TICKET TARIFFS MODELLING IN URBAN AND REGIONAL PUBLIC TRANSPORT

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Abstract:

Ticket tariff is an important factor influencing the demand for public transport. Among basic problematics regarding ticket tariffs are designing new fare systems and optimization of current systems. The task of optimization is influenced by two main factors: ticket prices and the structure of the tariff. Both elements were researched in this article, based on eleven public transport organizers fare systems in Poland – metropolitan areas and cities of a different scale. The purpose of this article was to define basic tariff types used in urban and regional public transport with a presentation of their function models. Ticket tariffs split into two main groups: flat and differential. Differential group of tariffs covers: distance (usually are encountered fares based on a number of kilometres or stops travelled), quality (e.g. different fares on basic and express lines), time (minutes, hours or days of ticket validity, but also different tariff during on-peak and off-peak hours), sections (between which passenger travel on a transit route) and zones (transport network divided into areas, e.g. designated by municipalities boundaries) tariffs. The concept of this study was to transform as many tariffs as possible from tabular form to the mathematical function. Five types of functions were considered for each tariff schematic: linear, power, polynomial, logarithmic and exponential. Functions and associated with them R-squared parameters were obtained as a result of regression analysis. The paper indicates that for time, distance and flat tariffs conformity (R^2) was in most cases very high and above 0,90. The results indicate that the power function best describes time tariffs. In the case of distance tariffs, different kind of functions can be used: logarithmic, power or polynomial. The proposed function form of tariffs may speed up the process of creating new fare systems or upgrading existing ones. With general knowledge about the structure of tariffs and their function forms, it would be easier to determine the price of different kinds of tickets. New fare integration solutions could be also proposed in the future by using Big Data analysis.

Keywords: ticket tariff, fare system, fare planning, big data, public transport

To cite this article:

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1. Introduction

Modern public transport (PT in shortcut) systems are made up of five components: infrastructure, suprastructure (vehicles), ITS solutions, PT employees and organization (Chamier-Gliszczyński, 2012). Ticket tariff is an element of the organization, which can be implemented using PT employees and ITS solutions to provide service available on the infrastructure and in vehicles. It is an important criterion determining passenger demand, the others are the level of service provided by the organizer and competing transit modes (Vuchic, 2007). The level of service provided by the organizer can be assessed by checking network coverage, travel times, passenger's path choices, service frequency, etc. The level of competing services is mainly based on the accessibility of private motorization, congestion, fuel prices etc. All those criteria's specify demand elasticity on public transport services, which can be determined during the multi-criteria decision-making process (Cieśla, 2020). Those criteria can be divided into economic, environmental and social criteria (Chamier-Gliszczyński, 2017). Further on, the demand is also influenced by the maximum level of supply, which in the case of public transport can be limited by strict capacity constraints. Overcrowding effects have an impact on path choices in public transport models (Drabicki, 2017).

Among basic problematics regarding ticket tariffs are designing new fare systems and optimization of current systems. Organizers have to choose a fare system from a wide range of tariff types and adapt it to local conditions. Maximization of demand (ridership), revenue, transport performance (e.g. passenger-km), profit or social welfare are mainly chosen as the objectives of fare systems (Ling, 1998, Borndörfer, 2012). Different objectives are selected on networks covering large cities, where transport performance is more important and small communities, where social accessibility via public transport is more crucial (Danesi and Tengattini, 2020). The task of optimization is influenced by two main factors: ticket prices and the structure of the tariff. Connections between those elements were researched in this article.

The profitability of the tariffs was researched in studies held on passengers' willingness and the possibility to pay. Models have shown that passengers are most willing to pay for a journey in the cumulative pricing schematic (Otto, 2017). A robust positive relationship between workers' income and travelcard possession among low-income people was indicated. The conclusion was, that if subsidies for public transport are made because of low-income groups, the ticket pricing must be affordable for those people (Bondemark, 2020). Researches were also conducted on the topic of fare evasion by different groups of passengers. They indicated that age and gender are the main predictors of this phenomenon. Younger and male travellers have the highest likelihood to evade fares (Cools, 2018).

Problematic of ticket tariffs is inseparably connected with the unprofitability of public transport by itself, especially in systems operating in Europe. However, profitability is not the main reason for public service functioning, instead of inclusion of all social groups in society, realization of city transport policy and providing basic access to all services (work, education, culture) in the selected area. Fees from tickets are just one of the three main sources of revenues in this system, the others are subsidies or grants from city budgets and funding from the national budget or EU programs (Popović, 2018). A major influence on profitability have also ticket discounts for selected social groups, which are or not refundable by the government. Some of the transport organizers even resign from ticketing systems by the implementation of free-fare public transport, which effects depend on the quality of service, public transport usage and level of fees that existed before implementation of the system. Well identified and researched case of free-fare public transport is a city of Tallinn in Estonia (Cats, 2017).

