

# VERIFICATION OF BOUNDARY CONDITIONS OF NUMERICAL MODELING OF THE TRACK SUBSTRUCTURE THERMAL REGIME – INFLUENCE OF THE SNOW COVER

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**Abstract:**

The initial part of the paper briefly characterizes a long-term experimental activity at the Department of Railway Engineering and Track Management (DRETМ). The research of the DRETМ focuses, besides other research activities and specific problems in the field of railway engineering (application of new structures and construction materials in conventional and modernized railway tracks, modernisation and rehabilitation of existing railway tracks for higher speeds, track diagnostics, influence of track operation on noise emissions and design of structural measures, possibility of application of recycled ballast bed material in the track substructure, ballast recycling technologies, ecological assessment of recycled material of the track substructure), on various factors affecting track substructure freezing. In 2012-2017, in the campus of the University of Žilina (UNIZA), an Experimental stand DRETМ was built for the research purposes. The experimental stand DRETМ consists of 6 types of track substructure placed in an embankment or a cut, in the 1:1 scale. Besides conventional building materials (crushed aggregate), these structures also include various thermal insulation materials (Liapor concrete, Styrodur, foam concrete). A significant part of the paper deals with numerical modeling of the freezing process of track substructure (an embankment with the embedded protective layer of crushed aggregate, fr. 0/31.5 mm) for various boundary conditions (air frost index, average annual air temperature), using SoilVision software. The aim of this research is to identify the thermal insulation effects of different thicknesses of snow cover on the depth of penetration of the zero isotherm into the track substructure (railway track). The paper conclusion specifies the influence of different snow cover thicknesses, or *nf* factor (factor expressing the dependency between the mean daily air temperature and the temperature on the ballast bed surface) and various climatic conditions (frost indexes and average annual air temperatures), affecting the railway infrastructure, on the resulting depth of freezing of the track substructure (railway track). These outputs will be in the further research used for the design of nomogram for determining the thickness of the protective layer of the frost-susceptible subgrade surface of the track substructure.

**Key words:**

railway track substructure, track substructure freezing, frost index, snow cover thickness, *nf* factor, numerical modeling of thermal regime of the track substructure

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## 1. Introduction

The research of the Department of Railway Engineering and Track Management (DRETM) focuses, besides other research activities and specific problems in the field of railway engineering, on various factors affecting track substructure freezing. In the campus of the University of Žilina (UNIZA), an Experimental stand DRETM was built for the research purposes.

This experimental stand contains 6 various types of track substructure in the 1:1 scale. These models are the most frequent types of track substructure as well as newly designed track substructures. The aim of this research is to increase their deformation resistance and to improve their thermal insulation properties. Within the experimental activities, the basic physical properties of building materials, applied or planned to be applied in the track substructure after verification, are determined. The respective properties are thermal conductivity coefficients, thermal capacity, density, bulk density, temperature and moisture values of building materials.

The climatic characteristics (mean daily air temperature  $\theta_s$ , average annual air temperature  $\theta_m$ , air frost index  $I_F$ , depth of track substructure freezing  $D_F$ ), are essential prerequisites for a correct track substructure design of non-traffic load (effects of climatic factors, especially of water and frost). Due to this reason, they also need to be verified.

The determination of all the relevant boundary conditions affecting the penetration of the zero isotherm into the track substructure for various Slovak regions is time-consuming and costly. Because of this, computing support was applied, in the form of numerical modeling of various track substructure freezing by *SoilVision* software, specifically *SVHeat* programme.

To gain relevant output values of the parameter depth of track substructure freezing  $D_F$ , it is necessary to enter correct input parameters in the *SVHeat*. Many of these parameters were collected within the extensive experimental activity, recently conducted at DRETM, in the field of non-traffic load of railway track, on railway track models in 1:1 scale.

However, some input parameters must be determined on the basis of knowledge and experience and verified by numerical modeling. Such parameters include the influence of snow cover

thickness as an active thermal insulation layer on the railway track surface on track substructure freezing. This paper is dedicated to the identification of influence of the snow cover thickness on the track substructure freezing.

As this influence will be verified by numerical modeling of the thermal regime or track substructure freezing, it is necessary to state the method of entering the snow cover thickness. Subsequently, various methods of entering must be compared and then the design values for snow cover thickness for particular Slovak regions based on frost index  $I_F$  and average annual air temperature  $\theta_m$  must be specified. In this way, it is possible to create conditions for identification of relevant influence of the snow cover thickness  $t_{snow}$  on the depth of track substructure (railway track) freezing  $D_F$ . Based on this influence, a design nomogram for dimensioning the structural thickness of the protective layer of the frost-susceptible subgrade surface  $t_{pl}$  can be created.

