GENERALIZED ROUTE PLANNING APPROACH FOR HAZARDOUS MATERIALS TRANSPORTATION WITH EQUITY CONSIDERATION

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Abstract:

Hazardous materials transportation should consider risk equity and transportation risk and cost. In the hazardous materials transportation process, we consider risk equity as an important condition in optimizing vehicle routing for the long-term transport of hazardous materials between single or multiple origin-destination pairs (O-D) to reduce the distribution difference of hazardous materials transportation risk over populated areas. First, a risk equity evaluation scheme is proposed to reflect the risk difference among the areas. The evaluation scheme uses standard deviation to measure the risk differences among populated areas. Second, a risk distribution equity model is proposed to decrease the risk difference among populated areas by adjusting the path frequency between O-D pairs for hazardous materials transportation. The model is converted into two sub models to facilitate decision-making, and an algorithm is provided for each sub model. Finally, we design a numerical example to verify the accuracy and rationality of the model and algorithm. The numerical example shows that the proposed model is essential and feasible for reducing the complexity and increasing the portability of the transportation process.

Key words:

hazardous materials transportation, route optimization; risk equity, multi-objective optimization, NSGA-II algorithm, genetic algorithm

1.   Introduction

In hazardous materials transportation, risk equity (Keeney, 1980) is an important factor along with the cost and risk of transport, because the distances between road segments and densely populated areas vary and the selection frequencies of road segments for transportation are different. Based on these two conditions, significant changes in risk differences are induced on populations near road segments. Thus, a reasonable path scheme can distribute the risk of a densely populated area equitably in the hazardous materials transportation network.

The study aims to provide a long-term routing solution with risk equity consideration for the hazardous materials transportation process from one or more origins to destination (O-D). A transportation routing scheme is designed to minimize the total transportation cost and risk and reduce the risk difference of different population densities in the transportation network to achieve a fair risk distribution.

Three innovation aspects are considered to improve the hazardous materials transport process. (1) A measuring risk equity method is proposed. By considering the risk assessment model, the method quantifies the risk per person in the densely populated area of the transportation network. The risk of all areas per capita is calculated, and then the risk equity is evaluated using the standard deviation. (2) The model is built to achieve the three goals by adjusting the path selected frequency. The model minimizes the transportation risk difference in the populated area of the transportation network. The long-term transport process can be repeated in accordance with the program. The model is a multi-objective nonlinear integer program, which is difficult to solve and provides many solutions for decision-makers. The model can be improved by employing model decomposition. The complexity of the model can be reduced by the relaxing partial objective function to the constraint condition. (3) The algorithm is designed according to the model. Several sub-models belong to the main model: multi-objective shortest path problem, multi-objective non-linear integer programming problem, and single-objective nonlinear integer programming problem. The multi-objective A* algorithm (NAMOA*) (Mandow and De La Cruz, 2010) can be used to solve the first problem, while the non-dominated sorting genetic algorithm-II (NSGA-II)-based multi-objective optimization

and genetic algorithms can be used to solve the second and third problems, respectively.

This paper has four sections. Section 2 presents several related works, which include the risk measure method, risk equity evaluation method, and the transport routing optimization model. In Section 3, standard deviation is used to measure the risk equity among densely populated areas. In part 4, the mathematical model is built, and the optimal routing goal is achieved by adjusting the path selected frequency after the model is improved twice and the algorithm for each sub-model is designed. Finally, the model and algorithm are verified by experiments, and the experimental results are analyzed.

2.   Literature Review

We introduce three research aspects: risk evaluation method, risk equity evaluation method, and transport routing optimization method.

2.1. Risk evaluation method

The first issue is assessing the risk induced on the population by hazardous materials vehicles traveling on various segments of the road network, although no consensus exists regarding the best way to design the risk model. Any formulation includes two elements: the probability of an accidental hazardous materials release and its associated consequences (Bronfman *et al.*, 2015). Alp (2016) proposed a traditional risk model to minimize the probability of a hazardous materials vehicle traveling along a path. The incident probability and the population exposure models (ReVelle *et al.*, 1991) can be viewed as two extreme cases of the traditional risk model. The perceived risk model (Abkowitz *et al.*, 1992), which is similar to the traditional risk model (Sivakumar *et al.*, 1995), uses alternative criteria and criteria weighting to balance the safety and operating efficiency of route selection. The traditional and perceived risk models can be viewed as single-attribute models although they have two attributes, namely, probability and consequence. In contrast with these single-attribute models, the conditional risk model is a multiplicative multi-attribute model with two attributes: expected risk and accident probability. This model evaluates the probability of the occurrence of the first accident and suggests the necessary suspension of a path between an O–D pair after a catastrophic accident. Erkut and Ingolfsson (2000) suggested that avoiding a catastrophe may be a relevant

issue in routing hazardous materials and introduced three different models of catastrophe avoidance. In the first model, catastrophe avoidance is achieved by minimizing the maximum population exposure. In the second model, the variance of the route consequence is incorporated in the decision. In the third model, a clear disutility function is used. Bell (2007) proposed a mixed-route model under a completely uncertain accident probability, which aims to reduce the maximum risk by sharing shipments among routes. Assael et al.(2015) discussed the quantification of hazardous effect of fires and explosions, and each described with a case study. Conca et al. (2016) proposed an integrated approach for the study of routing problems considering safety, to analyze the interactions between road traffic flow and frequency of accidents.

Another risk assessment method incorporates distance in models dealing with hazardous materials transportation. List and Mirchandani (1991) considered the distance from a hazardous materials route to a population. Erkut and Verter (1995) proposed two models for quantitative risk assessment in hazardous materials transport. The first model assumes that population is concentrated at points on a plane, whereas the second model treats population centers as two-dimensional objects. Carotenuto et al. (2007) introduced an approach that assesses incident consequence by distance as distance-sensitive damage functions based on the expected risk model.

