

Multi-Criteria Evaluation of Transportation Systems in Supply Chains

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Abstract

The considerations included in the paper apply to the problem of the multi-criteria evaluation of transportation systems with constraints on delivery times in supply chains. For this reason, the identification of the transport network model and the nature of the configuration of the supply chain itself was carried out. In addition, a review of issues of the multi-criteria evaluation were done with respect to transportation systems. Later, the problem of the transport service of a distribution system were formulated and then, the results were analysed using the multi-criteria method. Finally, the paper examines the impact of the threshold of compliance and the threshold of non-compliance on of the final outcome of the solution.

Keywords: transportation systems, supply chains, multi-criteria evaluation methods, distribution system

1. Introduction

Variable environmental conditions, increasing globalization and technological progress in various fields of economy cause companies today to look for solutions concerning the tools which could contribute to cost savings while maintaining the quality level offered in the different functional areas of the company. General considerations on transport are closely related to satisfying one of the basic human needs, which is the need to overcome spaces using transport. In the process of moving goods from producers to consumers, many people (logistics operators, freight forwarders, carriers, etc.) who are active in the area of production and

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procurement or distribution take part, so it becomes necessary to coordinate their actions [12], [23]. For the smooth flow of goods, it is necessary to integrate the physical flow of goods and related information within a well-functioning supply chain [1], [11], [27].

In literature, a supply chain is defined as an integrated process in which a group of many organizations, such as suppliers, manufacturers, distributors and retailers cooperate to obtain the raw material, process it in several stages of technological process to the delivery of a finished product to the final consumer [6]. The simultaneous consideration of the processes occurring in a supply chain with transport planning as an element which directly affects the efficiency of logistics processes performed, significantly improves the performance of both processes. The issues concerning the optimization of supply chains, taking into account the different functional areas of logistics and transport planning, can be found in many literature items, both domestic and foreign [6], [9], [12], [23].

2. Supply Chain Configuration

A supply chain is an element of division based on the fundamental concept, which is the logistics chain¹, which is a set of elements made up of companies interconnected by a temporal and spatial sequence to achieve a specific objective closely related with logistics².

The literature refers to both the so-called external and internal logistics chains. An external supply chain is formed by independent companies pursuing a common objective, while an internal supply chain is formed by functional units of the company to realize a specific objective of the company.

Configuration of external and internal logistics chains can be very diverse. This follows from the purpose and strategy of the operation of a logistics chain, the rate of its complexity and operating range, and the internal organization of its member companies, as well as their size and scope of operation.

The complexity of flows and their various configuration inspires the use of the term **supply network** instead of **supply chain**. The supply network is built around three key elements: the flow of goods, information flow and supplier's response time to demand. In the centre of the network, there is a base company around which both the suppliers and the purchasers (Fig. 1) are arranged and connected to it. Thus, the supply network is a system of interdependent processes in which an action taken at any point affects the functioning of all network components.

¹ A logistics chain is a network of interrelationships between enterprises involved in the processes and activities whose purpose is the flow of physical goods adding value to the purpose of this flow.

² Logistics is the process of planning, implementing and controlling the effective and efficient transportation and storage of raw materials, intermediate products and finished products, and related information from the point of origin to the point of consumption for the purpose of conforming to customer requirements. – *Council of Logistics Management (CLM)*, Oakbrook – Illinois, 1985.

The stimulus determining the functioning of the supply network is end buyer demand.

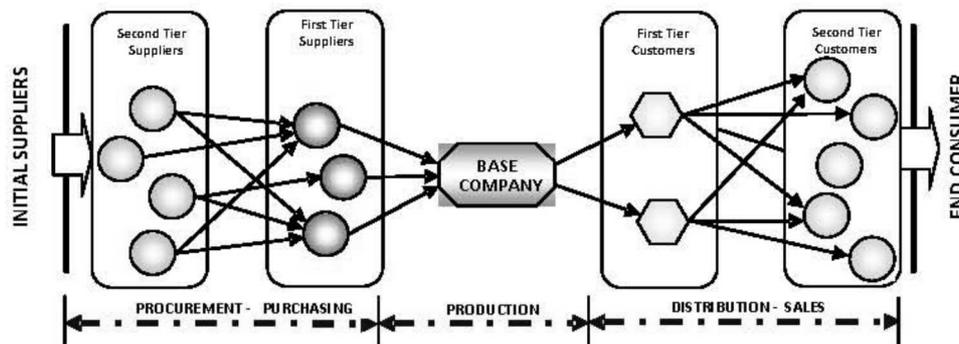


Fig. 1. Supply network

Source: Author's own study based on: Harrison A., Remko van Hoek., *"Zarządzanie logistyką"* (*"Logistics management"*), Polskie Wydawnictwo Ekonomiczne, Warszawa 2010.

