MODELING AND SIMULATION OF A CONTROL SYSTEM OF WHEELS OF WHEELSET

Andrzej CHUDZIKIEWICZ¹, Juraj GERLICI², Magdalena SOWIŃSKA³, Anna STELMACH⁴, Wojciech WAWRZYŃSKI⁵

¹ Faculty of Transport, Electrical Engineering and Computer Science, Kazimierz Pulaski University of Technology and Humanities in Radom, Radom, Poland

² Department of Transport and Handling Machines, Faculty of Mechanical Engineering, University of Žilina, Zilina, Slovakia

³ The Cold Flow Test Turbine Facility "Polonia Aero", Zielonka, Poland

^{4,5} Faculty of Transport, Warsaw University of Technology, Warsaw, Poland

Abstract:

Modern light rail vehicles, such as a tram or rail bus, due to the need to provide mobility for the elderly or disabled people and the requirements of operators operating passenger rail transport or transport in urban areas must have a 100% low floor. Structurally, this is associated with the use of wheelset with independently rotating wheels (IRW) in such vehicles. It is also possible to use a bogie structure without the use of a wheelset axle by mounting the wheels directly in the side parts of the bogie frame. This construction is more complex and will not be discussed in this article. Bearing in mind the dynamic behavior of such vehicles during operation (lateral stability, profile wear) in various driving conditions (curve traffic, crossovers) and taking into account operating costs, it becomes necessary to install wheel rotation control systems to maintain center movement mass of the wheelset around the centerline of the track. The subject of the article will be considerations on modeling and simulation of rail vehicle bogie motion with IRW sets including the wheel control system. Nominal and mathematical models of the analyzed vehicle will be presented, as well as a controlled strategy based on the comparison of the angular velocities of the wheels of the wheelset A review of works on solutions of such systems will be presented, and a control concept will be proposed. The summary contains conclusions regarding the possibility of practical use of the proposed method of steering wheels of a wheelset in the case of independently rotating wheels.

Keywords: rail vehicle, independently rotating wheels, control system, modelling, simulation

To cite this article:

Chudzikiewicz, A., Gerlici, J., Sowińska, M., Stelmach, A., Wawrzyński, W., 2020. Modeling and simulation of a control system of wheels of wheelset. Archives of Transport, 55(3), 73-83. DOI: https://doi.org/10.5604/01.3001.0014.4234



Contact:

¹⁾ chudzikiewicz.andrzej@gmail.com [https://orcid.org/0000-0003-4767-4056], 2) juraj.gerlici@fstroj.uniza.sk [https://orcid.org/0000-0003-3928-0567], 3) msowinsk89@gmail.com, 4) ast@wt.pw.edu.pl [https://orcid.org/0000-0002-2301-6908], 5) wwa@wt.pw.edu.pl [https://orcid.org/0000-0002-2523-8520]

1. Introduction

The issue of wheel control of a wheelset has been studied and analyzed for many years. The beginnings of these studies concerned railway vehicles and were related to works on high-speed railways. In the case of conventional vehicles, i.e. vehicles with wheelsets in which the wheels are connected by a rigid axle, the design requirements for maintaining vehicle stability at high speeds and obtaining good dynamic properties on curves are contradictory (Shen et al., 1977). In addition, the issue of wheel and rail profile wear and contact problems also proved to be important in such cases (Kalker et al., 1991) and (Yu Sun et al., 2018). The proposed solution proved to be a wheelset with independently rotating wheels (IRW). Experimental and simulation studies conducted in the 80 s of the 20th century (Eickhoff et al., 1989) showed that the phenomenon of kinematic oscillations has been eliminated in IRW sets, which in the case of high speeds may lead to loss of stability, but there has been a tendency to remain in contact with the rail rim. This phenomenon may lead to excessive wear of the wheel and rail profiles and may also be the cause of vehicle derailment. One of the ways to eliminate this phenomenon was to introduce a wheel control system in such wheelsets. This issue was mainly dealt with by British. Japanese and Chinese scientists (Maoru et al., 1008). The ongoing work was rather focused on railway applications. The possibility of using IRW wheelsets in streetcar vehicles was noticed at the turn of the 20th and 21st centuries, when works were started in the scope of development of the construction of low-floor trams. The reason for starting work on this type of vehicles was the development of trends in urban transport, emphasizing such development of vehicles and infrastructure, which in the least invasive way affect the urban landscape and are also friendly and functional in accessing the aging society in urban agglomerations. The low floor of the tram vehicle, located on the same level, is a necessary requirement considering the creation of disabled persons or mothers with baby prams using public transport. In this case, it is not possible to use a conventional wheelset on the vehicle's trolley. A solution with a wheelset having independently rotating wheels must be used in the construction of the truck. As research has shown (Chudzikiewicz et al., 2016). in such a case, in order to prevent long-term, onesided contact of the wheel with the rail rim during movement, it is necessary to use the wheel control system in the bogie, according to the adopted algorithm. The subject of the article will be considerations on modeling and simulation of such a system, carried out in the case of work on a tram prototype with independently rotating wheels.