2.2. Risk equity evaluation method

Many previous researches on hazardous materials shipments planning also incorporated equity concepts. Gopalan et al. (1990) developed a model for hazardous materials transport from a single origin to a single destination that minimizes total travel risk and spreads the risk fairly. Lindner-Dutton et al. (1991) proposed a suitable solution to the sequencing problem of hazardous materials transportation between an O–D pair. List and Mirchandani (1991) minimized the risk and total costs by selecting suitable facility locations and associated routing. Current and Ratick (1995) proposed a model for minimizing transportation and storage risks by considering them proportional to the exposed population. They also represented equity by minimizing the maximum total transportation exposure or the exposure derived from stored materials to all individuals. Giannikos (1998) proposed a multi-objective model

to optimize operating cost, perceived risk, maximum individual perceived risk, and the equitable distribution the disutility caused by the operation of treatment facilities. Carotenuto et al. (2007) investigated the problem of minimizing risk in hazardous materials transportation and distributing the risk equitably by constraining the maximum risk sustained by the population living near the network. Bianco et al. (2009) established a linear two-layer programming model for hazardous materials transportation network design by considering total risk minimization and risk equity agreement. Kang et al. (2014) applied the Value-at-Risk (VaR) model in designing a path plan for the hazardous materials transportation network, and risk equity constraint was considered in the establishment of the model.

2.3. Transport routing optimization model and method

Hazardous materials route planning has attracted the attention of many operations research (OR) researchers. Over the past decade, Bonvicini and Spadoni (2008) developed a model called OPTIPATH to solve the routing problem of hazardous materials transportation; it is integrated in the TRAT4-GIS software for transportation risk analysis. Dadkar et al. (2008) developed a k-shortest path algorithm to identify a small set of routes for use in the routing of hazardous material shipment. Zografos and Androutsopoulos (2008) presented an integrated decision support system for routing hazardous materials and locating first-response mobile units within a specified coverage time from hazardous material routes. A bi-criterion path-finding problem (Androutsopoulos and Zografos, 2010) was presented in hazardous material delivery problems, and an algorithm was presented for determining the non-dominated scheduled route-paths. Lozano et al. (2011) performed a blind and heuristic search analysis on multi-objective hazardous materials transportation problems. Xie and Waller (2012) proposed an alternative optimization approach for the multi-objective hazardous materials routing problem. They constructed a decomposition scheme to convert a multi-objective routing problem into a number of bi-objective problems and designed an efficient parametric optimization method for them. Toumazis and Kwon (2015) proposed a method for mitigating risk in routing hazardous materials based on the conditional VaR (CVaR) measured on time-dependent vehicular

networks. Pradhananga et al. (2014) presented a Pareto-based bi-objective optimization of hazardous materials vehicle routing and scheduling problem with time windows and demonstrated its application to a realistic hazardous material logistics instance. Bronfman et al. (2015) proposed an approach for addressing the hazardous materials routing problem on a transport network. The proposed approach maximizes the weighted distance between the route and it closest vulnerable center to minimize the potential consequences for the most exposed population. Romero et al. (2016) designed a formulation and solution procedure for the facility location and routing problem of hazardous materials. The model is multiobjective and minimizes total canister-miles and transportation accident risk. It also includes the possibility of considering equity in the selected sites and recommended transportation routes.

3. Risk equity evaluation
3.1. Unit-length segment risk
Two evaluation methods are used to measure risk equity in hazardous materials transportation. One is road segment-based risk equity evaluation method, and the other is region-based risk equity evaluation method. However, most of the studies described the differences of transportation risk that were undertaken by road segments or regions. The relationship between population and risk value in the road segment or region was not considered. This section discusses the unit-length segment risk evaluation method. To measure the risk difference among different regions, a method for evaluating the risk equity of the difference of per capita risks in the populated area is proposed. The method quantifies the risk of each route and the risk of the populated area in the transportation network and calculates the standard deviation of the risk of each population area to measure the risk equity among regions.

A transportation network is represented as $G = (N, A)$, where N is the set of n nodes, and A is the set of links. For any original node r ($r \in N$) and destination node s ($s \in N$), K_{rs} denotes the set of all links between r and s , and S denotes all populated areas. If an accident occurs in the unit length segment x in the transportation network, then the accident consequence of area y is define by

$$u(x, y) = p_y e^{-\alpha [d(x,y)]^2} , \tag{1}$$

where p_y is the population in area y ; $e^{-\alpha[d(x,y)]^2}$ is the distance-sensitive damage function; α is the impact factor, which indicates the effect of accident consequences with the increase of distance and decline of speed; the value of α depends on the hazardous materials characteristics; and $d(x, y)$ is the Euclidean distance between segment x and all particles in y .

According to the traditional risk model definition (Jin and Batta, 1997; Alp, 2016), considering the accident probability on the unit length segment and the accident consequence to the populated area, the measure of risk influence is defined as

$$\sigma_x^y = P_x u(x, y) , \tag{2}$$

where P_x denotes the accident probability on unit length segment x .