For the purposes of the article, for the clarity of terms in the following part of the paper, the term **supply chain** will be used. Considering the earlier findings, operational processes of the base company must be coordinated with the processes of other partners in the supply chain. The supply chain has a hierarchical structure, which means that on the supply side and distribution side, near and far groups of partners can be distinguished in different tiers of the flow of physical goods. In a supply chain, one can distinguish suppliers and customers – consumers, both the first and second hierarchical tiers in relation to the base company considered. The term **first tier supplier** designates an entity delivering products or providing services directly to the base company, while a second tier supplier means an entity supplying products or providing services directly to the first tier supplier serving the base company. In the supply chain, a first tier customer is a customer served directly by the base company, while a second tier customer is a customer served directly by the first tier customer in relation to the base company.

An integral part of the supply chain is the transportation system whose intended purpose of operation is to optimally – in terms of the criteria adopted - perform the transportation tasks reported in the area by implementing the transport process.

To accomplish the objective mentioned, it is necessary for the system to have a defined structure, set characteristics of its components, specified task sizes, and a specific organization [29]. Regionalisation problems on a macroscale, in particular the transportation volume between multiple points of the shipment of goods and multiple points of their destination, taking into account the task performance time, require a lot of criteria to be defined when selecting the organization of transport.

The problem of the optimal planning of the spatial relationship of points of shipment and the destination of the mass of goods is the routing problem and belongs to the essential issues of the management of the means of transport connected with performing transportation tasks in transport companies. The task of routing consists in determining the schedules of the movement of means of transport in such a way that every customer is served, and that the capacity of any of the vehicles is not exceeded.

The problem of determining the schedules of the movement of means of transport is a complex optimization problem whose complexity stems from the fact that multiple points of view are taken into account. Therefore, it is requested that the decision be the best with respect to the adopted subcriteria. Making decisions taking into consideration various criteria is made possible by the multi-objective optimization called a polyoptimization. The considerations included in the article concern the issues of the evaluation of transport systems in terms of a multi-criteria approach.

3. Model of the Transport Network in Relation to Supply Chain

For the purposes of modelling the transport systems in supply chains, the structure of the transport system is presented using a graph mapping the nodes of the transport system and the connections between these nodes. Graph vertices are interpreted as follows (depending on the specific issue involved): as railway stations, bus stops or, in the case of distribution systems – distribution warehouses, etc. Graph arcs represent the existing direct links between the nodes of the transport system (graph vertices).

In this approach the structure of the transport system can be presented in the form of an ordered triple: $G = \langle \mathbf{I}, \mathbf{U}, \hat{\mathbf{R}} \rangle$ where: \mathbf{I} – is a finite set of transport nodes the transport network, i.e., $\mathbf{I} = \{1, 2, \dots, i, j, \dots, I\}$; \mathbf{U} – is a set of transport connections of the transport network, $\mathbf{U} \subset \{u(i, j) : i, j \in \mathbf{I}, i \neq j\}$, wherein (i, j) – an ordered pair defining arc direction between the tail and head of an arc; $\hat{\mathbf{R}}$ – is the 3-ary relation, $\hat{\mathbf{R}} \subset \mathbf{I} \times \mathbf{U} \times \mathbf{I}$.

We assume that the quantitative characteristics, such as the connection length, connection capacity, travel time along the connection, etc are determined for the nodes and connections.

To describe such a situation, the notion of the network understood in the sense of graph theory is used. Such a network is defined as an ordered triple: $S = \langle G, \mathbf{F}_I, \mathbf{F}_U \rangle$, where: $G = \langle \mathbf{I}, \mathbf{U}, \hat{\mathbf{R}} \rangle$ is the graph, while $\mathbf{F}_I = \{f_1, f_2, \dots, f_n, \dots, f_N\}$ is the set of functions defined on the set of nodes \mathbf{I} of the graph G , i.e. $f_n: \mathbf{I} \rightarrow \mathfrak{R}^+$ and $f_n(i) \in \mathfrak{R}^+$, $n = 1, 2, \dots, N$. $\mathbf{F}_U = \{g_1, g_2, \dots, g_k, \dots, g_K\}$ is the set of functions defined on the set of arcs \mathbf{U} of the graph G , i.e.: $g_k: \mathbf{I} \times \mathbf{I} \rightarrow \mathfrak{R}^+$ and $g_k(i, j) \in \mathfrak{R}^+$,

$k = 1, 2, \dots, K$ and in addition a strictly defined interpretation is given to the quantities $f_n(i)$ and $g_k(i, j)$.

4. Problems of Multi-Criteria Evaluation of Transportation Systems

The decisions taken in the field of the organization of transport in the supply chain consist in resolving complex decision problems. Decision-making methods including decisions regarding the organization of transport in the supply chain are divided into single-criterion and multi-criteria methods. The single-criterion methods focus on the optimization problem, while the multi-criteria methods are divided in respect to the number of variants and adopted criteria. Among the criteria that can be taken into account in a decision-making process, one can distinguish, among others, the criterion of the performance cost of a transport service, its performance time or the number of the used means of transport. Thus, one strives for the adopted solution to ensure the best performance of all subcriteria taken into account in the decision-making process. It is important in so far as the subcriteria may be contradictory to each other, e.g. decision-making concerning transport services should depend, among others, on the transport costs, quality of the service rendered, and duration of transportation [29].