## 2. Review of literature

In the past, the primary importance in the track design procedure was exclusively placed on the railway superstructure quality. Only later it was generally accepted that to secure quality and safe roadway, the attention also needs to be paid to the track substructure, especially the track composition and the selection of suitable materials. The problem of soil behaviour in various boundary conditions (deformation resistance, thermal insulation properties) has been studied by many experts (Vanapalli and Oh, 2010), (Sussmann and Hyslip, 2010), (Akrouch et al., 2016), (Pentland, 2000), (Farbrot et al., 2013). An important factor affecting the quality of structural layers of the track subgrade is the effects of frost (Tang et al., 2018), (Hansson et al., 2004), (Nassar et al., 2000), (Panteleev et al., 2017), (Newman, 1995), (Wang et al., 2014), (Watanabe and Wake, 2009), often reduced by the occurrence of a snow cover (Ottaviani et al., 2015). All the achieved knowledge was applied in the *SoilVision* software which enables numerical modelling of complex cases of soil behaviour in subgrade of the line transport structures (Fredlund et al., 2011). A prerequisite for developing a quality numerical model is the knowledge of relevant input data. This problem has been studied by the Department of Railway Engineering and Track Management since 2003 (Ižvolt et al., 2013), (Ižvolt

and Dobeš, 2014), (Ižvolt et al., 2014), (Ižvolt and Dobeš, 2015), (Ižvolt et al., 2016), (Ižvolt et al., 2018a), (Ižvolt et al., 2018b).

### 3. Numerical modeling of the thermal regime at the Experimental stand DRETM

For the purposes of numerical modeling of the thermal regime of railway track and the track substructure, and verification of the influence of the snow cover thickness  $t_{snow}$  on the value of railway track freezing  $D_F$ , a 2D railway track model (Fig. 1) was built. A mesh generation module constructs a triangular finite element mesh over an arbitrary twodimensional model domain. Finite element basis is set to quadratic (number of elements is approximately 1000). This model is identical to the measuring profile no. 1 of the Experimental stand DRETM. The input parameters, which characterize the building materials of the measuring profile no. 1, were determined on the basis of the procedure in (Ižvolt and Dobeš, 2014).

Within the numerical modeling of the thermal regime of the railway track in the Experimental stand DRETM (measuring profile no. 1), the frostiest period was also determined. In terms of the achieved air frost index, it was winter 1986/1987 in Poprad ( $I_F = -741$  °C, day over the past 50 years). Frost index  $I_F$  is determined by adding up mean values of the day air temperatures  $\theta_s$  in the winter season based on the equation:

$$I_F = \sum_{t_b}^{t_f} \theta_s \quad (1)$$

The data – input parameters (mean daily air temperatures  $\theta_s$ ) were provided by the Slovak

Hydrometeorological Institute (SHMÚ). Mean daily air temperatures  $\theta_s$  were determined based on the equation:

$$\theta_s = \frac{\theta_7 + \theta_{14} + 2\theta_{21}}{4} \quad (2)$$

where  $\theta_7$ ,  $\theta_{14}$  a  $\theta_{21}$  represent the ambient air temperature as measured at 7 a.m., 2 p.m. and 9 p.m. of the Greenwich Mean Time at 2 meters above the terrain (Ižvolt, 2008).

The mean daily air temperature values  $\theta_s$  in this winter period were evenly reduced to reach the air frost index value  $I_F = -800$  °C , day. It is the maximum  $I_F$  value used for track substructure dimensioning for non-traffic load for the Slovak territory (valid for the area of High Tatras). The frost index value (-800 °C, day) was subsequently modified by an even increase of mean daily air temperatures  $\theta_s$ , to achieve other air frost index design values  $I_F$ , i.e. -700, -600, ..., -300 °C, day, with identical winter period course (number, course and intensity of the frost and thaw periods).

The respective analysis of the thermal regime of the railway track is conducted due to the necessity of collecting input parameters for numerical modeling of its thermal regime in the monitored winter period. Its final output is the new design nomogram for determination the structural thickness of the protective layer of the frost-susceptible subgrade surface  $t_{pl}$ . Due to this purpose, the frost index values  $I_F$  were assigned the values of average annual air temperature  $\theta_m$ , which is another parameter affecting the depth of track substructure freezing  $D_F$  by the effects of frost.