Another major problem of ticket tariffs is the integration of different systems on a city and regional level. In most studies, it is connected with reduction of transaction costs (Takahashi, 2017), but the problem of integration is extremely challenging in fragmented public transport systems involving multiple stakeholders, such as different level of local government, transport organizers and carriers using completely different tariff systems (Iwanowicz and Szczuraszek, 2019). National legal conditions are also very limiting the possibilities for integration. New opportunities in the case of fare integration were opened by intelligent transportation systems and Big Data analysis. Systems enabled central management of income distribution to organizers and personalized fare calculation for passengers using

e.g. NFC technology (Alan and Birant, 2018). Smart cards help also to better understand demand by identifying travel patterns of individual transit riders, which enables to create large databases for service optimization (Ma, 2013). Still it must be remembered, that payment systems are just a small element of complex services provided by public transport management systems and ITS solutions generally (Karoń and Żochowska, 2015; Żochowska et al., 2018).

As it was shown above, designing ticket tariffs is a basic organizational issue of public transport. The purpose of this article is to define basic tariff types with a presentation of their function models, based on fare systems implemented in Polish public transport networks. The authors presented an original approach to the modelling of tariffs by assignment of function formulas to tariffs of a different kind. The article is divided into 6 sections. Section 2 covers a critical analysis of literaturę, regarding ticket tariff types specified all around the world; different models, which are used to determinate their form and the role of Big Data analysis. Section 3 describes the area of the research. Analysed transport organizers were listed with an indication of tariffs, which were used by them. Section 4 includes the modelling method. First, were presented methodological assumptions and then tariffs were analysed one by one. Section 5 lists the results of the study in a table presenting used tariff types with their functions and match. Finally, section 6 is devoted to the discussion of the results and conclusions.

2. Literature review
2.1. Ticket tariff types

In urban and regional public transport are used different kinds of ticket tariffs split into two main groups: flat and differential (Ling, 1998). The first is very simple because the passenger pays the same amount of money per trip, regardless of the circumstances. Among the second group, there were indicated several structures of transit fares (Grey, 1975; Nash 1982; Lovelock 1987), but all can be reduced to 5 major tariffs based on: distance (usually are encountered fares based on a number of kilometres or stops travelled), quality (e.g. different fares on basic and express lines), time (minutes, hours or days of ticket validity, but also different tariff during on-peak and off-peak hours), sections (between which passenger travel on a transit route) and zones

(transport network divided into areas, e.g. designated by municipalities boundaries).

The flat tariff is presented by mentioned above authors as extremely unfair especially for passengers in greater urban areas, because both passengers travelling from one end to another of the city and the other travelling just for two stops, will pay the same amount of money. Distance tariff based on kilometres is classified as proper for intercity and regional transport (Tsai, 2008) and on the other way inconvenient in urban areas with dense stop localization. Distance tariff based on the number of travelled stops can be used in cities, but passengers must validate the ticket both while entering and exiting the vehicle, which is in turn not convenient for them. The quality tariff also disturbs passengers patience, as it must be remembered, that different services offered by the same transit organizer can cost differently. Even connections departing in the same direction on a similar route could have different prices, because one will be a basic line stopping on every stop, the other will be an express line stopping only on major interchanges. Time tariff is very popular in city and metropolitan areas, especially used for interchanging and by regular, everyday passengers of public transport. Zone tariff also uses differential ticket fees by a distance of the journey, but by assigning stops to areas (zones) and strictly depends on the localization of start and end of travel stations (origin-destination pairs) in zone system (Babel, 2003).

In-depth studies were mainly held on three types of above differential tariffs: distance, time and zone (Jørgensen and Pedersen, 2004; Otto, 2017; Yang, 2020). The major issue was, whether it is possible to achieve both a simple and efficient tariff structure. In the case of distance tariff, research has shown that dependency between fare and travel distance is less steep the higher the weight the organizer puts on profit (Jørgensen and Preston, 2007). In the case of time tariff, the one with different prices on-peak and off-peak, it is shown as a solution for overloaded transport system during peak hours. The differential tariff will influence passenger demand by reducing the number of travels during peak, in favour of off-peak hours (Marabucci, 2019). A major influence of season tickets on demand for public transport was also indicated, especially for acquiring additional everyday users (FitzRoy and Smith, 1999). In the case of zone tariff, many transport organizers use an

excessively large number of zones, which are too difficult to administrate and for passengers to navigate in. An example showed, that the ticketing system in Oslo can be simplified from 88 to only 10 zones, without any loss in functionality of the system and projected incomes (Jansson and Angell, 2012).

2.2. Tariff models

Selection of the optimal fare system is one of the basic issues for designing public transport system. Flat and differential tariffs were compared in case of the change of total revenue between those two general types of ticketing. The conclusion of the (Ling, 1998) research based on mathematical derivation was, that the optimal fare structure is affected by the fare elasticity of demand for short and long trips and the number of trips. (Sun and Szeto, 2019) claimed that there was no study with a comparison of more than two kinds of differentiated fare structures. Other proposed methods of assessment was a multicriteria analysis based on the Vitas method, which results in the ranking of the zonal fare systems (Popović, 2018). Researchers mainly conduct analysis based on one prepared model of the differential tariff. Each model is constructed differently and that's the problem for comparative analysis.