Multi-criteria decision problems are based on two basic postulates [14],[16],[17],[24],[25],[35],[38], which are as follows:

- **postulate of dominance** – *if we have two proposals of acceptable solutions and we recognize that one of them is better than the another due to at least one criterion, and in all other respects it is not the worse, we should consider the former to be better.*
- **postulate of transitivity** – *if, by comparison, we consider that variant A is better than B, and, in turn, B better than C, then we should consistently consider that variant A is better than C.*

The above-mentioned postulates indicate the need to comply with the valuation system once adopted in multi-objective optimization. Because in the event that it lacks in the situation when discretionary valuation is applied, no reference system is formed.

In the multi-objective optimization, in the set of acceptable solutions the non-dominated (strongly non-dominated) solutions can be distinguished [24],[25],[35],[38], i.e. Pareto-optimal (the biggest elements), weakly non-dominated solutions, and dominating (strongly dominating), so-called optimal in the usual sense (the maximum elements) and weakly dominating. Due to the way preferences are expressed by the decision maker, solution methods of multi-criteria decision problems can be divided into [14],[16],[24],[25],[38]:

- **methods, in which preferences are expressed *a priori***: in the form of a utility function, in the form of a hierarchy of objectives, in the form of objective achieving levels;
- **methods, in which preferences are expressed gradually**, so-called interactive (dialogue, conversational) programming;
- **methods, in which preferences are expressed *a posteriori***: selection of the compromise function.

One of the dialogue methods is the method of ranking the ELECTRE-class variants, which is based on the concept of outranking relations. The application of the ELECTRE method for the model resulted in the arrangement of the variants from the most preferred to least preferred. This arrangement follows from the outranking relation whose construction is based on the so-called concordance and discordance tests in respect to the preference evaluations for each pair of variants. Thus, the ELECTRE method can be used in problems with finite and a countable number of acceptable solution variants. A similar approach in determining the optimal solution (also for a countable number of variants), in an application for optimising transportation systems is presented by the authors of papers [16],[17] and [36].

5. The Problem of Organising a Transportation System in the Supply Chain

5.1. Formulation of the supply problem

Organising a transportation system in the supply chain requires a series of to be taken, which are connected with the performance of transport tasks reported in a given area. Such an approach reduces the problem of the organization of transport services to the formulation and solution of a usually single-criterion optimization problem. The basic criterion for the solution of the single-criterion optimization problem is the value of the criterion function adopted. However, in many cases, the solution of an optimization problem does not solve the whole problem. As a result of the solution, we obtain various characteristics describing the transportation system, which we would take into account when assessing the quality of the solution, e.g. the transport cost, number of new routes, number of vehicles involved in transport services or vehicle utilisation rate.

The decision situation applies to the transport service provided to the Distribution Centre "DC" located in central Poland. The Centre is a part of a wider supply chain and mediates the movement of cargo between the main point components of the supply chain, and, in addition, it serves the customers located within the influence radius adopted by the company, performing deliveries according to strictly specified assumptions, such as just-in-time delivery. For the purposes of this paper, the general formulation of the problem was applied, whereas the problems of formulating and

solving single-criterion optimization problems in this field have been described in detail in Reference [30].

Taking into account the assumptions and mapping of components of the transport system in the supply chain are described in Reference [30], a formal description of the optimization problem of the transport service for the distribution system with constraints on time delivery can be presented as follows.

For the data: the set of numbers of transport service points – \tilde{W}^* ; the set of road connections – L^* ; the set of vehicle numbers – V ; the vector of the earliest times to begin the customer service – A ; the vector of the latest times to complete the customer service – B ; the vector of customer service times – \bar{T} ; the matrix of transport connections of the transport network – C ; the matrix of travel times along transport connections of the transport network – P ; the matrix of vehicle loading capacities – Q ; the matrices characterising the loading space of the means of transport – accordingly BE^1 the loading space lengths, BE^2 the loading space widths, BE^3 the loading space heights, BE^4 the loading space volumes; the vector of fixed costs of the means of transport – K^1 ; the set of cargo numbers – Λ ; the vectors of cargo characteristics: GA^1 cargo weights, GA^2 cargo volumes, EP^1 cargo lengths, EP^2 cargo widths, EP^3 cargo heights; the matrix of customers' demands – E ; utilisation rate of loading space volumes – η_v ; regular service time of a means of transport in DC – \bar{T}_v^* ; loading time of a means of transport – $\bar{T}_{v\alpha}^{**}$; coefficients λ_α^1 and λ_α^2 increasing loading time and resulting from the cargo weight and volume; weights of importance parameters of component functions: the utilisation cost parameter of a means of transport f_1 , route length parameter f_2 , loading capacity utilisation parameter f_3 and volume utilisation parameter of a means of transport f_4 ;