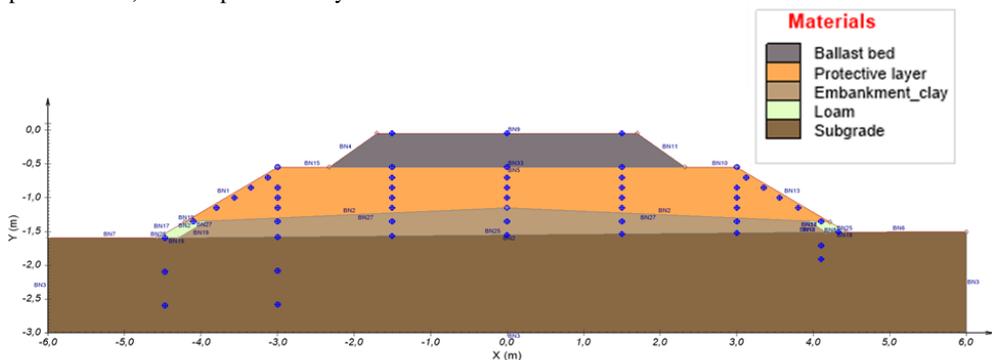


Fig. 1. Model of the Experimental stand DRETM – measuring profile No.1

The frost indexes  $I_F$  and average annual air temperatures  $\theta_m$  were combined to determine the maximum but real effects (in the Slovak territory) of frost on the depth of the railway track freezing  $D_F$  in a particular winter period. For the purposes of creating a new design map of frost indexes  $I_{Fd}$  for the Slovak territory (Ižvolt et al., 2018a), the SHMÚ provided the Department, besides values of mean daily air temperatures  $\theta_s$  at all meteorological stations in Slovakia in the coldest (frostiest) winter period 1986/87, also with data for the coldest year 1980. These data were requested to draw a new design map of average annual air temperatures  $\theta_{md}$  for the Slovak territory (Ižvolt et al., 2018b). Based on these inputs, combining frost indexes  $I_F$  and average annual air temperatures  $\theta_m$  was not demanding.

Besides climatic characteristics, in the numerical model it was also necessary to define the physical and thermal technical parameters of particular structural layers of the track substructure. These parameters were determined on the basis of laboratory measurements (Ižvolt and Dobeš, 2014) and experimental monitoring at the Experimental

stand DRETM (Ižvolt et al., 2014), (Ižvolt et al., 2016), (Ižvolt and Dobeš, 2015).

All the necessary input parameters for numerical modeling of the thermal regime – freezing – of the railway track model are clearly stated in *Tab. 1* and *Tab. 2*.

In the numerical model, the time units are set as TIME=1. This setting always matches the time moment of January 1. The TIME=365 is the day of December 31, (to consider the influence of average annual air temperature  $\theta_m$ ), TIME=333 -448 is used for 115-day winter period in 1986/87. The last day of the model, marked TIME=465 matches April 10, which was 3 days after the snow cover on the surface of the structure melted.

As previously mentioned, a partial aim of the research of the influence of non-traffic load on the railway track structure is the determination of the snow cover thickness  $t_{snow}$ . Specifically, we need to determine the effects, for which the depth of railway track freezing  $D_F$  would be identical to the thermal insulation effects of snow cover entered using the  $nf$  factor. (The  $nf$  factor expresses the conversion ratio of the mean daily air temperature  $\theta_s$  to the mean daily air temperature on the ballast bed surface  $\theta_{bb}$ ).

Table 1. Input parameters for numerical modeling of the thermal regime– freezing – of the railway track model (Ižvolt and Dobeš, 2014)

Material characteristics					
Structural part/ characteristics	Ballast bed	Protective layer	Embankment	Embankment subgrade	Humus
Layer material	gravel fr. 31.5/63	crushed aggregate fr. 0/31.5	clay	clay	soil
Temperature (°C)	2	5	6	7	1
Moisture $w_m$ (%)	1	6	20	20	20
Heat capacity $c_0$ (J.kg <sup>-1</sup> .K <sup>-1</sup> )	980	1088	1495	1495	1639
Bulk density $\rho_0$ (kg.m <sup>-3</sup> )	1908	1928	1646	1646	1320
Thermal conductivity coefficient $\lambda$ (W.m <sup>-1</sup> .K <sup>-1</sup> )	1.20	1.93	1.05	1.05	1.13