The distance tariff model was presented by the relationship between demand and total costs for transport operator as a linear function influenced by the number of passengers transported and their average travel distance (Jørgensen and Preston, 2007). In the earlier study, author discussion was also conducted between transport operators emphasis on profit versus consumer surplus. The conclusion of the article was that it is not clear how travel distance influences fare, quality of transport supply and generalized travel costs, in a situation, when fare and quality are controlled variables by transport operators (Jørgensen and Pedersen, 2004).

Time fare was modelled using demand distribution between peak and off-peak hours and estimation of influence on ticket sales with different values of elasticity. Research has shown, that implementation of the tariff with more expensive tickets for peak hours and cheaper tickets for the off-peak period will increase the overall income (Marabucci, 2019). Nevertheless, there is a lack of studies taking into consideration a wide range of time tickets, on the scale of minutes, hours and days of validity. Researchers

focus e.g. on single-use or season tickets separately and those offers are directed to different kinds of passengers (FitzRoy and Smith, 1999).

Model for zone tariff was first proposed using a graph-theoretical point of view on the public transportation network (Hamacher and Schöbel, 1995). Theoretical results for special networks and heuristic algorithms for the general problem were also presented (Babel, 2003). It was also indicated, that still there is a need for more sophisticated heuristics for the zone tariff problem. Zone models can vary in the way zones are designated (cut): ring structure vs. connected zones; and way the fares are calculated: counting zones, cumulative pricing and maximum pricing (Otto, 2017). The simplification of the problem was designing a zone model on rail transport networks with limited area coverage and number of stops. Optimal zone division for linear connections was also analyzed, which in the case of classification should be attributed to the sectional fare system (Yang, 2020).

The sectional fares model implies that the transit route is divided into sections. Payment is reflected in the number of sections crossed by the passenger, between boarding and alighting stops. This kind of model is adopted widely by bus companies in Hong Kong. According to the authors of the study, compared with a flat fare structure the sectional one is always better and comparing with distance-based tariff, it depends on the geometry of the network, demand distribution and maximum allowable fares (Sun and Szeto, 2019).

2.3. Big Data application during modelling of ticket tariffs

Access to data is a factor that can determine the position and competitive advantage of companies operating in urban and regional public transport. The concept of large, variable and diverse sets of data (Big Data) has been developed by technological solutions allowing information to be collected, then processed and finally used to construct sets of information suitable for defining the demand for service provision. In the case of public transport - in terms of designing tariff systems tailored to the specifics of a given market or optimizing existing systems (Gao et al., 2020).

Big Data in terms of application in public transport can cover many areas, including (Jabłoński, 2019): management, i.e. of substantial data sets taken, for

example, from carriers' reservation systems; aggregation; cleaning, i.e. verification of large data sets; analytics understood as an analysis of large data sets; machine learning, i.e. on large data sets. In turn, the analysis of the above datasets itself may include various techniques: Data Mining; Web Mining - using internet repositories; Visualization Methods; Machine Learning; Optimization Methods and Social Network Analysis.

Digital data are being continuously collected from millions of devices. They are downloaded from the applications and analyzed to make decisions about the direction of urban transport lines (passenger route selection), frequency of trips (service provision), means of transport used, customer preferences and choices, and thus used to optimize offers, increase their efficiency, personalize them, predict user behaviour, or make real-time business decisions (Pelletier et al., 2011). Today, companies' market value is mostly based on intangible resources, including access to data, which are becoming increasingly important and start to be treated as a type of capital. Data on users and their behaviour are taken by companies, e.g. from telecoms, financial institutions involved in the ticketing process. However, they can also be generated and collected due to using products and services provided by companies providing public transport, e.g. cameras, sensors, recordings, ticket machines, applications, etc. (Gschwender, 2016).

It should be noted that very often, in this situation, we are dealing with digital data that are unstructured and cannot be stored in traditional databases. The way to use them will be using Big Data technologies to store, but above all - to process them. Therefore, data with specific parameters will be of crucial importance in supporting the definition of tariff systems. These include, among others: availability, size of data (currently calculated in terabytes or even larger units, i.e. petabytes) and the possibility to generate and process them quickly, value (separation of relevant data from those that will not be analyzed), diversity (referring to the diverse nature of both structural and non-structural data), the type and nature of data, and at the same time reliability of collected data (its veracity).

3. Area of analysis

For this study, 11 Polish public transport organizers have been selected and their tariffs have been listed in table 1 (Decree of Inowrocław Mayor, 2020; Decree of PKS Gdynia CEO, 2021; Gromadzki, 2011; Resolution of GZM Management Board, 2020; Resolutions of City Councils: Krakow, 2020; Lodz, 2020; Poznan, 2020; Szczecin, 2019; Torun, 2020; Warsaw, 2017 and Ząbki, 2015). In Poland, the establishment of the ticket tariff takes place mainly through resolutions of area councils or private enterprise owners. After the resolution, the tariff is implemented by the transport organizer.