— **It is necessary to determine** values of the decision variables written as:

$$X = [x_{ijv}]_{W+2 \times W+2 \times V}, \quad x_{ijv} \in \{0, 1\}, \quad Z = [z_{(i,\alpha)v}]_{W \times \Lambda \times V},$$

$$z_{(i,\alpha)v} \in \{0, 1\}, \quad \Phi = [\varphi_v^h]_{V \times H}, \quad \varphi_v^h \in \{0, 1\},$$

which meet the constraints on: service to every customer, behaviour of traffic flow; vehicle return to the Distribution Centre; vehicle arrival to the customer not later than the latest time to begin the customer service; beginning of the customer service not earlier than the time his/her arrival; vehicle return to the Distribution Centre not later than the latest time offered; vehicle departure from the customer not earlier than the service to him/her has been completed; placing the cargo to be received by the customers on the vehicle (cargo inseparability), complying with the vehicle loading capacity, complying with the vehicle volume, binary character of the decision variables

— **and condition** the minimum value of the criterion function written as:

$$F(\mathbf{X}, \mathbf{Z}, \Phi) = f_1 \sum_{v \in V} \sum_{j \in \tilde{W}^*} \frac{k_v^1 x_{0jv}}{\sum_{h \in H} \varphi_v^h} + f_2 \sum_{v \in V} \sum_{i, j \in \tilde{W}^*} c_{ij} x_{ijv} + f_3 \sum_{v \in V} \left(q_v \sum_{h \in H} \varphi_v^h - \sum_{\alpha \in \Lambda} \gamma_\alpha^1 z_{(i,\alpha)v} \right) +$$

$$+ f_4 \sum_{v \in V} \left(\eta_v \beta_v^4 \sum_{h \in H} \varphi_v^h - \sum_{\alpha \in \Lambda} \gamma_\alpha^2 z_{(i,\alpha)v} \right)$$

The detailed formulation of the problem concerning the organization of the transport service of a distribution system with constraints on delivery time has been presented by the author in Reference [30], therefore, the formulation presented above is very general and used for the further analyses of decision-making problems using the multi-criteria approach.

5.2. A Method of Solving the Optimization Problem

The optimization problem formulated in paragraph 5.1 is a combinatorial problem and belongs to the Travelling Salesman Problem (TSP) class. The proposed method of solving the problem formulated in this way belongs to heuristic methods, which enable an approximation of the optimum solution to be determined. The advantage of such methods, however, is that they allow a solution to the complex decision problems to be obtained in a short time.

The proposed solution method of the optimization problem of the transport system in the supply chain takes into consideration the problem of ensuring timely deliveries to customers and the problem of placing the cargo on the vehicle, taking into account the vehicle characteristics saved as matrices \mathbf{Q} , \mathbf{BE}^1 , \mathbf{BE}^2 , \mathbf{BE}^3 , \mathbf{BE}^4 and cargo characteristics presented in the form of vectors \mathbf{GA}^1 , \mathbf{GA}^2 , \mathbf{EP}^1 , \mathbf{EP}^2 and \mathbf{EP}^3 . Such an approach considering the hierarchy of the pursued objectives allows the calculation algorithm to be shortened, which, in turn, leads to quicker decision-making on the transport service.

The hierarchical method assumes the determination of five acceptable initial solutions, in accordance with the five adopted variants of preparing hierarchical list. The hierarchical list (HL) is a list consisting of customer numbers, arranged according to rules specified in a given variant. The rules for the preparation of HL are as follows:

- **Variant No. 1** – Placing customers on the list according to the earliest time to begin the service (a_i) (and according the latest time to begin the service (b_i)) to the i -th customer;
- **Variant No. 2** – Placing customers on the list according to the shortest distance to the next customer;
- **Variant No. 3** – The first customer placed on the list is the one who is farthest from the Distribution Centre, the remaining customers are placed according to the shortest distance from the previous customer;
- **Variant No. 4** - Placing customers on the list according to the shortest distance to the next customer (approx. 25 per cent of all customers), and then according to the earliest time to begin the service (a_i), (and according to the latest time to begin the service (b_i)) to the i -th customer;
- **Variant No. 5** - The first customer placed on the list is the one who is farthest from the Distribution Centre, the remaining customers are placed according to the earliest time to begin the service (a_i), (and according to the latest time to begin the service (b_i)) to the i -th customer.