Table 2. Input parameters for climatic characteristics

Climatic characteristics						
Air frost index $I_F$ (°C, deň)	-300	-400	-500	-600	-700	-800
Average annual air temperature $\theta_m$ (°C)	9	8.5	8	7	6	5

### 3.1. Influence of $nf$ factor on the depth of railway track freezing

The  $nf$  factor, entered in the numerical modeling of the thermal regime of the railway track for a respective winter period, expresses the ratio of conversion of the mean daily air temperature  $\theta_s$  to the mean daily air temperature on the ballast bed surface  $\theta_{bb}$  (hereinafter surface air temperature  $\theta_{bb}$ ). As already mentioned, entering the  $nf$  factor is one of methods of defining the thermal insulation effects of the snow cover in the numerical modeling of the thermal regime of the railway track. The  $nf$  factor in the numerical modeling can be either entered using a single average value or tables for particular days. This was also our case, we applied TIME 333 to TIME448. The individual limit values of  $nf$  factors, applied in the numerical modeling, were determined in the following way. The first step was the  $nf$  factor determination for the meteorological station Poprad (High Tatras area) and the winter period 1986/87. With respect to the negative (positive) values of the mean daily air temperature  $\theta_s$  and the negative (positive) surface mean daily temperatures  $\theta_{bb}$ , recorded in winter 1986/87, an average  $nf$  factor value = 0.58 was stated (Fig. 2 - left). As in the winter period 1986/87 the stated air frost index value

was  $I_F = -741$  °C, day, and for numerical modeling of the thermal regime of the track substructure the air frost index value was  $I_F = -800$  °C, day, the  $nf$  factor value was modified to 0.55.

The highest applied factor value was  $nf = 0.80$ . It was determined by the identical method, again for the winter period 1986/87, but for the town Topoľčany (southwestern Slovakia). There the reached air frost index was  $I_F = -300$  °C, day, coincidentally identical to the minimum design air frost index value  $I_{F,d}$ , considered for the Slovak territory.

The  $nf$  factor value  $nf = 0.81$  (Fig. 2 - right) was determined as the average value of all ratios of negative (positive) mean daily air temperatures  $\theta_s$  to negative (positive) surface daily temperatures  $\theta_{bb}$  in the entire winter period.

Tab. 3 provides a set of values of  $nf$  factors, assigned to the particular air frost index values  $I_F$  and the average annual air temperatures  $\theta_m$ , subsequently applied to the numerical modeling of the railway track.

The entered  $nf$  factor values for particular days of the numerical model can be observed in Fig. 3, which demonstrates the winter time period TIME 333 – TIME 448.

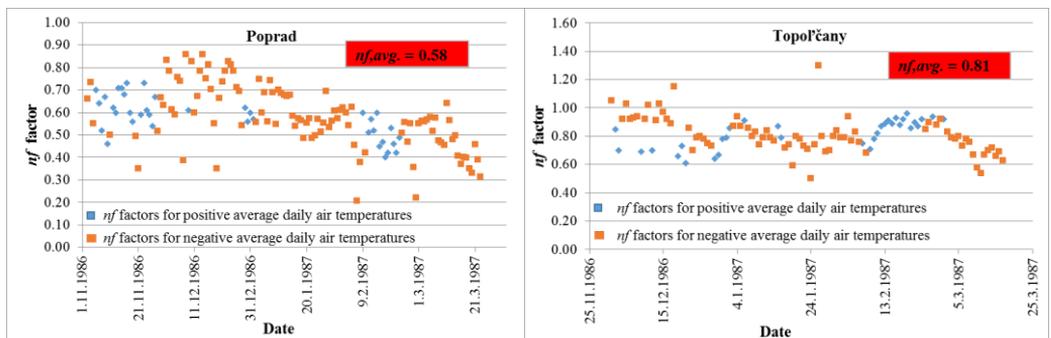


Fig. 2. Stated  $nf$  factor values for winter period 1986/87 (left – meteorological station Poprad, right – meteorological station Topoľčany)

Table 3. Design values of  $nf$  factors for the respective climatic conditions

$nf$ factor	0.80	0.75	0.70	0.65	0.6	0.55
Air frost index $I_F$ (°C, day)	-300	-400	-500	-600	-700	-800
Average annual air temperature $\theta_m$ (°C)	9	8.5	8	7	6	5