In most cases, organizers use a combination of different tariff types. Most common is a combination of 2 or 3 tariffs. The basic tariff type functioning in selected organizers is based on time.

Table 1. Examples of polish urban, metropolitan or regional transport organizers tariff types

Type of tariff \ Transport organizer	Flat	Distance (kilometres)	Distance (number of stops)	Quality	Time	Section	Zone
ZTM Warsaw					x		x
ZTP Kraków					x		x
UM Toruń					x		x
ZTM Poznań			x		x		x
ZDIT Łódź			x		x		x
MPK Inowrocław			x		x		
PKS Gdynia	x (municipality)	x (region)			x		
Project for LGOM (I variant)	x (cities)	x (region)			x		
ZTM GZM		x		x	x		
ZDiTM Szczecin				x	x		
UM Ząbki	x						

Passengers in that tariff have the possibility to buy short-term and long-term tickets. That fare system is used in 10 areas. In the case of large urban areas it can be combined with zoning (at least 2 zones in areas – case of Warsaw, Krakow, Torun, Poznan and Lodz), quality (e.g. more expensive tickets for airport lines – ZTM GZM situation or faster/express lines – case of ZDiTM Szczecin) or distance (based on kilometres in LGOM and ZTM GZM or number of stops travelled – cities: Poznan, Lodz and Inowroclaw) tariffs.

One private organizer – PKS Gdynia – operates on the regional area with distance fare based on kilometres, but the special feature of its tariff is a flat ticket rate inside borders of few municipalities. Also, the same fare solution is provided in the projected tariff for LGOM, but inside borders of cities (Legnica, Lubin and Głogów). The completely flat tariff was found in case of the Ząbki city, which is a small municipality inhabited by 35,000 people.

4. Modelling method
4.1. Methodological assumptions
For further modelling, all specified tariff types were subjected – flat, time, distance by a number of stops and distance by kilometres, zone and quality. The concept of this study was to transform tariffs from tabular form to the mathematical function:

$$y = f(x); \tag{1}$$

where:
y – ticket price [PLN],
x – feature of the tariff: time [h]; distance [number of stops]; distance [km]; quality or zone.

Functions were determined by using regression analysis. Five types of functions were considered for each tariff schematic:

Linear:
$$y = ax + b; \tag{2}$$

Power:
$$y = ax^b; \tag{3}$$

Logarithmic:
$$y = a \ln x + b; \tag{4}$$

Polynomial (max. of 4th degree):
$$y = a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0; \tag{5}$$

Exponential
$$y = ae^{bx}; \tag{6}$$

where:
a_n, b – parameters; n ∈ [0,4] and n ∈ ℕ.

For each tariff, the function form and coefficient of determination (R^2) were calculated. Functions with the highest R^2 were chosen as the closest possible fare model to the values presented in the tariff tables and showed on the figures with description.

4.2. Flat tariff
The first was modelled flat tariff of Ząbki city. In the fare, there is only one ticket type – single-trip – per 3 PLN (Resolution of Ząbki City Council, 2015). The function of this tariff is very simple: y = 3 and is not dependent on any parameter.

4.3. Time tariff
Time tariffs are constructed by public transport organizers in three-time intervals: minutes, hours and days. In this research four time tariffs were applied: ZTM Warsaw (Resolution of Warsaw Capital City Council, 2017), ZTP Krakow (Resolution of Krakow City Council, 2020), UM Torun (Resolution of Torun City Council, 2020) and ZDiTM Szczecin (Resolution of Szczecin City Council, 2019). It is worth noting, that those periods of ticket validity are different between the cities, some of the ticket types are used only in one of the cities: 15 minutes, 30 minutes, 45 minutes, 75 minutes, 120 minutes, 48 hours, 5 days, 10 days or 15 days valid tickets. But there are some general intervals used in every area, like tickets valid for 24 hours and 30 days. Tariffs with ticket prices were listed in table 2. For purpose of drawing a chart, validity periods have been unified to values expressed in hours.

Functions determined for the time tariffs were presented in Fig. 1.. Ticket price and time of ticket validity were shown on a logarithmic scale with a base of 2, because of multiple increments of the parameter in the case of subsequent tickets in the tariff. In all cases, time tariffs were described by power functions with the highest possible R^2 value. Parameter "a" of functions achieved value from interval 3,672 to 6,0512 and "b" from 0,4887 to 0,5085. Graphs of the functions marked in the diagram are very similar to each other

