w \left[\left(\frac{hm_w b}{2m_a a} + \frac{kl_0}{2\pi n_1}\right)^2 + h^2\left(1 + \frac{m_w l_w^2}{J_{wy}}\right)\right] \omega_\varepsilon^2. \tag{24}$$

2.3.4. Car body kinetic energy loss

We determine the kinetic energy loss of the car body due to the shock impact using the expression given below:

$$_\Delta T_w = T_{w2} - T_{w1}. \tag{25}$$

We receive the wanted formula for $_\Delta T_w$ after replacing T_{w1} and T_{w2} with their equivalents from (23):

$$_\Delta T_w = \frac{1}{2} m_w A_\varepsilon \omega_\varepsilon^2, \tag{26}$$

where the following replacement is performed:

$$A_\varepsilon = \left(\frac{2hm_{wi}b}{m_a a} + \frac{kl_0}{2\pi n_1}\right)^2 + h^2\left(1 + \frac{m_w l_w^2}{J_{wy}}\right) - \left(\frac{l_0}{2\pi n_1}\right)^2.$$

We analyze the case, when the wagon has four wheelsets, i.e. $m_{wi} = m_w/4$. Transforming the expression above we get to its final form:

$$_\Delta T_w = \frac{1}{2} m_w \left(A_w m_w^2 + B_w m_w + C_w \right) \omega_\varepsilon^2, \tag{27}$$

where the constants A_w, B_w and C_w can be calculated using the replacements:

$$A_w = \left(\frac{hb}{2m_a a} \right)^2, \quad B_w = \frac{2hbkl_0}{m_a a \pi n_1}, \quad C_w = \left(\frac{l_0}{2\pi n_1} \right)^2 \left(k^2 - 1 \right) + h^2 \left(1 + \frac{3l_w^2}{L_w^2 + H_w^2} \right).$$

2.3.5. Leading bogie kinetic energy

We calculate the kinetic energy of the leading bogie before and after the shock impact using the expressions below:

$$\begin{gathered} T_{bl1} = \frac{1}{2} m_b V_{res}^2 . \\ T_{bl2} = \frac{1}{2} m_b \left(U_{b1x}^2 + \dot{z}_{b1}^2 \right) + \frac{1}{2} J_{by} \dot{\phi}_{b1}^2, \end{gathered} \tag{28}$$

In case of four-axle wagon the velocities in the upper expression are replaced with their equivalent values, presented in Table 1. We obtain the following form after some transformations:

$$T_{bl2} = \frac{1}{2} m_b \left[\left(\frac{h^2 b^2}{4m_a^2 a^2} + \frac{h^2 l_b^2}{4m_b J_{by}} \right) m_w^2 + \frac{hbkl_0}{2m_a a \pi n_1} m_w + \left(\frac{kl_0}{2\pi n_1} \right)^2 + h^2 \right] \omega_\varepsilon^2 . \tag{29}$$

2.3.6. Leading bogie kinetic energy loss

We determine the kinetic energy loss of the leading bogie using the dependence given below:

$$\Delta T_{bl} = T_{bl2} - T_{bl1}. \tag{30}$$

We get the following equality by replacing T_{bl1} and T_{bl2} with their equivalent values:

$$_\Delta T_{bl} = \frac{1}{2} m_b \left[\left(\frac{h^2 b^2}{4m_a^2 a^2} + \frac{h^2 l_b^2}{4m_b J_{by}} \right) m_w^2 + \frac{hbkl_0}{2m_a a \pi n_1} m_w + \left(\frac{kl_0}{2\pi n_1} \right)^2 + h^2 - \left(\frac{l_0}{2\pi n_1} \right)^2 \right] \omega_\varepsilon^2. \tag{31}$$

The expression above is presented in the following short form:

$$_\Delta T_{bl} = \frac{1}{2} m_b \left(A_{bl} m_w^2 + B_{bl} m_w + C_{bl} \right) \omega_\varepsilon^2, \tag{32}$$

where the following replacements are made:

$$A_{bl} = \frac{h^2 b^2}{4m_a^2 a^2} + \frac{h^2 l_b^2}{4m_b J_{by}}, \quad B_{bl} = \frac{hbkl_0}{2m_a a \pi n_1}, \quad C_{bl} = \left(\frac{l_0}{2\pi n_1} \right)^2 \left(k^2 - 1 \right) + h^2.$$