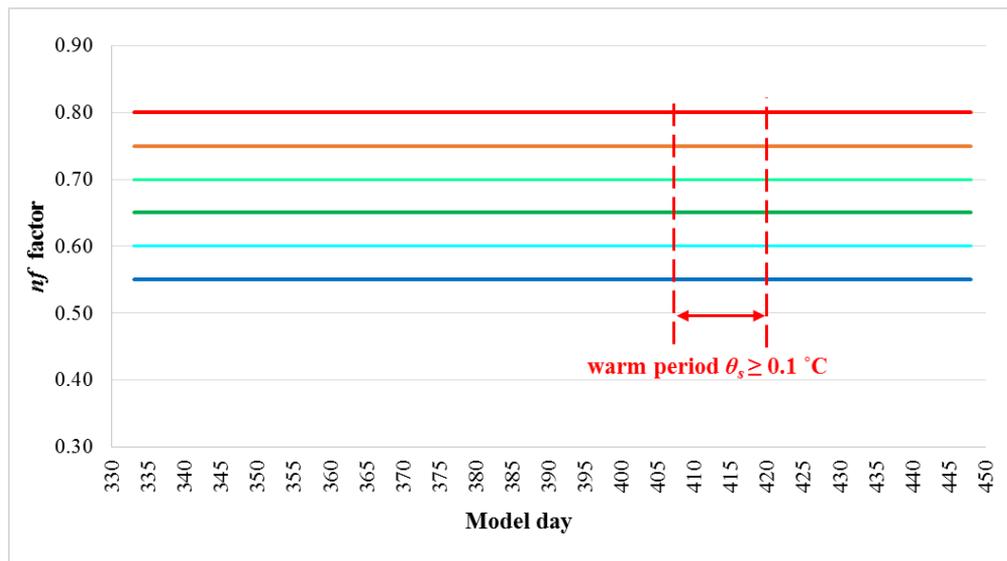


Fig. 3.  $nf$  factor values for numerical modeling

After summarizing all the necessary input parameters; numerical modeling of railway track freezing, and identification of the achieved depth of its freezing in the *SoilVision* software could be conducted. *Fig. 4* demonstrates a graphical output of the numerical modeling of the railway track freezing for the limit boundary conditions (air frost index  $I_F$

$= -800$  °C, day, average annual air  $\theta_m = 5$  °C and  $nf = 0.55$ ).

The output of the numerical modeling of railway track freezing for values specified in *Tab. 3* is the identification of the penetration (position) of the zero isotherm into this structure. The achieved values can be found in *Fig. 5*.

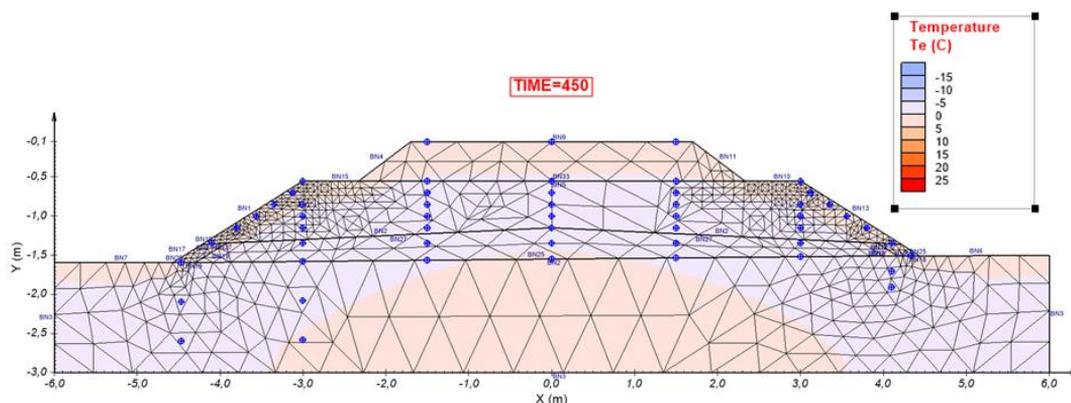


Fig. 4. Graphical output of modeling the railway track freezing for limit boundary climatic conditions ( $I_F = -800$  °C, day,  $\theta_m = 5$  °C,  $nf = 0.55$ )