Table 2. Summary of ticket prices in time tariff [PLN]

| Ticket validity period	Time [h]	Ticket price [PLN]			
		ZTM Warsaw	**ZTP Krakow**	**UM Torun**	**ZDiTM Szczecin**
15 minutes	0,25	-	-	-	2
20 minutes	0,33	3,4	4	-	-
30 minutes	0,5	-	-	-	3
45 minutes	0,75	-	-	3,4	-
60 minutes	1	-	6	-	4
75 minutes	1,25	4,4	-	-	-
90 minutes	1,5	7	8	6	-
120 minutes	2	-	-	-	5
24 hours	24	15	17	12	12
48 hours	48	-	35	-	-
72 hours	72	36	50	-	-
5 days	120	-	-	-	35
7 days	168	-	56	43	-
10 days	240	-	-	-	60
15 days	360	-	-	53	-
30 days	720	110	148	88	100
90 days	2160	280	-	240	260
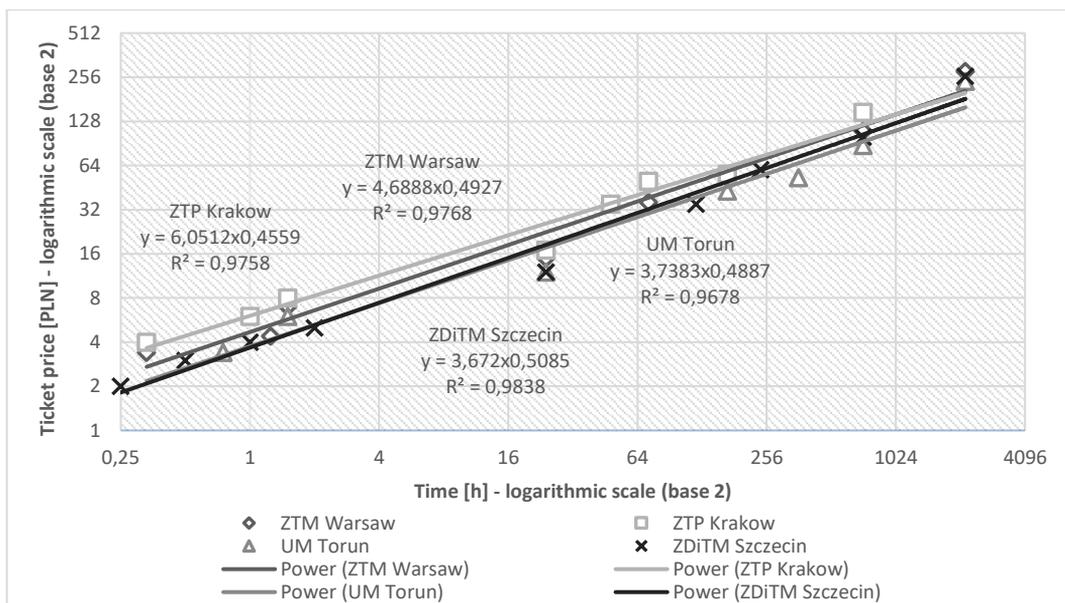
Fig. 1. Regression analysis of tariffs based on time

4.4. Distance (number of stops) tariff

Distance tariff with counting number of stops are constructed by public transport organizers in two different ways: by adding a fee to every additional travelled stop or by setting a price for several stops travelled in ranges. In the analysis, the first option is used by ZDiT Lodz [34] and ZTM Poznan [35], the second – by MPK Inowroclaw [29]. Inowroclaw tariff is almost flat – there are only 3 price levels, which does not differ much. It is connected with a small size of the city (72,000 inhabitants) and rather short routes of the bus lines. In all cases, the fare system

limits a maximum of 30 possible stops on the route. Complete distance tariffs with ticket prices were listed in table 3. The further the passenger trip is heading, the less the ticket price is incremented. In each case, tariffs were described by other types of function. ZTM Poznan fare system was regressed into a logarithmic function, MPK Inowroclaw to a polynomial of 4th degree and ZDiT Lodz to a power function. In each situation, the R^2 parameter was above 0,93 and very close to 1 in the case of logarithmic and power regression (respectively 0,9932 and 0,996). Functions were presented in Fig. 2.

4.5. Distance (kilometres) tariff

Distance tariffs with the counting of kilometres travelled are mainly constructed based on distance ranges in which passengers pay the same fare for a selected number of kilometres. The analysis covered two examples of transport organizers with this kind of fare system: PKS Gdynia (Decree of PKS Gdynia CEO, 2021) and ZTM GZM (Resolution of GZM Management Board, 2020); and also one based on project proposition for the LGOM area (Gromadzki, 2011). In urban areas, this type of tariff is flatter, than in the case of regional transport, where the increment of fare is steeper.

Table 3. Summary of ticket prices in distance (number of stops) tariff [PLN]