3.2. Link risk
All road segments are composed of several unit length segments. When hazardous materials pass through a road segment, the number of unit-length segments can be used as the number of Bernoulli tests. Assuming road segment a is composed of unit-length segments a_1 , a_2 ,..., a_{q_a} , the accident probability on any unit segment a_x is

$$(1 - P_{a_1})...(1 - P_{a_{x-1}})P_{a_x} . \tag{3}$$

In Formula (4), the accident probability on any unit segment x is minimal, that is, the probability of non-accident is close to 1, and the accident probability is p_x . When road segment a undertakes the transportation task, the risk to area y is approximately equal to the accumulated risk in all unit length segments, which is expressed as

$$r_a^y = \sum_{s=1}^{q_a} \sigma_{a_s}^y . \tag{4}$$

The discretization method can introduce relative error. Meanwhile, in the case of a certain accident probability on the unit-length segment, the length of the unit-length segment and the relative error are small (Erkut and Verter, 1995). According to the above evaluation method, the total risk on road segment a is defined by

$$r_a = \sum_{y \in S} r_a^y . \tag{5}$$

3.3. Region risk and risk equity

The risk in area y is the sum of risks that all road segments must accept for area y. Then, according to (5), the total risk of area y is expressed as

$$r^y = \sum_{a \in A} r_a^y . \tag{6}$$

Therefore, the standard deviation risk of each populated area can be used to measure the risk differences among different regions. The definition of equity evaluation index is as follows:

$$\kappa = \sqrt{\sum_{y \in S} (r^y - \mu)^2 / (|S| - 1)} , \tag{7}$$

where $\mu = \sum_{a \in A} r_a / |S|$ is the average risk of all areas in hazardous materials transportation.

4. Vehicle routing model

In a hazardous materials transportation network, the selected frequency of the relevant path contributes

in achieving balanced risk for each populated area when one or few "best" paths are obtained by optimization. In Fig. 1, the path selection frequency has an impact on risk equity. If path a_1 is selected, the risk to area 1 is 2 ($r_{a_1}^1 = 2$), and the risk to area 2 is 1 ($r_{a_1}^2 = 1$). If path a_2 is selected, the risk to area 1 is 1 ($r_{a_2}^1 = 1$), and the risk to area 2 is 3 ($r_{a_2}^2 = 3$). Fig. 1 indicates the different influences of risk equity by selecting different frequencies.

4.1. Basic model

When multiple transportations occur between r and s, the method adjusts the frequency selection path in the selected path set, which changes the risk distribution in the entire hazardous materials transportation and reduces the risk differences among various areas. Therefore, the frequency selection of path k ($k \in K_{rs}$) is the integer variable ξ_k^{rs} for a minimum transport cycle. Adjusting the value of ξ_k^{rs} minimizes the difference per capita risk among different populated areas in the transportation network. The definition of ξ_k^{rs} is

$$\xi_k^{rs} = \{0,...,M\} \quad k \in K_{rs}, r \in N, s \in N , \tag{8}$$

where M is the maximum selected number of paths (k) in a single cycle, K_{rs} is the set of all paths between r and s. The effect of the value of M on the results is analyzed in Section 5.
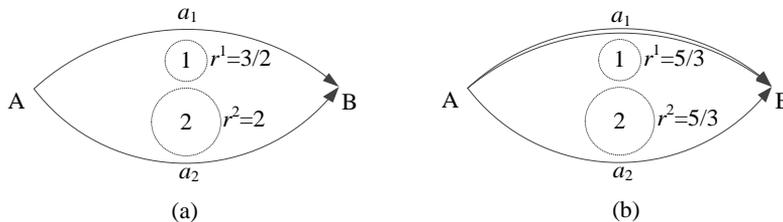

Fig. 1. Influence of different frequencies on risk equity (a) The ratio of frequency selection for paths a_1 and a_2 is 1:1, the average risk in areas 1 and 2 is $r^1 = 3/2$ and $r^2 = 2$, respectively, and the risk equity is $\kappa = 5/2$; (b) The ratio of frequency selection for paths a_1 and a_2 is 2:1, the average risk in areas 1 and 2 is $r^1 = 5/3$ and $r^2 = 5/3$, respectively, and the risk equity is.

When transportation tasks are performed between many O–D pairs at the same time, the number of transportation tasks will break the balance of risk distribution. To measure the risk equity accurately, we set the condition of equivalent transportation among all O–D pairs. Thus, the coefficient of the number of transport tasks between r and s (ε_{rs}) is introduced. It indicates that the count of transportation tasks between any O–D pair is equal. For any ε_{rs}, the following equation can be used:

$$\varepsilon_{rs} \sum_k \xi_k^{rs} = \varepsilon_{od} \sum_{k'} \xi_{k'}^{od}$$
$$k \in K_{rs}, k' \in K_{od}, \forall r \in N, s \in N, o \in N, d \in N$$
. (9)

One road segment may be selected by different transportation paths when O–D pairs are transporting at the same time, and the risk of the segment is the sum of the risks from each transport. The integer variable, $\delta_{a,k}^{rs}$, which denotes whether transportation path k between r and s passed road section a, takes the value of 1 when passed and 0 otherwise. For road segment a, when the number of transportations is $\varepsilon_{rs} \delta_{a,k}^{rs} \xi_k^{rs}$, the total risk of the road segment is

$$r_a = \sum_{r,s} \sum_k \sum_y \varepsilon_{rs} r_a^y \delta_{a,k}^{rs} \xi_k^{rs}$$
$$r \in N, s \in N, k \in K_{rs}, y \in S, \forall a \in A$$
. (10)

According to Equation (7), the total risk of area y when transporting between multiple O–D pairs at the same time is

$$r^y = \sum_{r,s} \sum_k \sum_a \varepsilon_{rs} r_a^y \delta_{a,k}^{rs} \xi_k^{rs}$$
$$r \in N, s \in N, k \in K_{rs}, a \in A, \forall y \in S$$
. (11)

In hazardous materials transportation, c_a denotes transportation cost in road segment a, which is another optimization objective of transportation cost, and the total cost in segment a is

$$w_a = \sum_{r,s} \sum_k \varepsilon_{rs} c_a \delta_{a,k}^{rs} \xi_k^{rs}$$
$$r \in N, s \in N, k \in K_{rs}, \forall a \in A$$
. (12)