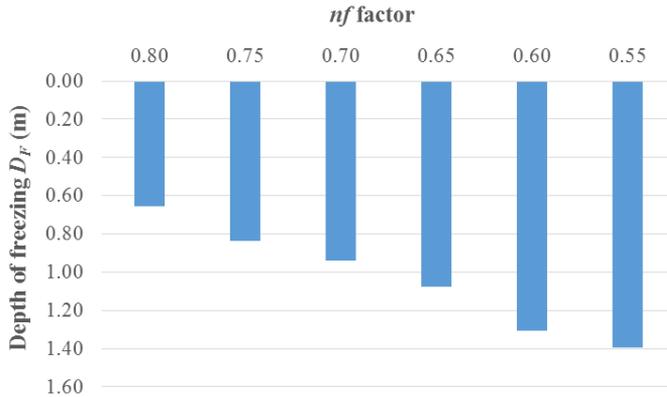


Fig. 5. Achieved values of the freezing depths  $D_F$  of track substructure in the railway track model for various boundary climatic conditions

### 3.2. Influence of the snow cover thickness on the depth of freezing of the railway track

Another possibility of conducting the conversion of air temperature to the air temperature on the rail track (ballast bed) surface in a numerical model is defining the snow cover parameters (thermal technical parameters of snow, bulk density of snow, snow cover thickness, etc.). The thickness of the snow cover  $t_{snow}$  can be defined as an average one, where the time period of its influence can be entered, or it can be entered as a table value, separately for particular model days (our case). Maintenance of the railway tracks has not been taken into account in the model. The snow cover thickness was entered as assumed and then modified with the aim of achieving values of the depth of structure freezing  $D_F$  identical to the values achieved by the numerical modeling with the use of the  $nf$  factor (Fig. 5).

Fig. 6 depicts a graphical output of numerical modeling of the railway track freezing for the limit (most adverse winter) boundary conditions (air frost index  $I_F = -800$  °C, day, average annual air temperature  $\theta_m = 5$  °C and the average snow cover thickness  $t_{snow} = 0.25$  m)

By numerical modeling of the railway track freezing, the differences in values of the achieved depths of structure freezing  $D_F$ , up to  $\pm 5$  mm, for identical boundary climatic conditions were identified.

The only variable here was the method of defining the thermal insulation function of the snow cover ( $nf$  factor vs. entering the snow cover thickness).

The final snow cover thickness values, entered for numerical modeling, can be found in Fig. 7.

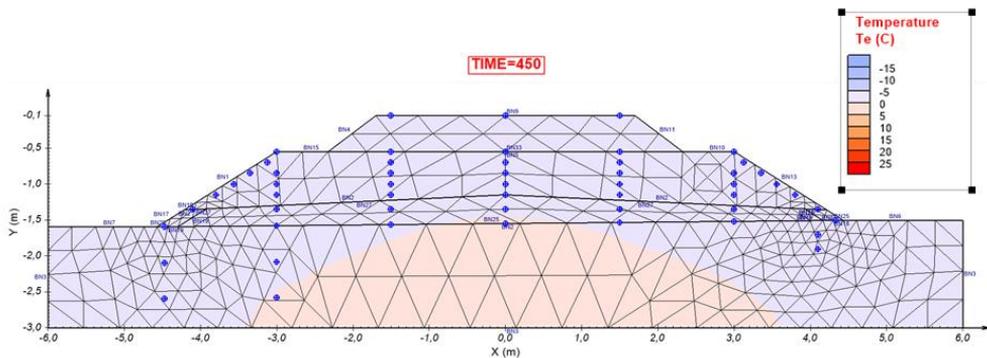


Fig. 6. Graphical output of modeling of the railway track freezing for limit (most adverse winter) boundary climatic conditions ( $I_F = -800$  °C, day,  $\theta_m = 5$  °C,  $t_{snow} = 0.25$  m)