2. Mathematical model of a bogie with independently rotating wheels

The basic subassembly of the bogie that determines the dynamic behavior of the vehicle is the wheelset. which transfers the forces generated in the contact of the wheel with the rail to the bogie frame. Of course, an important role in the dynamic behavior of this system is played by susceptible elements of the 1st and 2nd degree suspension system (Ren Luo et al., 2020). Between the set, called conventional and IRW set, with independently rotating wheels, in addition to differences in design, there are significant differences in the form of models describing the dynamics of these components. The IRW kit model additionally has variables describing the angle of rotation of the left and right wheels. Fig. 1 shows a schematic of a wheelset with independently rotating wheels - IRW. It indicates the rotation angle w around the z axis, rotation angle θ_L of the left and right wheels θ_R around the y axis, wheel conicity λ , wheel radius ro and wheel track equal to 21.



Fig. 1. Orientation of the coordinate system and differentiation of rotation angles for the wheelset IRW

Fig. 2 shows a diagram of two wheelsets mounted in a bogie frame. The location and markings of individual elastic-damping suspension elements are also presented.



Fig. 2. Diagram of the nominal model of the bogie with wheelsets IRW. C_{px(y)} - longitudinal (x) or lateral (y) coefficients of a damping, K_{px(y)} - longitudinal (x) or lateral (y) coefficient of a stiffness. Index p – primary suspension system, index s – secondary suspension system

The Fig. 2 indicates: three rotation angles ψ_b (angle of rotation around the axis from the bogie frame), ψ_1 (angle of rotation around the axis of the first wheelset), ψ_2 (angle of rotation around the axis of the second wheelset), y_1 (displacement along the y axis - in lateral direction - of the first wheelset), y_b (lateral displacement of the bogie frame) and y_2 (lateral displacement of the second wheelset). It should be noted that this is a model of a bogie with wheelsets having so-called broken axle, enabling it to seat

independently rotating wheels. The mathematical model of such a bogie, compared to the mathematical model of a bogie with conventional wheelsets, has four additional degrees of freedom associated with the description of the independent rotational movement of the wheels of the first and second wheelsets on the left and right side (Sowińska, 2018). Examples of four equations describing the motion of the first wheelset are presented below.

Equation of lateral movement of the first wheelset"

$$m_{w}\ddot{y}_{1} + \left(\frac{W\lambda}{a} + 2K_{py}\right)y_{1} + 2\left(\frac{f_{11}}{v} + \frac{f_{11}}{v}\frac{r_{0}\lambda}{a} + C_{py}\right)\dot{y}_{1} - 2f_{11}\psi_{1}$$

$$-2\frac{f_{12}}{v}\dot{\psi}_{1} - 2K_{py}y_{b} - C_{py}\dot{y}_{b} - 2K_{py}L_{1}\psi_{b} - C_{py}L_{2}\dot{\psi}_{b} = 0$$
(1)