2.3.7. Trailer bogie kinetic energy

We calculate the trailer bogie kinetic energy before and after the shock from the expressions given below:

$$T_{bt1} = \frac{1}{2} m_b V_{res}^2 .$$
$$T_{bt2} = \frac{1}{2} m_b \left(U_{b2x}^2 + \dot{z}_{b2}^2 \right), \tag{33}$$

The velocities in the upper expression are replaced with their equivalent values given in Table 1 and the following equality is obtained:

$$T_{bt2} = \frac{1}{2} m_b \left[\left(\frac{h^2 b^2}{4 m_a^2 a^2} + \frac{h^2 l_w^4}{J_{wy}^2} \right) m_w^2 + \left(\frac{hbkl_0}{2 m_a a \pi n_1} - \frac{2 h^2 l_w^2}{J_{wy}} \right) m_w + \left(\frac{kl_0}{2\pi n_1} \right)^2 + h^2 \right] \omega_\varepsilon^2 . \tag{34}$$

2.3.8. Trailer bogie kinetic energy loss

The expression used to determine kinetic energy loss of the trailer bogie is identical to the one for the leading bogie, i.e.

$${}_\Delta T_{bt} = T_{bt2} - T_{bt1} . \tag{35}$$

We replace the expressions for T_{bt1} and T_{bt2} with their equal values and we receive the following dependence for the kinetic energy loss.

$${}_\Delta T_{bt} = \frac{1}{2} m_b \left[\left(\frac{h^2 b^2}{4 m_a^2 a^2} + \frac{h^2 l_w^4}{J_{wy}^2} \right) m_w^2 + \left(\frac{hbkl_0}{2 m_a a \pi n_1} - \frac{2 h^2 l_w^2}{J_{wy}} \right) m_w + \left(\frac{kl_0}{2\pi n_1} \right)^2 + h^2 - \left(\frac{l_0}{2\pi n_1} \right)^2 \right] \omega_\varepsilon^2 . \tag{36}$$

This expression can be expressed in the following short form:

$${}_\Delta T_{bt} = \frac{1}{2} m_b \left(A_{bt} m_w^2 + B_{bt} m_w + C_{bt} \right) \omega_\varepsilon^2 , \tag{37}$$

where the following replacements are performed:

$$A_{bt} = \left(\frac{hb}{2 m_a a} \right)^2 + \left(\frac{h l_w^2}{J_{wy}} \right)^2 , \quad B_{bt} = \left(\frac{hbkl_0}{2 m_a a \pi n_1} - \frac{2 h^2 l_w^2}{J_{wy}} \right) , \quad C_{bt} = \left(\frac{l_0}{2\pi n_1} \right)^2 \left(k^2 - 1 \right) + h^2 .$$

2.4. Wagon complete kinetic energy loss

The complete kinetic energy loss can be calculated from the expression given below:

$${}_\Delta T = {}_\Delta T_w + {}_\Delta T_{bl} + {}_\Delta T_{bt} + 4 {}_\Delta T_a . \tag{38}$$

This expression measures the kinetic energy loss of a four-axle wagon when the first wheelset passes through a rail joint. Kinetic energy losses of the car body, the leading bogie, the trailer bogie and the first wheelset are determined using expressions (27), (32), (37) and (20), correspondingly.

This expression can be transformed by replacing each monomial with its equivalent value:

$$\begin{aligned} {}_{\Delta}T = &\frac{1}{2}m_w\left(A_w m_w^2 + B_w m_w + C_w\right)\omega_\varepsilon^2 + \frac{1}{2}m_b\left(A_{bl} m_w^2 + B_{bl} m_w + C_{bl}\right)\omega_\varepsilon^2 + \\ &+\frac{1}{2}m_b\left(A_{bt} m_w^2 + B_{bt} m_w + C_{bt}\right)\omega_\varepsilon^2 + 4\frac{1}{2}m_a\left(A_a m_w^2 + B_a m_w + C_a\right)\omega_\varepsilon^2 . \end{aligned} \tag{39}$$

We receive the following expression after some grouping of terms:

$$ {}_{\Delta}T = \frac{1}{2}\left[\begin{array}{r} (m_w A_w + m_b A_{bl} + m_b A_{bt} + 4m_a A_a)\, m_w^2 + (m_w B_w + m_b B_{bl} + m_b B_{bt} + 4m_a B_a)\, m_w + \\ + (m_w C_w + m_b C_{bl} + m_b C_{bt} + 4m_a C_a) \end{array}\right]\omega_\varepsilon^2 . \tag{40}$$