| Number of stops	1	2	3	4	5	6	7	8	9	10
ZTM Poznan	0,72	1,32	1,92	2,32	2,72	2,90	3,08	3,16	3,24	3,32
MPK Inowroclaw	2,30	2,30	2,50	2,50	2,50	2,70	2,70	2,70	2,70	2,70
ZDiT Lodz	1,00	1,20	1,40	1,60	1,80	2,00	2,20	2,30	2,40	2,50
Number of stops	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
ZTM Poznan	3,40	3,48	3,56	3,64	3,72	3,80	3,88	3,96	4,04	4,12
MPK Inowroclaw	2,70	2,70	2,70	2,70	2,70	2,70	2,70	2,70	2,70	2,70
ZDiT Lodz	2,60	2,70	2,80	2,88	2,96	3,04	3,12	3,20	3,28	3,36
Number of stops	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
ZTM Poznan	4,18	4,24	4,30	4,36	4,42	4,48	4,54	4,60	4,66	4,72
MPK Inowroclaw	2,70	2,70	2,70	2,70	2,70	2,70	2,70	2,70	2,70	2,70
ZDiT Lodz	3,42	3,48	3,54	3,60	3,66	3,72	3,78	3,84	3,90	3,96
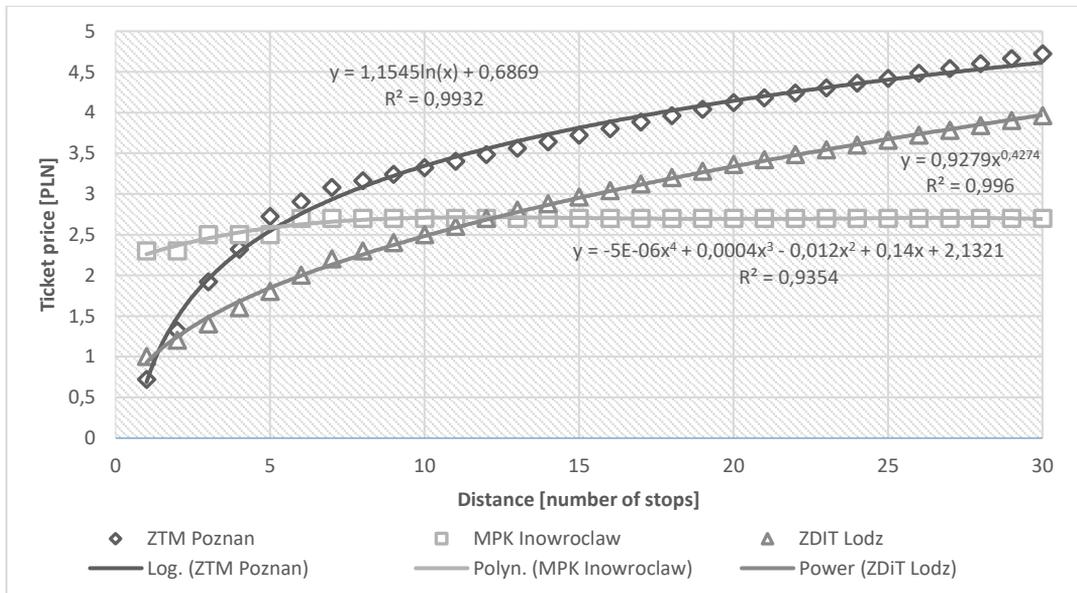
Fig. 2. Regression analysis of tariffs based on distance (number of stops)

The maximum travel distance in this tariff depends on the size and organization of the transport network.In the ZTM GZM network, the longest possible route is 50-kilometres long, in LGOM 80 kilometres and in PKS Gdynia it is limited to 140 kilometres. Only single-trip tickets were analyzed in this kind of tariff, but in the offer of some of the transport organizers, there are also season tickets connected with a distance range of trips. Complete distance tariffs with ticket prices were listed in Table 4.

Two tariffs were described by polynomial functions of 2nd (LGOM) and 4th degree (PKS Gdynia) and one by logarithmic function (ZTM GZM). This happened due to fare construction – rather systematic increase of ticket price with the length of the journey or large discount for short journeys and a rather flat rate for longer ones. In each situation, the R^2 parameter was above 0,9 and very close to 1 in the case of polynomial regressions (values of 0,9915 and 0,9885). Functions were presented in Fig.3.

4.6. Zone tariff

Zone tariff is often combined by Polish public transport organizers with time tariff. The second characteristic element of zoning in Poland is also division on a low number of zones: 2 or 3. Generally, they are cut on the area of city and suburban municipalities. In the case of Krakow, the third zone is designated on further metropolitan municipalities areas. Availability of different kind of time tickets in selected zones of the organizer is very low – tickets are mainly available only in the I zone (area of the main city) or in all zones. Best availability is represented by 30 days tickets. Zone tariffs with ticket prices were listed in Table 5.

Table 4. Summary of ticket prices in distance (kilometres) tariff [PLN]