Rotation equation for the first wheelset around a vertical axis:

$$I_{wz}\ddot{\psi}_{1} + 2\frac{a\lambda f_{33}}{r_{0}}y_{1} - \left(\frac{2f_{12}}{v} - I_{y}\frac{v\lambda}{r_{0}a} + \frac{2f_{12}r_{0}\lambda}{va}\right)\dot{y}_{1} + \left(2f_{12} - a\lambda W + 2K_{px}b_{1}^{2}\right)\psi_{1} + \left(2a^{2}\frac{f_{33}}{v} + \frac{f_{22}}{v} + 2C_{px}b_{1}^{2}\right)\dot{\psi}_{1} - \frac{f_{33}ar_{0}}{v}\dot{\theta}_{1L} + \frac{f_{33}ar_{0}}{v}\dot{\theta}_{1R} - 2K_{px}b_{1}^{2}\psi_{b} - 2C_{px}b_{1}^{2}\dot{\psi}_{b} = 0$$

$$(2)$$

Equation of rotation of the left wheel around the transverse axis of the first wheelset:

$$I_w \ddot{\theta}_{1L} - f_{33} \lambda y_1 - r_0 f_{33} \frac{a}{v} \dot{\psi}_1 + f_{33} \frac{r_0^2}{v} \dot{\theta}_{1L} = 0$$
(3)

Equation of rotation of the right wheel around the transverse axis of the first wheelset:

$$I_w \ddot{\theta}_{1R} + f_{33} \lambda y_1 + r_0 f_{33} \frac{a}{v} \dot{\psi}_1 + f_{33} \frac{r_0}{v} \dot{\theta}_{1R} = 0$$
(4)

Descriptions of parameters appearing in these equations and their values are generally available in such work as (Chudzikiewicz et al., 2019). These equations are part of the equations describing the entire vehicle model, which was used for simulation analyzes of the multi-component vehicle model including wheel control systems.

3. Review of control strategy

Research in the area of independently rotating wheels has been conducted since the mid-twentieth century and the first results were described in (Kaplan et al., 1970). The lack of self-steering of the wheelset equipped with independently rotating wheels caused further development of research on this mechanical system. The work was carried out in two directions. The first was related to the analysis of the system from the point of view of mechanics and was aimed at improving the kinematic and dynamic properties of the system, the second direction of research concerned control techniques. The basic works on the dynamics of the system with independently rotating wheels is publication (Frederich, 1989), where the problem of "self-centering" of the wheel was proposed to be solved by the use of gravity stiffness (Einzelrad-Einzel-Fahrverk - EEF). However, this solution has not been implemented for wide use due to the complicated geometry of the bogie. The "Stadtbahn 2000" project, which postulated its use, was not fully implemented.

In (Eickhoff et al., 1989), the results of experimental and simulation tests of vehicles with bogies with independently rotating wheels were compared. They confirmed the elimination of the phenomenon of oscillation motion of wheelsets, but also tendencies to keep the wheel flange in permanent contact with the rail. The paper (Eickhoff, 1991) presents a model for testing the lateral dynamics of a bogie used in articulated vehicles with independently rotating wheels. The method of generating automatic equations in symbolic form is presented. The work (Satou et al., 1991) presents a bogie developed at the Railway Technical Research Institute (RTRI) in Japan with independently rotating wheels, whose mass is twice less than the weight of a conventional bogie. This type of bogie was designed to improve running stability at high speeds when running on straight tracks and curves, as the next generation of the bogie used on Japanese Railway Lines with narrow tracks. The bogie prototypes were tested on roller stands and during field tests. The measurements verified the fact that thanks to the new wheel profile, the bogies have self-steering properties and the truck's behavior is more predictable. Analysis of the model's eigen values and simulation results showed better stability when driving bogies with independently rotating wheels.

The work (Dukkipati et al., 1992) is of a review nature. It discusses various methods to improve track guidance of wheelset with independently rotating wheels. Also, comparative studies of steady-state motion on the arc of unconventional bogies were carried out. The impact of the first stage damping on system dynamics was investigated and the dynamics of the system in which the rear set has independently rotating wheels and the front wheelset is a classic was analyzed.

Also noteworthy is work (Jawahar et al., 1990), which describes the construction of a non-linear mathematical model of transverse dynamics of the bogie. Lateral track stiffness was included in the tests. Lateral dynamics of the system for a bogie with classic sets and wheelsets with independently rotating wheels was analyzed.