This expression can be presented in a simplified way:

$$ {}_{\Delta}T = \frac{1}{2}\left(Am_w^2 + Bm_w + C\right)\omega_\varepsilon^2 , \tag{41}$$

where the following replacements have been made:

$$A = m_w A_w + m_b(A_{bl} + A_{bt}) + 4m_a A_a\,, \quad B = m_w B_w + m_b(B_{bl} + B_{bt}) + 4m_a B_a\,,$$

$$C = m_w C_w + m_b(C_{bl} + C_{bt}) + 4m_a C_a\,.$$

We can calculate the kinetic energy loss of the wagon using the expression (41) after the shock impact on the first wheelset. The entire kinetic energy loss can be obtained by calculating the kinetic energy loss when the four wheelsets pass through the splice of the rails. The final expression is given below:

$$ {}_{\Delta}T_\Sigma = 4\,{}_{\Delta}T = 2\left(Am_w^2 + Bm_w + C\right)\omega_\varepsilon^2 . \tag{42}$$

This expression measures the complete kinetic energy loss of the railway vehicles with one-stage spring suspensions. We consider a four-axle wagon. By analyzing this dependence, we find out that the kinetic energy loss depends on the masses, the mass moments of inertia of the vehicle and the sizes and the conditions of the springs and the dampers. These parameters are initially fixed or change just slightly. The kinetic energy loss depends mainly by the wagon mass. We can find such value of this parameter that will minimize the kinetic energy loss.

2.5. Energy losses optimization

We can establish the cases when the energy loss will be equal to zero or this loss will have a minimum value. The mass of the car body physically can not be equal to zero, i.e. $m_w \neq 0$. That is why we consider the following two cases, analyzing the expression (42):

- $\omega_\varepsilon = 0$

This condition is difficult of access because in this case the motion of the machine is aperiodic one. The basic way of shock loads protection is the lowering of the natural frequency. That is why we have to choose suitable values for c_Σ and β_Σ so that the natural frequency ω_ε has a minimum value.

- $Am_w^2 + Bm_w + C = 0.$

In this case we transform the expression above and receive the dependence (43):

$$m_w^3 + a_s m_w^2 + b_s m_w + c_s = 0, \tag{43}$$

where $a_s = \dfrac{m_b(A_{bl} + A_w) + 4m_a A_a + B_w}{A_w}$, $\quad b_s = \dfrac{2m_b B_{bl} + 4m_a B_a + C_w}{A_w}$

$$c_s = \frac{6m_b h^2 l_w^2 \left(1,5l_w^2 - L_w^2 - H_w^2\right)}{A_w \left(L_w^2 + H_w^2\right)^2} + \frac{2C_{bl} m_b + 4m_a C_a}{A_w}.$$

This is a cubic equation with respect to m_w. We can solve it through computer calculations, using the formulas of Cardano or graphically. For this purpose, the substitution $m_w = (x - a_s/3)$ is made and the next reduced equation is received:

$$x^3 + 3px + 2q = 0 \tag{44}$$

where $p = \dfrac{b_s}{3} - \dfrac{a_s^2}{9}$, $\quad q = \dfrac{a_s^3}{27} - \dfrac{a_s b_s}{6} + \dfrac{c_s}{2}$

We receive from this equation the value for the mass m_w when the loss of energy will be equal to zero, i.e. ${}_\Delta T_\Sigma = 0$. We must know that the mass of the car body accepts only positive values. We establish the conditions when the kinetic energy loss has a minimum value if these conditions can not be performed. To this end, the expression measuring the total kinetic energy loss of the railway vehicles with one-stage spring suspension is transformed in the following pattern:

$${}_\Delta T_\Sigma = A_2 m_w^2 + A_1 m_w^1 + A_0 m_w^0 + A_{-1} m_w^{-1} + A_{-2} m_w^{-2} \tag{45}$$

where the constants A_j $(j = -2 \div 2)$ are calculated through the expressions bellow:

$$A_2 = 2c_\Sigma A_w\,,$$

$$A_1 = 2c_\Sigma \left(m_b A_{bl} + m_b A_w + 4m_a A_a + B_w - \frac{A_w \beta_\Sigma^2}{4c_\Sigma} \right),$$