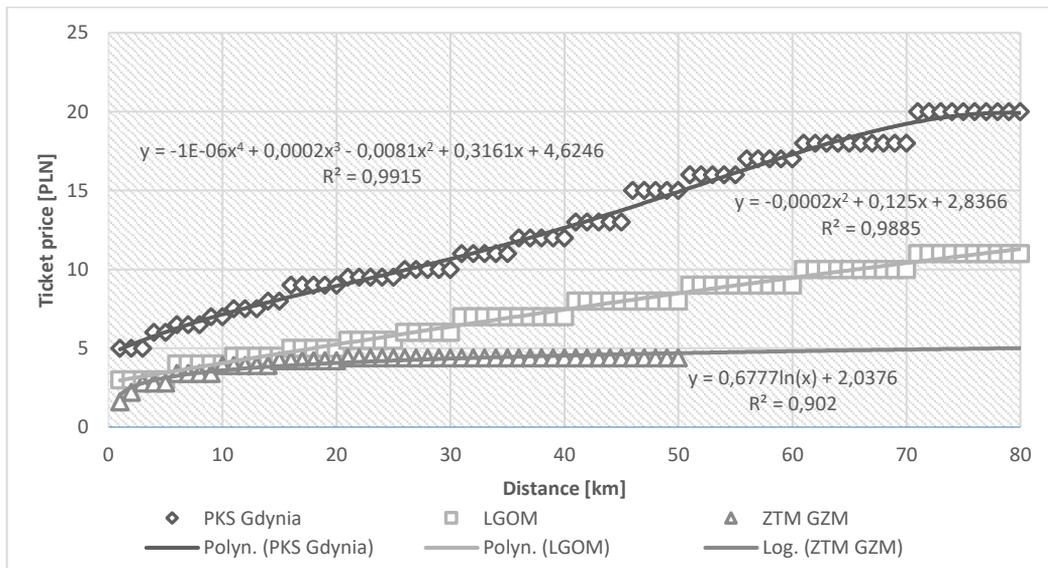| Distance [km]	1	2	3	4-5	6-8	9	10	11-13	14	15	16-20
PKS Gdynia	5,0	5,0	5,0	6,0	6,5	7,0	7,0	7,5	8,0	8,0	9,0
LGOM	3,0	3,0	3,0	3,0	4,0	4,0	4,0	4,5	4,5	4,5	5,0
ZTM GZM	1,6	2,2	2,8	2,8	3,4	3,4	3,9	3,9	3,9	4,2	4,2
Distance [km]	21-25	26-30	31-35	36-40	41-45	46-50	51-55	56-60	61-70	71-80	>80
PKS Gdynia	9,5	10,0	11,0	12,0	13,0	15,0	16,0	17,0	18,0	20,0	20,0
LGOM	5,5	6,0	7,0	7,0	8,0	8,0	9,0	9,0	10,0	11,0	-
ZTM GZM	4,4	4,4	4,4	4,4	4,4	4,4	-	-	-	-	-

Fig. 3. Regression analysis of tariffs based on distance (kilometres)

Table 5. Summary of ticket prices in zone tariff [PLN]

Ticket validity period	ZTM Warsaw			ZTP Krakow					UM Torun		
	Zone I	Zone II	Zone I+II	Zone I	Zone II	Zone III	Zone I+II	Zone II+III	Zone I+II+III	Zone I	Zone I+II
20 minutes	-	-	3,4	-	-	-	-	-	4	-	-
45 minutes	-	-	-	-	-	-	-	-	-	-	3,4
60 minutes	-	-	-	-	-	-	-	-	6	-	-
75 minutes	4,4	-	-	-	-	-	-	-	-	-	-
90 minutes	-	-	7	-	-	-	-	-	8	-	6
24 hours	15	-	26	17	-	-	-	-	22	-	12
48 hours	-	-	-	-	-	-	-	-	35	-	-
72 hours	36	-	57	-	-	-	-	-	50	-	-
7 days	-	-	-	56	-	-	-	-	68	-	43
15 days	-	-	-	-	-	-	-	-	-	53	-
30 days	110	112	180	148	79	37	158	96	179	88	126
90 days	280	282	460	-	-	-	-	-	-	240	-

Selective nature of zone tariff unable to perform modelling of a coherent function forms for this kind of fare system.

4.7. Quality tariff

The quality tariff is rare in urban public transport, but in an analyzed set of organizers, there are two specific situations, in which organizations use them. ZTM GZM for three ticket types has extended tariff including whole public transport network with special express lines going to the Katowice Airport. The tariff is available only with 24 hours, 30 and 90 days network tickets. Basic ticket prices are multiplied by 1,16 or 1,40 to achieve an extended tariff (Resolution of GZM Management Board, 2020). Differ in the quality tariff of the organizer is presented in table 6.

Table 6. Summary of ticket prices in quality ZTM GZM tariff [PLN]

Ticket validity	24 hours	30 days	90 days
Network without airport line	10	134	344
Network with airport express lines	14	160	400
Conversion factor	1,40	1,16	1,16

ZDiTM Szczecin network is divided into normal and express lines. The tariff is available only in case of minute and monthly tickets, but without a 120 minutes ticket, because there is no express line with such a long travel time of a single journey. Express lines tariff is achieved by multiplying the basic price by 1,62 (monthly tickets) or 2,00 (single tickets). Differ in the quality tariff of the organizer is presented in table 7.

The above-mentioned tariffs are very specific and it was not possible to designate a coherent function form of them.

5. Statement of results

Functions and R-squared obtained as a result of regression for different ticket fares were listed in table 8. The simplest function was obtained in the case of the flat tariff. Power functions determined mainly time fare systems, but with a very high R^2 parameter (above 0,96). Polynomial functions were best assimilated to the distance fares – both connected with counting stops or kilometres. In some situations, distance tariffs were also represented by logarithmic and power functions. The lowest obtained R^2 value was 0,9020 (distance tariff of ZTM GZM) and the biggest one 0,9960 (distance tariff of ZDIT Lodz), which is extremely close to the actual tariff form.