We design a multi-objective optimization model for simultaneous transportation between multiple O–D pairs that can adjust the selected frequency ξ_k^{rs} of path k to achieve the optimal risk equity in a short cycle and select several paths in path set K_{rs} between r and s. The result of the model provides an optimal periodic vehicle scheduling scheme. Model P is defined as follows:

$$\min \kappa ,$$
(13)

$$\min \sum_a w_a \Big/ \sum_{r,s} \sum_k \varepsilon_{rs} \xi_k^{rs} ,$$
(14)

$$\min \sum_a r_a \Big/ \sum_{r,s} \sum_k \varepsilon_{rs} \xi_k^{rs} ,$$
(15)

subject to

$$\sum_k \xi_k^{rs} > 1 \quad k \in K_{rs}, \forall r \in N, s \in N ,$$
(16)

$$\varepsilon_{rs} \sum_k \xi_k^{rs} = \varepsilon_{od} \sum_{k'} \xi_{k'}^{od}$$
$$k \in K_{rs}, k' \in K_{od}, \forall r \in N, s \in N, o \in N, d \in N$$
, (17)

$$\delta_{a,k}^{rs} = \{0,1\} \quad a \in A, k \in K_{rs}, r \in N, s \in N ,$$
(18)

$$\xi_k^{rs} = \{0,...,M\} \quad k \in K_{rs}, r \in N, s \in N ,$$
(19)

where objective function (13) is the risk equity to be minimized, and objective functions (14) and (15) are the minimized average transport cost and risk, respectively.

4.2. Improved models
4.2.1.Improved model I: Decomposing

Obtaining an optimal solution is difficult because P is a multi-objective non-linear integer programming model. Moreover, exhaustion is impossible because the combined number of path set K_{rs} increases with the nodes. An improved model is proposed to aid decision makers. First, we obtain double objective Pareto-optimal solutions of cost and risk between all O–D pairs. Second, we select parts or full paths in the solution and adjust the selected frequency to

achieve the optimal objective of risk equity in the network. Corollary 1 proves that the improvement is satisfactory for model P in terms of cost and risk. The improvement may lose some best results, which will be proven in Section 5. However, the improvement is valuable because of the problem's complexity is reduced and the control of decision makers is increased.

Corollary: $U = \{u_1, u_2, ..., u_r\}$ and $V = \{v_1, v_2, ..., v_r\}$ *are two objective vectors in the objective space. If* U *dominates* V ($U \prec V$), *for the objective vectors* $\bar{U} = \{u_1, u_2, ..., u_{r-l}\}$ and $\bar{V} = \{v_1, v_2, ..., v_{r-l}\}$ ($1 \le l < r$), *then* $\bar{U} \prec \bar{V}$ or $\bar{U} = \bar{V}$ *is satisfied.*

Proof of Corollary: $U \prec V \Leftrightarrow u_k \le v_k$ ($k = 1, 2, ..., r$) *and* $\exists l \in \{1, 2, ..., r\}$, $u_k < v_k$. *Thus, for* $u_k \le v_k (k = 1, 2, ..., r-l)$, *if* $\exists l \in \{1, 2, ..., r-l\}$, $u_k < v_k$, *then* $\bar{U} \prec \bar{V}$, *else if not exist* $l \in \{1, 2, ..., r-l\}$, $u_k < v_k$, *then* $\forall k = \{1, 2, ..., r-l\}$, $u_k = v_k$, *that is,* $\bar{U} = \bar{V}$.

Based on the above analysis, model P is composed of models $P1$ and $P2$, an interactive link from decision makers is added in the solution, and the range of path set K_{rs} is reduced to $\bar{K}_{rs}$. Model $P1$ is a dual-objective shortest path problem of cost and risk, which obtains non-dominated solutions by using multi-objective shortest path algorithms. Model $P1$ is defined as follows:

$$\min r_a \delta_{a,k}^{rs}, \tag{20}$$

$$\min c_a \delta_{a,k}^{rs}, \tag{21}$$

subject to

$$\delta_{a,k}^{rs} = \{0,1\} \quad a \in A, k \in K_{rs}, \forall r \in N, s \in N, \tag{22}$$

where objective function (21) is the risk to be minimized, and objective function (22) is the cost to be minimized.

Path set K_{rs} becomes $\bar{K}_{rs}$. For any path $k \in \bar{K}_{rs}$, if $\xi_k^{rs} > 0$, then path k is selected by decision makers, and for any segment a in path k, $\delta_{a,k}^{rs} = 1$; else if $\xi_k^{rs} = 0$, then $\delta_{a,k}^{rs} = 0$. Thus, variable $\delta_{a,k}^{rs}$ is eliminated when path set K_{rs} becomes to $\bar{K}_{rs}$. Model P can be simplified into the following model:

$$\min \sigma, \tag{23}$$

$$\min \sum_a w_a / \sum_{r,s} \sum_k \varepsilon_{rs} \xi_k^{rs}, \tag{24}$$

$$\min \sum_a r_a / \sum_{r,s} \sum_k \varepsilon_{rs} \xi_k^{rs}, \tag{25}$$

subject to

$$\sum_k \xi_k^{rs} > 1 \quad k \in \bar{K}_{rs}, \forall r \in N, s \in N, \tag{26}$$

$$\begin{aligned} &\sum_k \xi_k^{rs} = \varepsilon_{od} \sum_{k'} \xi_{k'}^{od} \\ &k \in \bar{K}_{rs}, k' \in \bar{K}_{od}, \forall r \in N, s \in N, o \in N, d \in N \end{aligned}, \tag{27}$$

$$\xi_k^{rs} = \{0, ..., M\} \quad k \in \bar{K}_{rs}, r \in N, s \in N, \tag{28}$$

where path set K_{rs} becomes to $\bar{K}_{rs}$ in every constraint of model P.