$$A_0 = 2c_\Sigma \left(2m_b B_{bl} + 4m_a B_a + C_w\right) - \frac{\left(m_b A_{bl} + m_b A_w + 4m_a A_a + B_w\right)\beta_\Sigma^2}{2},$$

$$A_{-1} = \frac{18m_b c_\Sigma h^2 l_w^4}{\left(L_w^2 + H_w^2\right)^2} - m_b B_{bl}\beta_\Sigma^2 - 2m_a B_a \beta_\Sigma^2 - \frac{12m_b c_\Sigma h^2 l_w^2}{L_w^2 + H_w^2} + 4m_b c_\Sigma C_{bl} + 8m_a c_\Sigma C_a - \frac{\beta_\Sigma^2 C_w}{2},$$

$$A_{-2} = \frac{3m_b h^2 l_w^2 \beta_\Sigma^2}{L_w^2 + H_w^2} - \frac{9m_b h^2 \beta_\Sigma^2 l_w^4}{2\left(L_w^2 + H_w^2\right)^2} - \left(m_b C_{bl} + 2m_a C_a\right)\beta_\Sigma^2 .$$

The diagram, which shows the dependence between ${}_\Delta T_\Sigma$ and m_w can be drawn, i.e.${}_\Delta T_\Sigma = f\,(m_w)$. We can draw different diagrams for different values of the summary damping coefficient β_Σ and the summary elastic constants of the spring suspension c_Σ, as well as for different lengths l_0 of the railway sections. The most suitable case can be chosen among the drawn diagrams. The summary elastic constant c_Σ and the summary coefficient of damping β_Σ take part in the expressions for ${}_\Delta T_\Sigma$. They are calculated for every concrete case. Usually, the springs from the one-stage spring suspension and the dampers are connected parallel in-between.

3. Numerical Solution

This part of the developed theory is applied for a concrete wagon. We use the following initial data [5, 6, 7, 8, 9, 20] to draw the diagrams shown in Fig. 8 to Fig. 11.

- initial data

$R = 0.46$ [m], $a = 0.096$ [m], $b = 0.45$ [m], $h = 0.01$ [m], $l_0 = 12.5$ (25) [m],
$l_b = 0.9$ [m], $l_w = 8.32$ [m], $L_w = 10.85$ [m], $H_w = 1.64$ [m], $k = 0.55$, $n_1 = 2$,
$m_a = 1450$ [kg], $m_b = 2600$ [kg], $J_{ay} = 120$ [kgm^2], $J_{by} = 4000$[kgm^2].

For this purpose we resolve the cubic equations (43) or (44). It is suitable these equations to be solved graphically. In this case, we can write the function $y(m_w) = m_w^3 + a_s m_w^2 + b_s m_w + c_s$. Figure 8 shows the graphic diagram of this function.

We can determine the values of the mass m_w, in which the function $y(m_w)$ is equal to zero. These values are the roots of the cubic equation (43), i.e. $m_w^3 + a_s m_w^2 + b_s m_w + c_s = 0$. In this case we measure the following values: m_{w1} =13880 [kg], $m_{w2} = -7250$ [kg], $m_{w3} = -288960$ [kg].

The reduced cubic equation (44) is presented as: $x^3 = -3px - 2q$. The roots of this equation are the abscissas of the points of intersection of the diagrams of the functions $y_1(x) = x^3$ and $y_2(x) = -3px - 2q$. We calculate in advance the