Initial solutions determined according to the above variants are corrected using a local search algorithm 2-OPT, which consists in swapping the places of customers in the designated routes. If a route with a shorter length is obtained as the result of the swapping and, at the same time, all the constraints are satisfied, the new solution becomes the current solution. It is assumed that the number of iterations depends on the number of customers ϑ_h in the h -th path and the following values are:

- $\vartheta_h \in [2; 3]$, where $\vartheta_h \in \mathbb{N}_+$ – 20 iteration cycles;
- $\vartheta_h \in [4; 7]$, where $\vartheta_h \in \mathbb{N}_+$ – $50\vartheta_h$ iteration cycles;
- $\vartheta_h \in [8; 34]$, where $\vartheta_h \in \mathbb{N}_+$ – $100\vartheta_h$ iteration cycles;
- $\vartheta_h \in [35; \infty)$, where $\vartheta_h \in \mathbb{N}_+$ – 2000 iteration cycles.

For a route, which consists of only one customer, no attempt is made to improve the solution.

6. A Case Study for the Distribution of Goods in the Supply Chain

6.1. Methodology of multi-criteria evaluation of transportation systems



Fig. 2. Welcome window of the OTR-OCD program
Source: Author's own study.

Analysis of the distribution system was carried out using the OTR-OCD application (Figure 2), which is a computer implementation of the hierarchical method. The investigated distribution system covers the central part of Poland, and concerns the services of 16 customers. The data used for the experiment is derived from Reference [30] in which the author identified in detail and parameterized the distribution system.

As the result of solving the optimization problem, the matrices \mathbf{X} , \mathbf{Z} and Φ were obtained, whose elements are values of decision variables characterizing the way

transport services of the distribution system under consideration are performed. On their basis, the number of routes, route alignment, route characteristics, and detailed realisation schedules were determined in variants, as well as the utilisation rates of the offered vehicle working time, loading capacity and volume were calculated for the vehicles involved in performing transport tasks. In addition, values of the criterion function for each of the service variants considered were determined. Specific indicator values for particular solution variants are presented in Table 1.

Table 1
Values of solution indicators of the optimization problem according to the hierarchical method

| Criterion name | | Solution results | | | | |
|--|-------------------|------------------|-----------|-----------|-----------|-----------|
| | | Variant 1 | Variant 2 | Variant 3 | Variant 4 | Variant 5 |
| Total costs | (-) | 6024.095 | 7953.407 | 6843.698 | 6614.495 | 8274.61 |
| Number of routes | szt. | 2 | 2 | 3 | 2 | 3 |
| Length of routes | km | 578.4 | 609.7 | 564.5 | 652.2 | 533.5 |
| Number of requisite vehicles | szt. | 2 | 2 | 3 | 2 | 3 |
| Vehicle loading capacity utilisation rate | (-) | 0.71 | 0.46 | 0.57 | 0.71 | 0.40 |
| Unused loading capacity of the vehicles engaged | (kg) | 1779.20 | 4929.20 | 3229.20 | 1779.20 | 6379.20 |
| Utilisation rate of the offered volume of the vehicles engaged | (-) | 0.53 | 0.42 | 0.39 | 0.53 | 0.33 |
| Unused offered volume of the vehicles engaged | (m ³) | 14.59 | 22.41 | 26.20 | 14.59 | 34.02 |
| Vehicle engagement costs | (zł) | 500 | 600 | 700 | 500 | 800 |

Source: Author's own study.

Alternative solutions of transportation systems are graphically compared in Figure 3. In order to evaluate the transportation system in the supply chain, the computer program Expert was used, which was developed for the purposes of decision-making in terms of a multi-criteria approach. The program provides an important tool in the process of decision making, in which a decision maker must consider several evaluation criteria, which are, in the majority, contradictory to each other. The problem of this kind also concerns the organization of transport systems in the distribution of goods where different preferences of the decision maker may occur. After defining the evaluation criteria and organization variants of transportation systems (Table 1), the evaluation of particular variants by the k -th criterion was begun.

Multi-criteria evaluation of the phenomenon identified as the k -th criterion becomes possible in different variants of the organization of transportation systems when we make the transformation of feature values (actual data) in order to standardize them. Transformed variables have no units of measurements any more and take values of a similar order of magnitude. Methods of transformation of the actual values of diagnostic features are called normalization methods. Generally speaking, normalization methods can be divided into two groups:

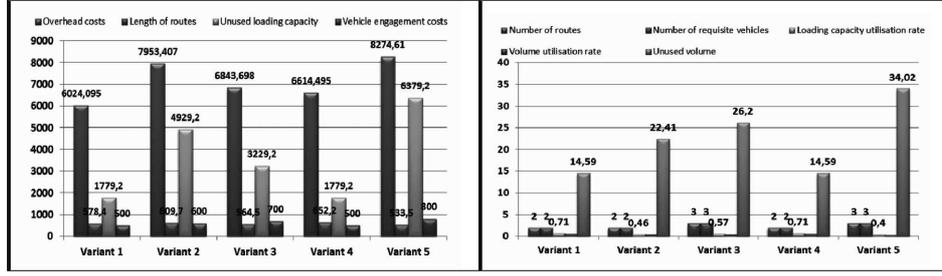


Fig. 3. Comparison of alternative solutions of transportation systems
Source: Author's own study.