4.2.2. Improved model II: Relaxation

Although the above improvement can obtain satisfactory routing solutions, the non-dominated solutions set still has many choices. Therefore, we try to relax the parts of objective functions in model $P2$ as constraint conditions and adjust the path selection frequency to obtain the Pareto-optimal routes set and the optimal solution of risk equity. Based on the above analysis, model $P2$ is changed into the following single objective optimization model $P3$:

$$\min \kappa, \tag{29}$$

subject to

$$\begin{aligned} &\sum_a w_a / \sum_k \varepsilon_{rs} \xi_k^{rs} \le \theta \max(\sum_{a'} c_{a'}) \\ &\theta \le 1, a \in A, a' \in A_k, k \in \bar{K}_{rs}, \forall r \in N, s \in N \end{aligned}, \tag{30}$$

$$\sum_a r_a / \sum_k \varepsilon_{rs} \xi_k^{rs} \leq \theta \max(\sum_{a'} r_{a'}) \tag{31}$$
$$\theta \leq 1, a \in A, a' \in A_k, k \in \bar{K}_{rs}, \forall r \in N, s \in N$$

$$\sum_k \xi_k^{rs} > 1 \quad k \in \bar{K}_{rs}, \forall r \in N, s \in N , \tag{32}$$

$$\varepsilon_{rs} \sum_k \xi_k^{rs} = \varepsilon_{od} \sum_{k'} \xi_{k'}^{od} \tag{33}$$
$$k \in \bar{K}_{rs}, k' \in \bar{K}_{od}, \forall r \in N, s \in N, o \in N, d \in N$$

$$\xi_k^{rs} = \{0, ..., M\} \quad k \in \bar{K}_{rs}, r \in N, s \in N , \tag{34}$$

where constraint (30) is the maximum value of average cost that is relaxed from objective function (24) and indicates that the average cost is less than θ times of the largest single transportation cost in the selected path. The value of θ can be set according to the risk preference of the decision maker. Constraint (31) is the maximum value of average risk, which is relaxed from objective function (25). The other constraints and decision variables are similar with those in model $P2$.

The improvement process is shown in Fig. 2. The two improvements reduce the complexity of the model and decision, and the entire improved process is constantly based on the optimal solution of total cost and total risk in hazardous materials transportation. Although we make concessions on risk equity, the entire improvement process is in accordance with the objective demand of hazardous materials transportation enterprises.

4.3. Solution approach

Model $P1$ is a multi-objective shortest path problem, and the edge attribute is the vector composed of cost and risk. A Pareto-optimal set is the set of shortest paths between each O–D pair, and the decision maker can select several paths from a selected path set $\bar{K}_{rs}$ according to his risk preference. Heuristic algorithm is an effective method for solving multi-objective shortest path problems. In this study, NA-MOA* algorithm (Mandow and De La Cruz, 2010) is used to realize model P , and Dijkstra algorithm is used to compute function $H(\bar{x})$ (Machuca Sánchez, 2012).

Model $P2$. is a multi-objective nonlinear-integer programming model. We use the branch and bound method or exhaustive search to obtain the optimal solution when few alternative paths exist. However, the exact solution algorithm is not valid when many alternative paths exist. In this study, we design an improved multi-objective optimization algorithm based on NSGA-II to solve the model. In the algorithm, the code format of individual is $\left\{ \xi_1^{r_1 s_1}, \xi_2^{r_1 s_1}, ..., \xi_{k_1}^{r_1 s_1}, \xi_1^{r_2 s_2}, \xi_2^{r_2 s_2}, ..., \xi_{k_2}^{r_2 s_2}, ... \right\}$.
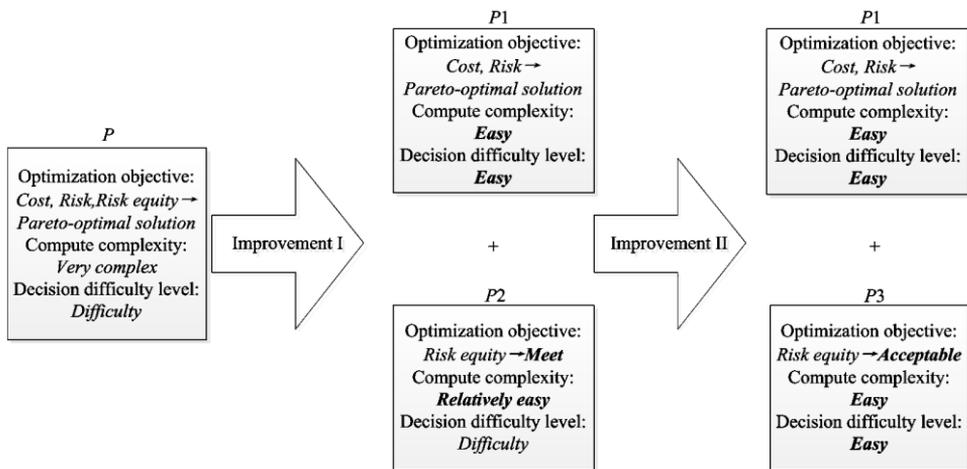

Fig. 2. Diagram of the improvement process mode

The main steps of the algorithm are explained in detail as follows:

Step 1: Initialize a population. Generate random parent solutions $P_i(t=0)$. The population size is *Popsize*.

Step 2: Create a new population. The crossover and mutation operations are used to create an offspring population Q_t.

Step 3: Perform non-dominated sorting. Combine the parent and offspring populations and create $R_t = P_t \cup Q_t$, perform a non-dominated sorting to R_t, and identify different fronts: F_i, $i = 1, 2, \ldots$.