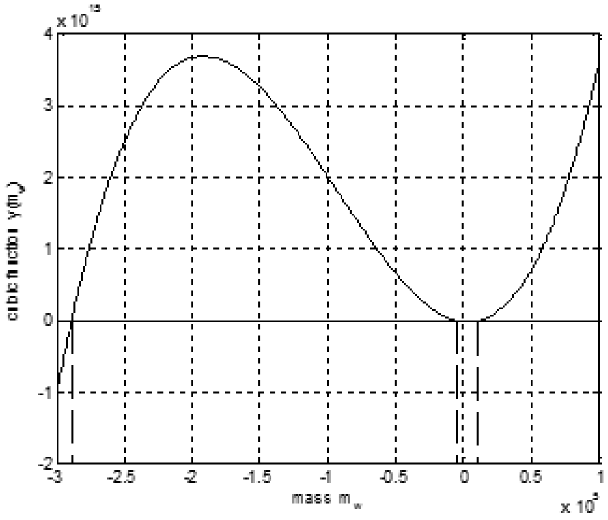


Fig. 8. Cubic function $y(m_w)$

constants p and q. In this case they accept the following values: $p = -9.5286e+009$, $q = 9.1382e+014$. Figure 9 shows the graphic diagrams of the functions $y_1(x)$ and $y_2(x)$. The roots of the cubic equation (44) are: $x_1 = 107990, x_2 = 86858, x_3 = -194848$. We calculate the mass m_w through the substitution $m_w = (x - a_s/3)$ and the received values for the independent variable x. These values are: $m_{w1} = 13879$ [kg], $m_{w2} = -7253$ [kg], $m_{w3} = -288959$ [kg].

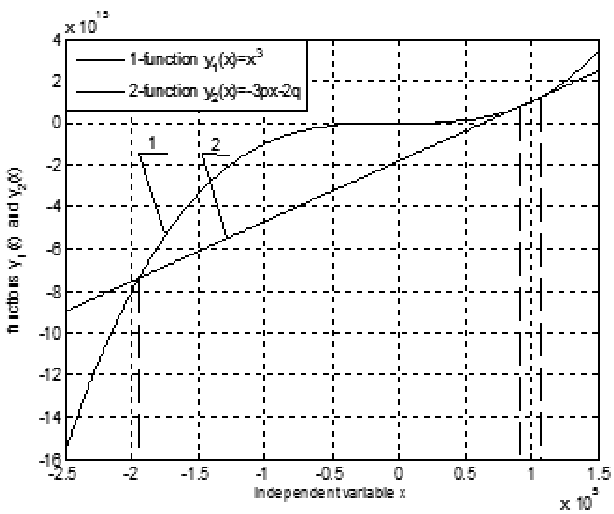
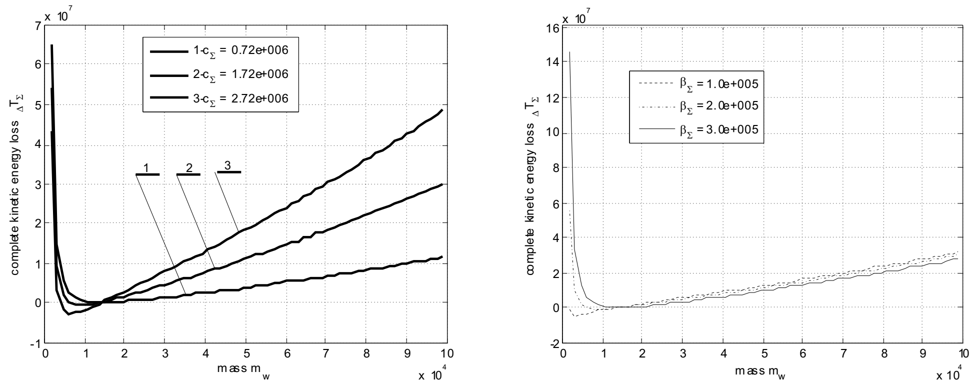


Fig. 9. Functions $y_1(x)$ and $y_2(x)$

The mass $m_{w1} = 13880\,[\text{kg}]$ is positive, but it means that the wagon can only be empty. We can determine at what loading the energy losses will be equal to zero. If these conditions can not be performed we establish the conditions when the kinetic energy loss has a minimal value. To this end, the diagrams which show the dependence between ${}_{\Delta}T_{\Sigma}$ and m_w are drawn, i.e. ${}_{\Delta}T_{\Sigma} = f(m_w)$. We can draw different diagrams for different values of the summary elastic constant of the spring suspension c_{Σ} and the summary damping coefficient β_{Σ}. These diagrams are shown in Fig.10a and Fig.10b. ($l_0 = 12.5$ m)



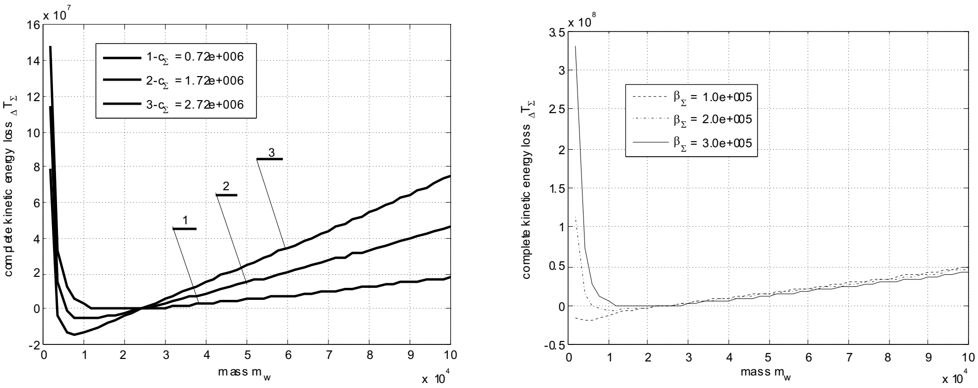
a) Different spring suspension b) Different damping