Issues related to the geometry of wheel profiles in unconventional bogies were dealt with in (Dukkipati et al., 1995). It compares, by analyzing simulation results, the behavior of several unconventional bogie designs. The simulation studies concerned the behavior of a rail vehicle when traveling over small radius curves and determining the permissible level of wheel profile wear. Vehicles traveling at high speeds as well as vehicles used in public transport were tested.

In article (Santamaria et al., 2004) the arrangement of the so-called coupled bogie (two uniaxially spring-loaded bogies) equipped with wheelsets with independently rotating wheels. The theoretical analysis of this system showed that it behaves perfectly on an arc and on a straight track. Analysis of system dynamics simulation results showed that a coupled bogie can significantly reduce the self-steering problems of such a system.

The concepts discussed in the previous two articles were developed in (Maoru et al., 2008). The publication describes a differential mechanism whose application improves the behavior on large-radius curves and straight tracks of a vehicle equipped with wheelsets with independently rotating wheels. It proposed coupling the wheels with a limited-slip differential type clutch. The differential consists of a clutch with an initial static tension that can lock the wheels relative to each other when driving on a large radius curve and unlock them on a small radius curve thus allowing it to travel over small displacements. Simulation studies focused on the analysis of self-steering of the system and its kinematic oscillations.

The concept of using "inverted geometry" wheels in a wheelset with independently rotating wheels is presented in (Maoru et al., 2009). The work describes experiments on a model built in 1:10 scale and on a mathematical model of a system built in the SIMPACK environment. The inversion of the wheel race conicity as well as the introduction into the system of a semi-active torsional vibration damper resulted in improved system stability without the necessity of introducing complicated changes in the structure of the bogie.

Particularly noteworthy are works related to the description of the dynamics of TALGO trains in which wheelsets with independently rotating wheels are used. Items such as (Xingwen et al., 2014) should be mentioned here. They describe the mathematical model of the vehicle using Newton-Euler formalism with Lagrange multipliers. Characteristic features of TALGO trains caused that original's programs were used for simulation tests, not commercial ones. Simulation studies concerned the vehicle's behavior on the curve.

The lack of self-centering of wheelsets with independently rotating wheels resulted in the development of work on steering the wheels.

Examples of work carried out by British scientists include (Carballeira et al., 2008), where optimal linear-square control was dealt with, which can be used

for both conventional and IRW bogies. However, in (Mei et al., 1999), an analysis of the movement of wheelsets with independently rotating wheels was performed, which showed the possibility of some quasi-kinematic oscillations in the system. It also specifies the possibilities of the type of control that can be used to stabilize the movement of a wheelset without affecting its stationary curve movement.

The robust $H\infty$ control was the subject of the work (Mei et al., 2000). Work (Goodall et al., 2000) was devoted to the basic problems of active control. The regulator discussed there is able to maintain stability and efficient operation at the same time, even if the system has variable parameters, which can be especially observed in the contact zone of the wheel with the rail. This control is resistant to uncertainties in the system that are not included in the model, e.g. those related to the dynamics of actuators. Furthermore, in practice the robust control system uses only sensors for measuring speed and acceleration, because the basic measurements required for active control, such as lateral displacement of the wheel relative to the rail, cannot be easily and cheaply measured in practice.

In (Mei et al., 2001), the problem of drivers used for vehicles traveling at low speeds was raised.

Whereas in (Mei et al., 2003) the authors formulated two conditions for perfect control: condition 1 is equal to the values of lateral forces on all wheelsets and condition 2 is zero longitudinal forces on all wheelsets.

The work (Li et al., 2003) presents tests of a new design of the induction motor and controller for IRW, which are aimed at ensuring stability required during operation in a rail vehicle. The developed model of mechanical and electrical system components was simulated and then tested on a real object. Studies have shown that wheelsets with active drive motor control with independently driven wheels have better stability compared to a conventional wheelset. It also turns out that regulators with indirect control by the field orientation method used for control systems of induction motors are suitable for them both in terms of response speed and controllability.