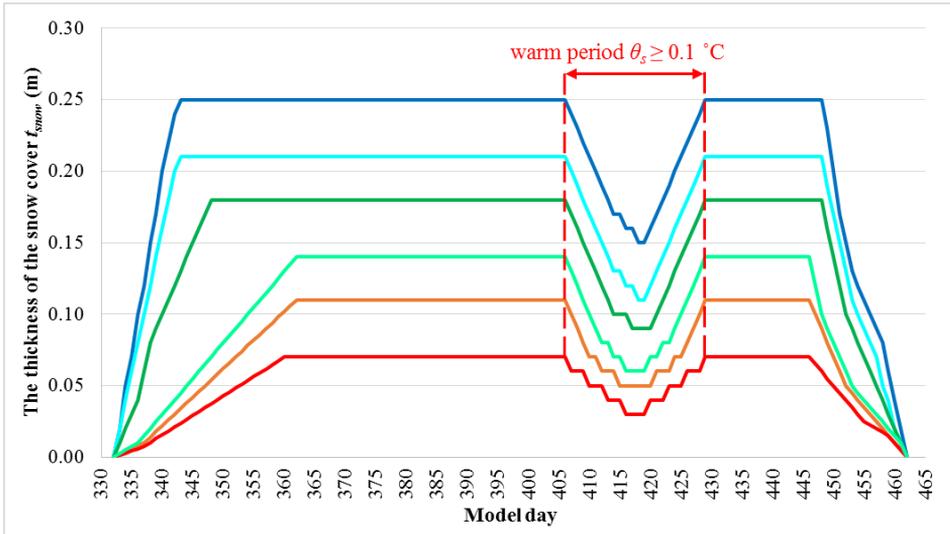


Fig. 7. Considered values of snow cover thickness in a numerical model

#### 4. Conclusions

Partial results of scientific research of the non-traffic load of railway track (track substructure), presented in this paper, include the determination of the influence of the thermal insulation effects of snow cover on the thermal regime, (the penetration of the zero isotherm into the track substructure), and the method of its correct entering in the *SoilVision* software. The first method of its entering was conducted using the  $nf$  factor, (which expresses the conversion of the mean daily air temperature  $\theta_s$  to the temperature on the ballast bed surface  $\theta_{bb}$  in the considered period of their influence). The second method focuses on entering the snow cover thickness  $t_{snow}$  on particular days in the identical period. The snow cover thicknesses  $t_{snow}$ , matching

the respective  $nf$  factor values, and the values of the depth of freezing of the railway track model  $D_F$  for both methods can be found in *Tab. 4*.

The determination of the snow cover influence on the track substructure model freezing is only a partial task of experimental activity at DRETM. The parameters in *Tab. 4* will be applied in the further research of influence of non-traffic load on the thermal regime and the achieved depth of railway track freezing  $D_F$  as entry parameters for numerical modeling. The output of this research will be a design nomogram for determination of necessary thickness of the protective layer of frost-susceptible subgrade surface  $t_{pl}$  in the methodology of track substructure dimensioning for non-traffic load.

Table 4. Values of snow cover thickness matching the respective  $nf$  factor values

Air frost index $I_F$ (°C, day)	-300	-400	-500	-600	-700	-800
Average annual air temperature $\theta_m$ (°C)	9	8.5	8	7	6	5
$nf$ factor	0.8 (1.15 <sup>*</sup> )	0.75 (1.15 <sup>*</sup> )	0.7 (1.15 <sup>*</sup> )	0.65 (1.15 <sup>*</sup> )	0.60 (1.15 <sup>*</sup> )	0.55 (1.15 <sup>*</sup> )
Average snow cover thickness $t_{snow}$ (m)	0.07	0.11	0.14	0.18	0.21	0.25
Depth of railway track model freezing $D_F$ (m)	0.655 (0.652 <sup>**</sup> )	0.835 (0.830 <sup>**</sup> )	0.940 (0.944 <sup>**</sup> )	1.075 (1.070 <sup>**</sup> )	1.305 (1.300 <sup>**</sup> )	1.395 (1.400 <sup>**</sup> )

Notes : values valid for positive mean daily air temperatures,  
<sup>\*\*</sup> values valid for shorter period of snow cover effects (Fig. 7 left),  
<sup>\*\*</sup> values valid for modeling with the snow cover.