Fig. 10. Complete kinetic energy loss ${}_{\Lambda}T_{\Sigma}$ – ($l_0 = 12.5$ m)

The most suitable case is chosen among the drawn diagrams. The summary elastic constant c_{Σ} and the summary damping coefficient β_{Σ} take part in the expressions for ${}_{\Delta}T_{\Sigma}$. They are calculated for every concrete case.

The diagrams for complete kinetic energy loss ${}_{\Lambda}T_{\Sigma}$ when the length of the railway section is $l_0 = 25\,[\text{m}]$ are shown in Fig. 11a and Fig. 11b.

These diagrams show the influence of the different lengths of the railway sections. Obviously, the kinetic energy loss is bigger when the length of the railway sections is higher, i.e. $l_0 = 25$ m, because in this case the resonance velocity V_{res} is increased. We can see from these diagrams that the spring suspension has a major influence to reduce the shock impact. In this case, the energy losses are the smallest for the following value of the summary elastic constant: $c_{\Sigma} = 0.72e + 006$. Energy losses increase when increasing the stiffness of the spring suspension. The dampers have no significant influence in order to reduce the shock impact. The losses of the kinetic energy are approximately the same for the different values of the summary coefficient of damping β_{Σ}. Obviously, we must choose such spring suspension, which corresponds to the state of the railroad. The spring suspension must have less stiffness as the shock load is greater. Thus, the requirements for minimum energy loss will be performed.



a) Different spring suspension b) Different damping

Fig. 11. Complete kinetic energy loss ${}_{\Delta}T_{\Sigma}$ – (l_0 = 25 m)

4. Conclusion

In this paper the influence of the shock loads on railway vehicles with one – stage spring suspension is investigated. These loads arise in normal operation. The motion kinematic components before and after the shock are different. This means that the kinetic energy of the units before and after the shock impact will be different. The expressions calculating the energy of the different units before and after the shock impact are marked in the following way: wheelset – T_{a1} and T_{a2}, car body – T_{w1} and T_{w2}, leading bogie – T_{bl1} and T_{bl2}, trailer bogie – T_{bt1} and T_{bt2}. Thus, the dependencies for the kinetic energy loss ${}_{\Delta}T_a$, ${}_{\Delta}T_w$, ${}_{\Delta}T_{bl}$, ${}_{\Delta}T_{bt}$ are derived. The obtained expressions for the kinetic energy of the different units take part. In such way the expression (42) is defined for calculating the loss of the kinetic energy ${}_{\Delta}T_{\Sigma}$ of the railway vehicle. This expression is derived as a sum of the expressions for the loss of kinetic energy of the four wheelsets, the two bogies and the car body. In this case the shock load affects the four wheelsets.

Cubic equations are analyzed to determine at what value of the mass of the wagon the energy loss is equal to zero. These expressions are investigated. The cases, when the impulses can be eliminated are analyzed.

The expression (45) is obtained if the condition for zero energy loss can not be fulfilled. In this way, we establish in what state the wagon (state of the spring suspension and the dampers) energy losses are minimal. We can take into account and other parameters as the length of the railway section, etc.

Numerical solution is proposed. The cubic equations are resolved graphically. In this way we obtain values of the mass of the car body, in which the energy loss is zero. These values are unreal. That is way the diagrams which show the dependence between ${}_{\Delta}T_{\Sigma}$ and m_w are drawn. These diagrams are drawn for different values of the summary elastic constant of the spring suspension c_{Σ} and the summary coefficient of damping β_Σ. We can determine the most favourable case in which the shock load is minimal.

In conclusion, we can say that the received results can be used to guarantee the effectiveness of the railway vehicle in normal operation. This study can be used as a basis for future work and modelling of the shock processes so that the influence of the shock impacts to be minimal.

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