The works of Chinese researchers include, among others: work (Mei et al., 2005) where a model of a bogie with independently rotating wheels consisting of two frames of elastically coupled single-axle bogies was described, and in (Liang et al., 2011) an analysis of this system was carried out, which showed that it behaves perfectly on the curve and then, after exiting the arc, on a straight track. Analysis of system dynamics simulation results showed that the use of a coupled bogie can significantly reduce control problems in the system.

The works of Japanese researchers include work (Xue et al., 2018).

The review certainly does not exhaust the issue, but signals the main trends in research on steering wheelsets.

4. Adopt control strategy

As shown by studies conducted by many researchers, as described in the previous chapter, and simulation analyzes made in (Sowińska, 2018), the best solution would be to adopt a strategy that allows the wheel set to move in such a way that its center of mass moves along the center line of the track. For this type of strategy, the most desirable input signal to the regulator would be the lateral displacement of the wheelset relative to the track, but direct measurement of such a size is impossible or requires a complex system. Therefore, an alternative was proposed in the form of a similarity of the unconventional wheelset to the classic one's with a fixed axis. The reason for this is that the bogies with classic wheelsets have the feature of centering on the track after exiting the curve. The control can impose a condition that will imitate the connection of two wheels with a fixed axis, i.e. set a control purpose in which the angular velocity difference on the curve is equal to zero. This condition can be written as follows:

$$\omega_{1r} - \omega_{1l} = 0 \tag{5}$$

where

 $\omega_{1r,l}$ – angular velocity of the wheel of the right (left) wheelset, indicated in Fig. 3:



Fig. 3. IRW wheelset subject to control

To develop a control strategy, let's consider the behavior of a conventional wheelset (wheels connected by a rigid axle) on a curved track with a regular radius of R_0 . The position of the wheelset is shown in Fig. 4.



Fig. 4. Wheelset geometry on an arc with a radius R_{0} .

The following designations were adopted:

- C wheelset center of mass
- L left wheel
- R right wheel
- Ro radius of the arc
- l half the distance between the nominal positions of the radii on the wheels of the wheelset
- r₀ nominal radius of the wheel
- Ψ angle of rotation of the set around the vertical axis OZ
- Θ wheel rotation angle around the OY axis
- ϕ track cant
- r = r_0 in the central position of the set on the track ω angular velocity

It can be shown that the velocity of the center of mass of the wheelset V (point C) is related to the angular velocity Ω of the wheelset (point C) moving on the arc with radius R₀ by the relationship:

$$\Omega = \frac{V}{R_0 - r_0 \sin \phi_0} \tag{6}$$

and then write the expressions into the linear speed of the contact points located on the left and right wheels in the form:

$$V_l = V \frac{R_0 + l\cos\phi_0}{R_0 - r_0 \sin\phi_0} \tag{7a}$$

$$V_r = V \frac{R_0 - l\cos\phi_0}{R_0 - r_0 \sin\phi_0} \tag{7b}$$

Assuming that ϕ_0 it is a small angle, the above equations will take the form:

$$\omega_l = \frac{V}{r_0} \left(1 + \frac{l}{R_0} \right) \tag{8a}$$

$$\omega_r = \frac{V}{r_0} \left(1 - \frac{l}{R_0} \right) \tag{8b}$$

Ultimately, the angular velocity difference will be:

$$\omega_l - \omega_r = \frac{2lV}{r_0 R_0} \tag{9}$$

Compliance with condition (5) coupled with condition (9) allows the wheelset to move along the track centreline. In practice, controlling the wheels of an IRW wheelset would involve applying torques to the right and left wheels (Fig. 1):

$$M_{1r} = K_{1r}\omega_{1r}$$

$$M_{1l} = K_{1l}\omega_{1l}$$
(10)

The strategy described above for controlling the movement of a wheelset so that the center of its mass, in motion along an arc of radius R_0 , moves along the track (curve) coinciding with the center line of the railway track, can be schematically shown in Fig. 5.