Table 7. Summary of ticket prices in quality ZDiTM Szczecin tariff [PLN]

Ticket validity	15 minutes	30 minutes	60 minutes	120 minutes	30 days	90 days
Normal lines	2	3	4	5	100	260
Express lines	4	6	8	-	162	422
Conversion factor	2,00	2,00	2,00	-	1,62	1,62

Table 8. Summary of regression functions and r-squares for different ticket fares

Transport organizer	Analyzed type of tariff (x)	The function of ticket price [PLN]	Type of regression	Match R²
UM Ząbki	**Flat**	**y = 3**	**-**	**-**
ZTM Warsaw	**Time**	$y = 4{,}6888x^{0{,}4927}$	**power**	0,9768
ZTP Krakow	**Time**	$y = 6{,}0512x^{0{,}4559}$	**power**	0,9758
UM Torun	**Time**	$y = 3{,}7383x^{0{,}4887}$	**power**	0,9678
ZDiTM Szczecin	**Time**	$y = 3{,}672x^{0{,}5085}$	**power**	0,9838
ZTM Poznan	**Distance (number of stops)**	$y = 1{,}1545\ln(x) + 0{,}6869$	**logarithmic**	0,9932
ZDIT Lodz	**Distance (number of stops)**	$y = 0{,}9279x^{0{,}4274}$	**power**	0,9960
MPK Inowroclaw	**Distance (number of stops)**	$y = -5E{-}06x^4 + 0{,}0004x^3 - 0{,}012x^2 + 0{,}14x + 2{,}1321$	**polynomial**	0,9354
PKS Gdynia	**Distance (kilometers)**	$y = -1E{-}06x^4 + 0{,}0002x^3 - 0{,}0081x^2 + 0{,}3161x + 4{,}6246$	**polynomial**	0,9915
Project for LGOM	**Distance (kilometers)**	$y = -0{,}0002x^2 + 0{,}125x + 2{,}8366$	**polynomial**	0,9885
ZTM GZM	**Distance (kilometers)**	$y = 0{,}6777\ln(x) + 2{,}0376$	**logarithmic**	**0,9020**

6. Conclusions

Problematic of ticket tariffs are complex and have many areas to be researched. The basic research issue is that public transport organizers use different kinds of ticket tariffs. As it was shown in this paper, in most cases, there is no one tariff implemented in the transport system, but a combination of different models on a single area. The most common is a combination of 2 or 3 tariffs. Only 1 on 11 selected organizers from Poland used just one tariff and it was a flat one. That kind of tariff is rather used in small cities and the whole set of other types of tariffs in larger areas. The group of differential tariffs was specified in the research and it included time, distance (kilometres or number of stops), section, zone and quality fare systems. In the case of demand analysis, which could be the continuation of this research, the situation of coexisting multiple ticket tariffs should be taken into consideration.

As it was shown in this research, flat, time or distance tariffs can be approximated from tabular to function form after making the regression. Their match is in most cases very high and above 0,90. The power function is best for describing time tariffs. In the case of distance tariffs, different kinds of functions can be used: logarithmic, power or polynomial. Function form of tariffs may speed up the process of creating the new fare system. With general knowledge about the structure of tariffs and their function forms, it would be easier to determine the price of different kinds of tickets, which would be

based on the experience of existing systems. So the results of this study can be used in designing the process of new tariff systems. This problem can be also researched further to find typical models describing each tariff and some basic, repeatable parameters.

The type of function achieved in regression of tariffs can be connected with its features, which are benefitting longer journeys with less addition to the travel cost (degressive tariff). Such an approach to fare policy is specific for urban and regional public transport. Deeper studies can be conducted on comparison of different tariff types used in one area – for example, time and distance tariffs in Poznan and Lodz. The literature review showed that researches are mainly conducted on one ticket tariff system, while the biggest problem is the interference of those solutions. The influence of both fare systems should be assessed as well as, which tariff is more attractive to passengers or more profitable for transport organizer.

Functions describing tariffs can be also used to create more types of tickets within existing tariffs, for the reason of creating a more user-friendly system. By using ITS solutions organizers can implement electronic pay as you go systems with a wide range of tickets, regardless of the offer which was created for paper tickets. The advantage of such a system would be better matching to passenger's needs, for example by creating an offer for any multiple number of days travelled. As it was indicated in the research, nowadays there are some specific periods of

ticket validity, limited by the possibility of performing operations with paper tickets.

The problem of tariff diversity is very complex, especially in the situation of few transport organizers functioning in the area, e.g. urban transport, railway and regional bus transport, and integration of them into one coherent system. This issue can be researched further using Big Data analysis for finding the solution for the integration problem, that will connect or create new tariffs. Models for integration should take into consideration existing law, the structure of incomes, areas of communes, demand for transport etc.

Data processing techniques could be applied, so the transport companies will build competitive advantage strategies and thus improve their business models to increase their operations' efficiency, including the improvement of their operational performance. Big Data, therefore, would help to gain a deeper understanding of the micro and macro environment of modern public transport, including its users, in order to be able to design a better fare system.

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