- methods based on the quotient transformation formula; they adopt various reference points, such as the standard deviation of a variable (standardization methods) or the range of a variable (unitarization methods), etc.;
- ranking methods.

Ensuring the comparability of evaluations of the analysed variants of the organization of the transportation systems in the analysed case study, they underwent normalisation, i.e. values of evaluation criteria $f(w, k)$ were determined with the formulae contained in Reference [16]: in the case of maximized criteria

$$f(w, k) = \frac{o(w, k)}{\max_{w \in \mathbf{W}} \{o(w, k)\}}, \text{ in the case of minimized criteria } f(w, k) = \frac{\min_{w \in \mathbf{W}} \{o(w, k)\}}{o(w, k)},$$

where: $o(w, k)$ means the evaluation of the w -th variant against the k -th criterion.

In the next step, individual criteria were assigned certain numerical values ξ_k interpreted as a relative importance of the k -th criterion. Weights were chosen taking into account the range of the criteria under study. We assume that \mathbf{Y} constitutes a finite set of variants w , hence: $\mathbf{Y} = \{Y(1), \dots, Y(w), \dots, Y(W)\}$, while $\mathbf{\Psi}$ is a set of subcriteria k of the evaluation of the transportation systems, i.e.: $\mathbf{\Psi} = \{\psi_1(\mathbf{Y}), \dots, \psi_k(\mathbf{Y}), \dots, \psi_K(\mathbf{Y})\}$.

In order to obtain the evaluation of the evaluation variants of the transportation systems, it was assumed that on the Cartesian product $\mathbf{Y} \times \mathbf{\Psi}$ the mapping μ in the following form is given:

$$\mu : \mathbf{Y} \times \mathbf{\Psi} \longrightarrow \mathfrak{R}^+$$

for which the quantity $\mu(w, k) \equiv \mu_{w,k} \in \mathfrak{R}^+$ is interpreted as an evaluation of the w -th variant of the transportation system $Y(w)$, by the k -th subcriterion ψ_k .

In addition, for individual subcriteria, $k \in \mathbf{K}$, the numerical values ξ_k were assumed, which are interpreted as their relative importances. It is assumed that the choice of weights is associated with the range of variant evaluations for individual criteria. In this connection, weights do not express the preferences (subjective feelings) of the decision maker, but they have a normalizing character. The range $\delta(\psi_k)$ of the k -th criterion is calculated using the formula:

$\delta(\psi_k) = \max_{\{(w,k)\}} \{\mu(w,k)\} - \min_{\{(w,k)\}} \{\mu(w,k)\}$. The value ξ_k of the relative importance of each criterion is a number from the interval $\langle 0, 1 \rangle$, wherein the bigger the value ξ_k , the smaller the range of the variant evaluation range of the k -th criterion. The value ξ_k of the relative importance of each k -th criterion is calculated using the following formula:

$$\xi_k = \frac{1}{\delta(\psi_k)} \left(\sum_{k=1}^K \frac{1}{\delta(\psi_k)} \right)^{-1}, \text{ wherein } \sum_{k=1}^K \xi_k = 1 \quad (1)$$

In the problem formulated in this way, the evaluation concordance indicator of the variant w with the variant w' was determined according to the formula:

$$z(w, w') = \frac{1}{\xi} \sum_{k \in K; \mu(w,k) > \mu(w',k)} \xi_k \quad (2)$$

where:

$$\xi = \sum_{k=1}^K \xi_k \quad (3)$$

In contrast, the evaluation discordance indicator of the variant w with the variant w' was formalized as follows:

$$n(w, w') = \frac{1}{K} \max_{(w,k); \mu(w',k) > \mu(w,k)} \{\mu(w',k) - \mu(w,k)\} \quad (4)$$

wherein:

$$\kappa = \max_{\{(w,k)\}} \{\mu(w,k)\} - \min_{\{(w,k)\}} \{\mu(w,k)\} \quad (5)$$

The next step of the multi criteria evaluation of the organization of transport systems in the distribution of goods is to adopt the evaluation concordance threshold α and the evaluation discordance threshold β . The thresholds adopted serve to define the outranking relations. It is set that the variant $Y(w)$ surpasses the variant $Y(w')$ when for $Y(w), Y(w') \in \mathbf{Y}$: $z(w, w') \geq \alpha \wedge n(w, w') \leq \beta$. Based on the outranking relation, the dominance matrix is built according to the principle that:

$$m(w, w') = \begin{cases} 1, & \text{when } z(w, w') \geq \alpha \wedge n(w, w') \leq \beta \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

This approach helps to identify non-dominated variants in the set of acceptable solutions, i.e. Pareto optimal, which constitute the solution to the problem. The full description of the problem of multi-criteria evaluation can be found among others in References [16],[17] and [22].

6.2. Analysis of evaluation results of transportation systems

The results obtained with the OTR-OCD program in the variants (Table 1) were used in the further part of the analysis of the problem and entered as input data in the EXPERT program. The normalization of the evaluation results of the analysed variants (Table 2) was carried out and the value of relative importance of each k -th criterion (Table 3) was determined.