Fig. 5. Diagram of the IRW wheel control system

In this case, a bogie with IRW wheelsets, has been assumed that two motors on both sides of the bogie drive two wheels independently. One engine two left wheels, the other two right wheels. Measuring the rotational speed of the left and right wheels of the lead set is not technically a problem. Information about the actual values of the angular velocity of the wheels ω_l and ω_r is transmitted to the module and is compared with the value of the difference $(\omega_l - \omega_r)$ based on calculations using the values of such parameters as: l_0 , r_0 , R_0 and taking into account the actual current vehicle speed V and then the control procedure calculates the signals controls that supplied to the engines will generate corrective moments for the movement of the wheelset, so that the distance of the trajectory of point C (center of mass of the wheelset) from the track center line is minimal. Based on this idea, a procedure has been developed for the system controlling the motors driving the wheels of the wheelset.

5. Simulation analyzes and sample results

Using the Matlab and Simulink packages, based on the developed mathematical models of the vehicle and the control procedure, a simulation program was built. The control system uses the PI controller - proportional - integral controller working in a feedback loop.

In order to obtain different torque values from the induction motor, two separate control blocks corresponding to the left and right sides of the bogie were inserted into the model built in the Simulink package. The block controlling the winding angle of the left wheel takes the sum as the desired value and compares it with the speed of rotation of the left wheel resulting from the simulation. The control wheel winding angle for the right wheel takes the sum as the desired value and compares it with the speed of the right wheel. All these dependencies result from the relationship (9). Fig. 6 shows a fragment of the program diagram with controllers built in the Simulink package.

The left and right wheel control blocks generate moments M_l and M_r , which are attached to the equations of the winding wheel of the set.

The use of PID controller is the best solution in case of incomplete knowledge about the control object. Please note that this control method is not optimal control. By adjusting the controller settings appropriately, a regulation adapted to a given object is obtained. In our case, the regulator's derivative term is not included. Then the transfer function of the independent PI controller has the form (11):

$$G_{PID}(s) = K_p + K_i \frac{1}{s}$$
(11)
where:

 K_p - amplification of the proportional part,

 K_i - amplification of the integrating part,

s - complex variable in Laplace transformation.

PI regulators are quite common because differentiation is sensitive to measuring noise, and the possible lack of an integrating element may prevent the system from reaching the set value.



Fig. 6. Fragment of the schematic of the program with controllers built in the Simulink package

The selection of PI controller settings consists in determining the optimal values of the parameters of individual sections of the controller so as to obtain the desired control. Usually, the initial settings obtained using the available methods must be corrected several times by conducting computer simulations until the system works as expected or a compromise solution is accepted. The process of selecting the controller setting in the case of our tests was carried out "manually" in several simulations. An example of the result showing the comparison of transverse displacements of the IRW wheelset, when driving on an arc with a radius of R = 50 m at a speed of 50 km/h, for different values of setting the parameter K_p is shown in Fig. 7.

For K_p values greater than 80, there are practically no significant differences between the lateral displacement of the IRW wheelset relative to the center track line. Therefore, the value $K_p = 80$ was adopted as the controller setting for further simulation analyzes of the bogie model with IRW wheelsets. An example of the simulation result showing the lateral displacements of the center of mass of the IRW wheelset for the application of control and without control, for driving on curves with different radii at a speed of V = 20 km/h, is shown in Fig. 8. In each of the simulated cases, the use of control causes compared to a set without control, reducing the lateral displacement of the center of mass of the wheelset and the time after which the center of mass of the wheelset reaches the center line of the track. Simulations conducted at higher speeds showed that the behavior of the IRW wheelset on curves did not deteriorate taking into account the lateral displacement of the center of mass of the wheelset and the way along which the center of mass of the wheelset reaches the center line of the track.



Fig. 7. Comparison of lateral displacements of the IRW wheelset for different K_p values



Fig. 8. Examples of lateral displacements of an IRW wheelset, with and without steering (rolling bogie), for various arc radii, for speed V = 20 km/h.