Step 4: Perform sorting on $\succ_c$. Perform the Crowding-sort (F_i, $\prec_c$) procedure and include the widely spread solutions by using the crowding distance values in sorted F_i to P_{t+1}.

Step 5: Terminal condition. If t reaches the maximum evolution times (*Iteration*), return P_i; else $i = i + 1$, go to Step 2.

$P3$ is a non-linear integer programming model that can be solved by using a genetic algorithm.

For creating new population in models $P2$ and $P3$, we design crossover and mutation operators based on the coding characteristic of the selected path frequency. In the crossover operation, two individuals are selected randomly by using the roulette method. Then, the section (*pos*1 to *pos*2) is randomly generated. Two new individuals are generated after mutually exchanging the selected sections (Fig. 3). In

the mutation process, the new value in the code section is obtained from M by subtracting each original value. Then, the new values replace the old values. Subsequently, a new individual is formed (Fig. 4).
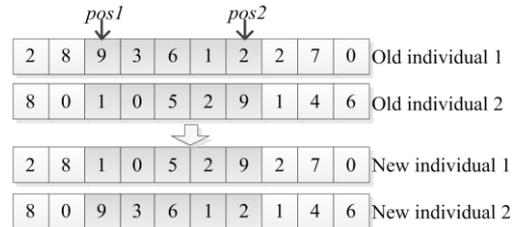
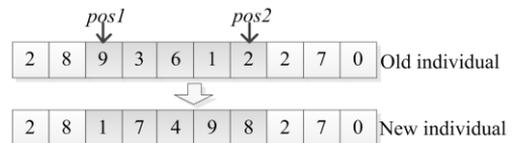

Fig. 3. Illustration of crossover operator

Fig. 4. Illustration of mutation operator

5. Experimental analysis
5.1. Transportation network

A concrete example is used to test the optimization process that considers risk equity in hazardous materials transportation. Fig. 5 illustrates the transportation network and population distribution for testing. The network is composed of 10 nodes (A–J) and 18 links (1–18).
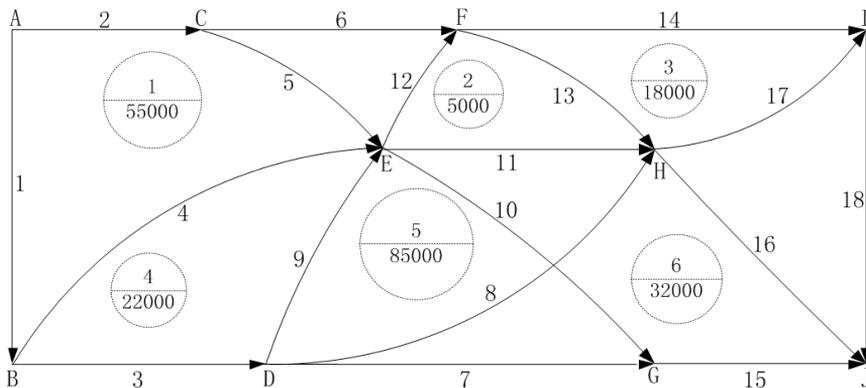

Fig. 5. Test transportation network

In the figure, the dotted circle denotes the population area. Two numbers are found in each circle. One is above the line and denotes the area number. The other is below the line and denotes the total population in the area. This process aims to design a transportation plan for hazardous materials shipments from nodes A to J and from B to I to ensure optimal risk equity in the population area and ensure that the total transportation cost and risk are within the acceptable range.

In the transportation network in Fig. 5, the length of unit segment is 1 km. The road segment distance and the Euclidean distance of unit-length segment and areas are measured in AutoCAD. Then, the values of P_x are generated randomly in the range [0.5×10^{-4}, 0.2×10^{-3}], and α is set to 0.03. The risk value in each segment and the total risk value in all segments are estimated in Table 1. The average cost of transportation in each segment is 80 \$/km.

5.2. Solutions and discussions

All algorithms are realized through C#, and the operating platform is a personal computer with Intel(R) Core(TM) i5-3470 3.20GHz CPU and 4.0 GB memory. According to the model in subsection 4.1, we obtain the routing solution between two O–D

pairs (A–J, B–I) by using NSGA-II. The result is different with different upper bound M values in the model (M is the upper bound of decision variable ξ_k^{rs}).

Table 2. Different CPU times with different M values

M Value	CPU time (s)
2	4.39
5	72.69
10	103.25

The comparison of CPU program times shows that the CPU time of the program increases gradually with the increase in M value. This is because the individual coding range increases with the M value and the computing time of fitness function values.

However, M value is high and the quality of solutions is good from the solution distribution perspective, which indicates that the distribution of solutions in the target space is close to the origin of the coordinate point.

To verify the improvement effect of the model, the shortest paths between two O–D pairs (A–J, B–I) are computed by NAMOA*. The two Pareto-optimal route sets are listed in Tables 3 and 4.