Table 2

Normalized evaluation results for the variants analysed

| Variant number | k1 | k2 | k3 | k4 | k5 | k6 | k7 | k8 | k9 |
|----------------|------|------|------|------|------|------|------|------|------|
| W1 | 1.00 | 1.00 | 0.92 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| W2 | 0.76 | 1.00 | 0.88 | 1.00 | 0.65 | 0.36 | 0.79 | 0.65 | 0.83 |
| W3 | 0.88 | 0.67 | 0.95 | 0.67 | 0.80 | 0.55 | 0.74 | 0.56 | 0.71 |
| W4 | 0.91 | 1.00 | 0.82 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| W5 | 0.73 | 0.67 | 1.00 | 0.67 | 0.56 | 0.28 | 0.62 | 0.43 | 0.63 |

Source: Author's own study.

Table 3

Value of relative criterion importance

| Parameter | k1 | k2 | k3 | k4 | k5 | k6 | k7 | k8 | k9 |
|------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| Criterion weight | 0.1369 | 0.1117 | 0.2046 | 0.1117 | 0.1201 | 0.0516 | 0.0987 | 0.0652 | 0.0993 |

Source: Author's own study.

Furthermore, using formulas (2) and (4), the values of concordance indicator and discordance indicators were calculated. The values of the concordance and discordance indicators of the variant w with the variant w' are shown in Tables 4-5.

Table 4

Variant concordance indicator

| Variant number | W1 | W2 | W3 | W4 | W5 |
|----------------|------|------|------|------|------|
| W1 | 0.00 | 0.78 | 0.80 | 0.34 | 0.80 |
| W2 | 0.00 | 0.00 | 0.49 | 0.20 | 0.80 |
| W3 | 0.20 | 0.51 | 0.00 | 0.20 | 0.58 |
| W4 | 0.00 | 0.58 | 0.80 | 0.00 | 0.80 |
| W5 | 0.20 | 0.20 | 0.20 | 0.20 | 0.00 |

Source: Author's own study.

Table 5

| Variant discordance indicator | | | | | |
|-------------------------------|------|------|------|------|------|
| Variant number | W1 | W2 | W3 | W4 | W5 |
| W1 | 0.00 | 0.00 | 0.04 | 0.00 | 0.11 |
| W2 | 0.89 | 0.00 | 0.26 | 0.89 | 0.17 |
| W3 | 0.63 | 0.46 | 0.00 | 0.63 | 0.07 |
| W4 | 0.14 | 0.08 | 0.18 | 0.00 | 0.25 |
| W5 | 1.00 | 0.46 | 0.38 | 1.00 | 0.00 |

Source: Author's own study.

In further analysis, the determination of the evaluation of concordance and discordance thresholds $\alpha = 0,77$ and $\beta = 0,19$, respectively, took place. These are essential to choose an efficient variant of the transportation system for the analysed case study. The principle of determination of the concordance and discordance thresholds is presented in References [16], [17], it being assumed that the quantity α interpreted as a concordance threshold takes values closer to unity, while the quantity β interpreted as a discordance takes values closer to zero. For the analysed case study of multi-criteria evaluation, considering the concordance and discordance thresholds adopted, an outranking relation was obtained, as shown in Figure 4.

The first level consists of the first and fourth decision variants, because none of them has been surpassed by any other decision variant, and also no mutual outranking relation occurs between them. The second level creates the second variant, which is surpassed by the first variant, however, it itself surpasses the fifth variant, which is at a lower level. At the third level, there are the third and fifth variants. These variants have been surpassed by the all variants from the first and second levels, while they themselves do not exceed any of the options analysed.

It is hardly possible to identify the best variant on the basis of the analysis of outranking relations. The fourth variant seems to be worse than the first variant, because although it remains at the first level, it surpasses only one variant of the lowest level. In such a case, a decision to lower the concordance threshold can bring clarification of these doubts.

A reduction of the concordance threshold to a value of 0.33 allowed a clearer determination of preferences in relation to all decision variants. The first level is only made up by the first decision variant, which has not been surpassed by any other decision variant. The fourth variant, which was at the first level for the concordance threshold with a value of 0.77, is found on the second level and has been surpassed by the first variant (Figure 5).

Moreover, comparisons can be made of variants that are on the first level, which will allow the decision maker to take a decision. In Figure 6, decision variants are compared for the analysed case study with respect to two criteria: overhead costs and length of routes operated. For other criteria, solution variants are identical, it was, therefore, decided not to compare them, because they do not bring any information relevant to the decision maker.