6. Conclusions

The complex mechanical system, which is a rail vehicle with wheelsets having independently rotating wheels, despite numerous analyzes and studies, both theoretical and experimental, is still an interesting research object. As demonstrated by numerous new designs of rail vehicles equipped with wheelsets with IRW, solutions that meet safety requirements are still being sought considering the higher travel speeds and lower impacts on humans and the environment (noise and vibrations). Obtaining such solutions would not be possible without the use of mathematical modeling and computer simulation methods in research. The results presented in the article are proof of this. The aim of the analysis was to find a simple but effective control strategy that allows maintaining stable motion of a rail vehicle with IRW wheelsets while driving on a straight track and on a curve. The proposed control strategy is based on a comparison of the angular velocities of wheels of wheelsets installed in a given bogie. From a practical point of view, this strategy is not difficult to apply in practice, because the measurement and comparison of the angular velocities of the wheels is not difficult, taking into account the current technical possibilities. Also, in terms of control procedures and technical possibilities of generating control signals, this problem is currently practically no difficulties.

The simulation experiments carried out indicate that with the assumed control strategy, the controller setting values cannot be accepted a priori for any situations occurring while driving. In fact, the controller settings will have to be selected on the basis of "trial and error" depending on the speed and radius of the arc, taking into account the general rules for safe driving. At the same time, it should be noted that even these limited and simple attempts to introduce control into the system with independent wheels are able to improve its properties related to the self-centering of the wheelset relative to the track centerline. The developed methods of analysis and the results obtained were used in the design and then construction of a prototype tram vehicle with a 100% low floor by one of the rail vehicle manufacturers operating on the Polish market.

References

- Carballeira, J., Baeza, L., Rovira, A., & García, E. (2008). Technical characteristics and dynamic modelling of Talgo trains. Vehicle System Dynamics, 46(S1), 301-316.
- [2] Chudzikiewicz, A. et al., (2019). Pojazdy tramwajowe z niezależnie obracajacymi się kołami. Warsaw University of Technology Publishing House.
- [3] Chudzikiewicz, A., & Kalker, J. J. (1991). Calculation of the Evolution of a Railway Wheel Profile Trough Wear, International Series of Numerical Mathematics, Vol. 101. Birkha user Verlag Basel, pp. 71-84.
- [4] Chudzikiewicz, A., Sowińska, M. (2016). Modelling and simulations of dynamics of the lowfloor tramcar with independently rotating wheels, in: Communications, Vydavatelstvo Zilinskej Univerzity Edis, 17(4), 45-52.
- [5] Dukkipati, R. V., Narayana Swamy, S., & Osman, M. O. M. (1992). Independently rotating wheel systems for railway vehicles-a state of the art review. Vehicle System Dynamics, 21(1), 297-330.
- [6] Eickhoff, B. M. (1991). The application of independently rotating wheels to railway vehicles. Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit, 205(1), 43-54.
- [7] Eickhoff, B. M., & Harvey, R. F. (1989). Theoretical and experimental evaluation of independently rotating wheels for railway vehicles, in: Vehicle System Dynamics: International Journal of Vehicle Mechanics and Mobility, Suppl. 19, 190–202.
- [8] Eickhoff, B. M., Harvey, R.F. (1989). Theoretical and experimental evaluation of independently rotating wheels for railway vehicles, in: Vehicle System Dynamics: International Journal of Vehicle Mechanics and Mobility, Suppl. 19, 190–202.
- [9] Frederich, F. (1989). Dynamics of a bogie with independent wheels. Supplement to Vehicle System Dynamics, 19, 217-232.
- [10] Goodall, R., & Li, H. (2000). Solid axle and independently-rotating railway wheelsets-a control engineering assessment of stability. Vehicle System Dynamics, 33(1), 57-67.
- [11] Jawahar, P. M., Gupta, K. N., & Raghu, E. (1990). Mathematical modelling for lateral

dynamic simulation of a railway vehicle with conventional and unconventional wheelset. Mathematical and Computer Modelling, 14, 989-994.