Table 1. Quantitative results and risks of each road segment length

| Road segment	Cost($)	r_a	r_a^y					
			r_a^1	r_a^2	r_a^3	r_a^4	r_a^5	r_a^6
1(A→B)	1,788.00	5.09	4.70	0.00	0.00	0.39	0.00	0.00
2(A→C)	964.00	30.19	30.19	0.00	0.00	0.00	0.00	0.00
3(B→D)	1,424.00	17.62	0.00	0.00	0.00	17.62	0.00	0.00
4(B→E)	2,612.00	22.98	3.40	0.00	0.00	19.52	0.06	0.00
5(C→E)	1,168.00	1.12	0.50	0.21	0.00	0.00	0.41	0.00
6(C→F)	1,360.00	0.74	0.00	0.65	0.00	0.00	0.10	0.00
7(D→G)	2,320.00	12.20	0.00	0.00	0.00	0.00	11.91	0.28
8(D→H)	3,396.00	63.96	0.00	0.00	0.00	0.00	57.63	6.33
9(D→E)	1,276.80	32.84	2.52	0.02	0.00	0.78	29.52	0.00
10(E→G)	2,072.00	76.20	0.00	0.32	0.10	0.00	57.27	18.50
11(E→H)	1,568.00	32.10	0.00	4.81	0.94	0.00	26.11	0.24
12(E→F)	755.20	7.01	0.01	3.03	0.00	0.00	3.97	0.00
13(F→H)	820.00	9.83	0.00	2.82	6.27	0.00	0.68	0.05
14(F→I)	2,160.00	12.71	0.00	0.01	12.69	0.00	0.00	0.00
15(G→J)	1,320.00	3.49	0.00	0.00	0.00	0.00	0.00	3.49
16(H→J)	1,520.00	13.40	0.00	0.00	0.11	0.00	0.00	13.29
17(H→I)	1,288.00	6.00	0.00	0.00	5.89	0.00	0.00	0.11
18(I→J)	1,788.00	0.39	0.00	0.00	0.05	0.00	0.00	0.34
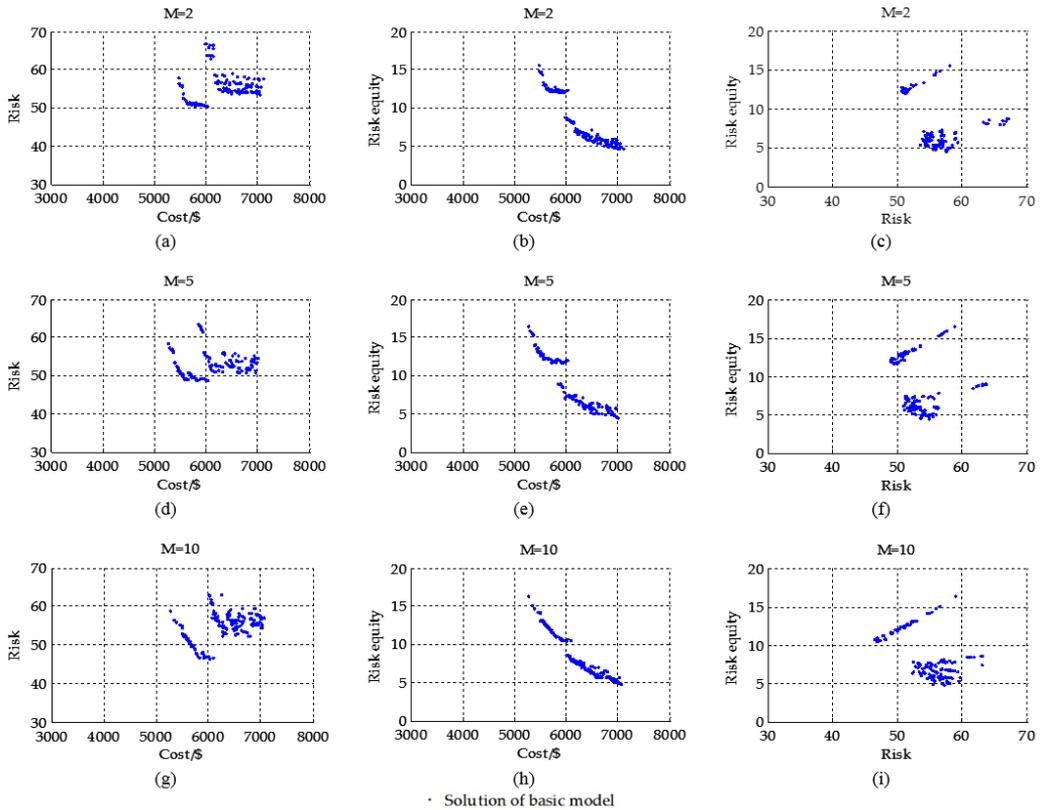
Fig. 6. Different values of *M* affecting the quality of the basic model (*Popsize* = 200, *Iteration* = 100)

Table 3. Pareto-optimal set of transportation costs and risk A–J

Path No.	Path	Road segments	Cost($)	Risk
1.1	A→B→D→G→J	1→3→7→15	6,352.00	38.40
1.2	A→C→F→I→J	2→6→14→18	6,272.00	44.03
1.3	A→C→F→H→J	2→6→13→16	5,164.00	54.16
1.4	A→C→E→G→J	2→5→10→15	5,024.00	111.00

Table 4. Pareto-optimal set of transportation cost and risk B–I

Path No.	Path	Road segment	Cost($)	Risk
2.1	B→E→F→I	4→12→14	5,527.20	42.70
2.2	B→E→F→H→I	4→12→13→17	5,475.20	45.81
2.3	B→E→H→I	4→11→17	5,468.00	61.08

Assuming that paths 1.1–1.4 are selected as transportation paths for A–J, paths 2.1 and 2.2 are selected for B–I, path 2.3 is discarded because the increase in transportation risk is higher than that in path 2.2. When $\theta = 1$, the average costs and risks are unlimited. When $M = 10$, the same path in the

transport cycle is up to 10 times. Transportation tasks are found between two O–D pairs (A–J, B–I) at the same time. NSGA-II is used to solve improved model I, and the genetic algorithm is used to solve improved model II. The population sizes (*Popsize*) in the algorithm are 200 and 500, and the maximum

iterations (*Iteration*) are 50 and 100, respectively. Table 5 shows the runtimes of the program with different parameter values. Fig. 7 lists the distribution of solutions with different parameter values.