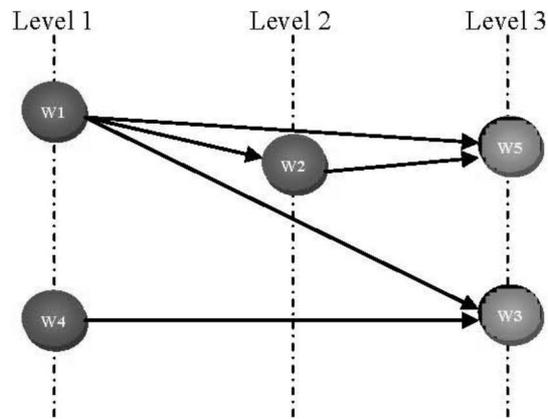


Fig. 4. Outranking relations for concordance threshold $\alpha = 0.77$ and discordance threshold $\beta = 0.19$
Source: Author's own study.

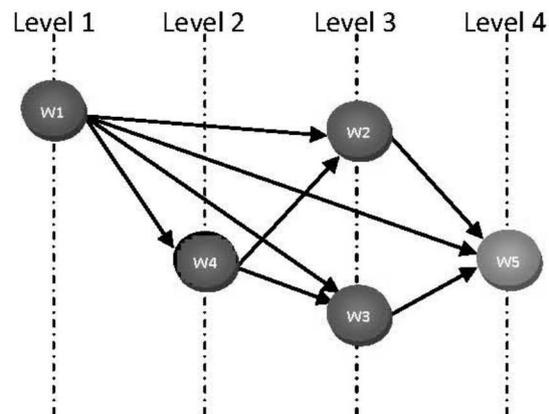


Fig. 5. Outranking relations for concordance threshold $\alpha = 0.33$ and discordance threshold $\beta = 0.19$
Source: Author's own study.

For the case study analysed, the best variant with respect to the subcriteria adopted is the first variant, which exceeds the second, third and fifth variants, while it itself is unsurpassed by any of the variants analysed.

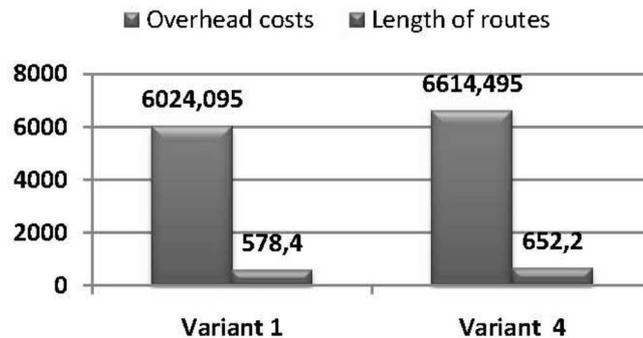


Fig. 6. Comparison of the first and fourth variants with respect to two evaluation criteria
Source: Author's own study.

7. Conclusions

The problem presented in the paper may be subjected to further analysis and testing. For the purposes of the paper, experiments were conducted in the area of the impact of the change in value of the concordance and discordance thresholds on the outcome of the final solution. Three values were adopted for the concordance threshold and four values for the discordance threshold and this was dictated by the fact that for other simulations, no significant changes were obtained in the selection of a decision making variant. In Figure 7, detailed outranking values for the analysed variants of transportation systems for various concordance and non-concordance thresholds are presented.

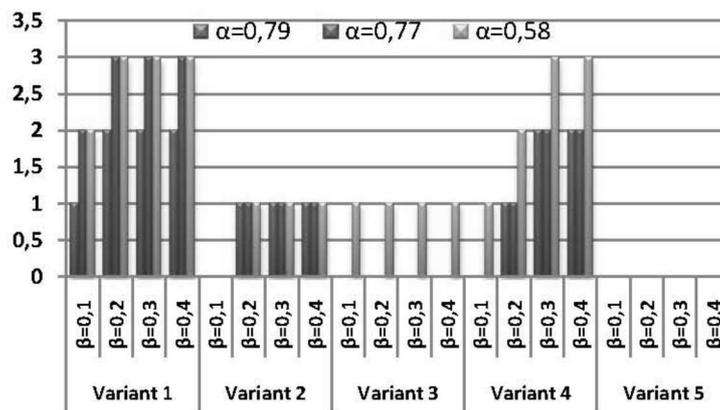


Fig. 7. Outranking values of the analysed variants of transportation systems for various concordance and discordance thresholds
Source: Author's own study.

It follows from the simulation carried out that the best variant is the first variant, while the worst variant of the fifth variant. Computer simulation allows a decision maker to take decisions satisfactory to him. This is possible by changing the values of the concordance and discordance thresholds, the thresholds α and β play the role of a sort of sieve letting through only those variants $Y(w)$ of the transportation system from the set \mathbf{Y} , that simultaneously meet the concordance threshold α and discordance threshold β .

The approach proposed in the paper can be a tool assisting in decision-making regarding forming supply chains. Moreover, this approach may be used by decision makers in other functional areas of logistics, for example, in the design of point elements of logistics networks and the assessment of logistics service providers. The computer package used in the analysed case study allows the assessment of multiple variants with different disturbances in the process realisation.

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