- [12] Kaplan, A., Hasselman, T. K., & Short, S. A. (1970). Independently rotating wheels for high speed trains. SAE Transactions, 2496-2508.
- [13]Li, J., Goodall, R. M., Mei, T. X., & Li, H. (2003). Steering controllers for rail vehicles with independently driven wheel motors. Electronic System and Control Division Research, 46, 4-6.
- [14] Liang, B., & Iwnicki, S. D. (2011). Independently rotating wheels with induction motors for high-speed trains. Journal of Control Science and Engineering, 2011, pp. 1-7.
- [15]Lin, X., Dong, H., Yao, X., & Cai, B. (2018). Adaptive active fault-tolerant controller design for high-speed trains subject to unknown actuator faults. Vehicle system dynamics, 56(11), 1717-1733.
- [16] Luo, R., Shi, H., Guo, J., Huang, L., & Wang, J. (2019). A nonlinear rubber spring model for the dynamics simulation of a high-speed train. Vehicle System Dynamics, 58(9), 1367-1384.
- [17] Maoru, C., Jing, Z., Wenhao, G., Weihua, Z., Xuesong, J. (2009). Analysis on steering capability of a new bogie with independently rotating wheels. International Journal of Railway, 2(4), 164-169.
- [18] Maoru, C., Weihua, Z., Yiping, J., Huanyun, D. (2008). A Self-Acting Radial Bogie with Independently Rotating Wheels, in: International Conference on Computational & Experimental Engineering and Sciences, 7(3), 141-144.
- [19] Maoru, C., Weihua, Z., Yiping, J., Huanyun, D. (2008). A Self-Acting Radial Bogie with Independently Rotating Wheels. International Conference on Computational & Experimental Engineering and Sciences, 7(3), 141-144.
- [20] Mei, T. X., & Goodall, R. M. (1999). Optimal control strategies for active steering of railway vehicles. IFAC Proceedings Volumes, 32(2), 2915-2920.
- [21] Mei, T. X., & Goodall, R. M. (2000). LQG and GA solutions for active steering of railway vehicles. IEE Proceedings-Control Theory and Applications, 147(1), 111-117.
- [22] Mei, T. X., & Goodall, R. M. (2001). Robust control for independently rotating wheelsets on

a railway vehicle using practical sensors. IEEE Transactions on control systems technology, 9(4), 599-607.

- [23] Mei, T. X., & Goodall, R. M. (2003). Practical strategies for controlling railway wheelsets independently rotating wheels. J. Dyn. Sys., Meas., Control, 125(3), 354-360.
- [24] Mei, T. X., Shen, S., Goodall, R. M., & Pearson, J. T. (2005). Active steering control for railway bogies based on displacement measurments. IFAC Proceedings Volumes, 38(1), 586-591.
- [25] Santamaria, J., & Vadillo, E. G. (2004). Equivalent conicity and curve radius influence on dynamical performance of unconventional bogies. Comparison analysis. In The Dynamics of Vehicles on Roads and on Tracks Supplement to Vehicle System Dynamics: Proceedings Of The 18th Iavsd Symposium Held In Kanagawa, Japan August 24-30, 2003 (Vol. 18, pp. 133-142). CRC Press.
- [26] Satou, E., & Miyamoto, M. (1992). Dynamics of a bogie with independently rotating wheels. Vehicle System Dynamics, 20(sup1), 519-534.
- [27] Shen, G., & Goodall, R. (1997). Active yaw relaxation for improved bogie performance. Vehicle system dynamics, 28(4-5), 273-289.
- [28] Sowińska, M. (2018). Analiza właściwości biegowych wózków lekkich pojazdów szynowych na przykładzie wózków z niezależnie obracającymi się kołami. Ph.D. Thesis. Poznan University of Technology.
- [29] Sun, Y., Zhai, W., & Guo, Y. (2018). A robust non-Hertzian contact method for wheel-rail normal contact analysis. Vehicle System Dynamics, 56(12), 1899-1921.
- [30] Swamy, S. N., Dukkipati, R. V., & Osman, M. O. M. (1995). Analysis of modified railway passenger truck designs to improve lateral stability/curving behaviour compatibility. Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit, 209(1), 49-59.
- [31] Xingwen, W., Maoru, C., Jing, Z., Weihua, Z., Minhao, Z. (2014). Analysis of steering performance of differential coupling wheelset. Journal of Modern Transportation, 22(2), 65–75.