Table 5. CPU time of the program with different parameter values

Popsize	Iteration	CPU time (s)		
		Basic model	Improvement II	Improvement II
200	50	58.62	31.24	0.18[a]
200	100	112.36	45.60	--
500	100	173.27	98.16	--

Note:[a] As the optimal solutions obtained with *Popsize* = 200 and *Iteration* = 10 in improved model II, increasing the scale of population and the number of iteration is not required.

Compared with the consumption time in Table 5, the CPU time of the program increases whether the pop-

ulation size is increased or the iteration of the program. When *Popsize* = 200 and *Iteration* = 10, the optimal solution of improved model II is obtained. When the population size and the number of iteration in Fig. 7 increase, the quality of solutions of the base model and improved model I improve and the number of non-dominated solutions increases. However, the real optimal solution cannot be determined when the population size and the number of iterations increase. Furthermore, many solutions exist. To address this problem, we provide an acceptable solution for the decision-maker through improvement model II.

Table 6 shows the proportional relation of selected paths for A–J and B–I while transporting simultaneously. From the results, paths 1.1, 1.3, and 1.4 are selected with the ratio of 2:4:1. Path 1.2 is not selected for A–J, and path 2.2 is selected for B–I. Meanwhile, the optimal risk equity is achieved.
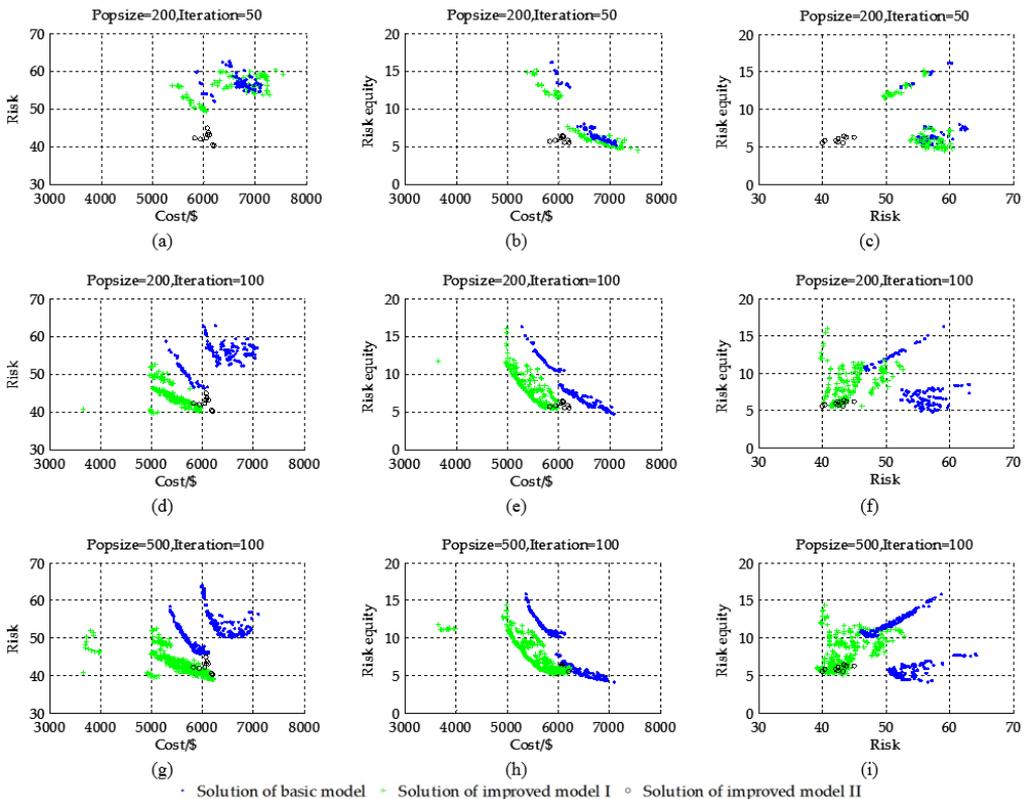

Fig. 7. Different *popsize* and iteration affecting the quality of three models (*M*=10)

Table 6. Best routing solutions between two O–D pairs (A–J and B–I) simultaneously (Top 5)

Solution No.	Frequency of path selection, average cost, and risk										Risk equity
	A–J						B–I				
	Path 1.1	Path 1.2	Path 1.3	Path 1.4	Ave. cost($)	Ave. risk	Path 2.1	Path 2.2	Ave. cost($)	Ave. risk	
1	2	0	4	1	5,483.43	57.77	0	1	5,475.20	45.81	7.1493
2	4	0	7	2	5,508.00	58.05	0	1	5,475.20	45.81	7.1500
3	3	0	7	2	5,437.66	59.69	0	1	5,475.20	45.81	7.1674
4	4	0	9	2	5,462.13	57.53	0	1	5,475.20	45.81	7.1775
5	2	0	3	1	5,536.67	58.38	0	1	5,475.20	45.81	7.1959

6. Conclusion

In this study, a measure of regional risk equity is proposed, the optimization model for vehicle routing based on risk equity is designed and improved, and then the optimal routing scheme for hazardous materials transportation is obtained in multiple O–D pairs.

The risk assessment method based on population distribution can quantify the risk impact on transportation path to each area. The risk equity index for region risk difference is proposed, and the index can reflect the risk equity in the hazardous materials transportation network. The routing optimization model can achieve the goals for transportation cost, risk, and risk equity. The cyclical routing scheme among multiple O–D pairs is obtained to adjust the selected frequency of different paths in a cycle. Case studies proved the effectiveness of the model. The analysis of the improvement model indicates that the complexity of the problem should and could be reduced for decision makers although the opportunity to obtain the optimum solution is lost.

This study did not consider the risk equity of multi-commodity hazardous materials though transporting multiple categories through the hazardous materials transportation network is common and complex. The risk equity of multi-commodity hazardous materials will be the research target of future studies.

Acknowledgment

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