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## References

- [1] AKROUCH, G. A., BRIAUD, J. L., SANCHEZ, M. & YILMAZ, R., 2016. Thermal Cone Test to Determine Soil Thermal Properties. *Journal of Geotechnical and Geoenvironmental Engineering*. 142 (3).
- [2] FARBROT, H., ISAKSEN, K., ETZELMÜLLER, B. & GISNÅS, K., 2013. Ground Thermal Regime and Permafrost Distribution under a Changing Climate in Northern Norway. *Permafrost and Periglacial Processes*. 24 (1), 20-38.
- [3] FREDLUND, M., 2011. *SoilVision. A knowledge-based soils database*. Canada: SoilVision Systems Ltd.
- [4] HANSSON, K., SIMUNEK, J., MIZOGUCHI, M., et al, 2004. Water flow and heat transfer in frozen soils. Numerical solution and freeze-thaw applications. *In Vadose Zone Journal*. 3. 693-704.
- [5] ĽVOLT, L., 2008. *Railway substructure – stress, diagnostics, design and implementation of body construction layers of railway subgrade*. University of Žilina. Slovakia .
- [6] ĽVOLT, L., KUPČULIAK, P. & PULTZNEROVÁ, A., 2013. Monitoring of the temperature regime and thermotechnical properties of railway subgrade materials. *In Scientific Journal of Silesian University of Technology. Series Transport*, 80, 41-52.
- [7] ĽVOLT, L. & DOBEŠ, P., 2014. Test procedure impact for the values of specific heat capacity and thermal conductivity coefficient. *In 23<sup>th</sup> Russian - Polish - Slovak seminar. Theoretical Foundation of Civil Engineering*, Wroclaw, Poland. 91. 453-458.
- [8] ĽVOLT, L., DOBEŠ, P. & PITOŇÁK, M., 2014. Some experience and preliminary conclusions from the experimental monitoring of the temperature regime of subgrade structure. *In 14<sup>th</sup> International Conference on Railway Engineering Design & Operation*, Rome, Italy. 135. 267-278.
- [9] ĽVOLT, L. & DOBEŠ, P., 2015. Experimental monitoring of moisture changes in railway track structure. *In TRANSCOM 2015. In 11<sup>th</sup> international scientific conference of young scientists and Ph.D. students*. University of Žilina, Žilina, Slovakia. 46-51.
- [10] ĽVOLT, L., DOBEŠ, P. & PULTZNEROVÁ, A., 2016. Monitoring of moisture changes in the construction layers of the railway substructure body and its subgrade. *In Procedia Engineering*. 161. 1049-1056.
- [11] ĽVOLT, L., DOBEŠ, P. & PIEŠ, J., 2018. Updating the design map of frost indexes as a prerequisite for relevant assessment of track substructure for non-traffic load. *Civil and environmental engineering*. 14 (2) (in press).
- [12] ĽVOLT, L., DOBEŠ, P. & PIEŠ, J., 2018. Contribution to the modification of input data of subgrade structure dimensioning for non-traffic load according to the ŽSR methodology. *In 16<sup>th</sup> International Conference on Railway Engineering Design & Operation*, Lisabon, Portugal (in press).
- [13] NASSAR, I. N., HORTON, R. & FLERCHINGER, G. N., 2000. Simultaneous Heat and Mass Transfer in Soil Columns Exposed to Freezing/Thawing Conditions. *Soil Science*. 165 (3). 208-216.
- [14] NEWMAN, G. P., 1995. *Heat and mass transfer in unsaturated soils during freezing*. M. Sc Thesis, University of Saskatchewan, Canada.
- [15] OTTAVIANI, M., VAN DIEDENHOVEN, B. & CAIRNS, B., 2015. Photopolarimetric retrievals of snow properties. *The Cryosphere*. 9. 1933-1942.
- [16] PANTELEEVA, I., KOSTINA, A., ZHELNIN, M., PLEKHOV, A. & LEVIN, L., 2017. Intellectual monitoring of artificial ground freezing in the fluid-saturated rock mass. *Procedia Structural Integrity*. 5. 492-499.
- [17] PENTLAND, J. S., 2000. *Use of a general partial differential equation solver for solution of heat and mass transfer problems in soils*. University of Saskatchewan, Canada.
- [18] SUSSMANN, T. R. & HYSLIP, J. P., 2010. Track Substructure Design Methodology and Data. *In Proceedings of the ASME Joint Rail Conference*, Illinois, USA.

- [19] TANG, L., WANG, K., JIN, L., YANG, G., JIA, H. & TAOUM, A., 2018. A resistivity model for testing unfrozen water content of frozen soil. *Cold Regions Science and Technology*. 153. 55-63.
- [20] VANAPALLI, S. & OH, W., 2010. Mechanics of unsaturated soils for the design of foundation structures. *In Proceedings of the 3rd WSEAS international conference on Engineering mechanics, structures, engineering geology*. 363-377.
- [21] WANG, A., XIE, Z., FENG, X. et al, 2014. A soil water and heat transfer model including changes in soil frost and thaw fronts. *Science China Earth Sciences*. 57 (6). 1325-1339.
- [22] WATANABE, K. & WAKE, T., 2009. Measurement of unfrozen water content and relative permittivity of frozen unsaturated soil using NMR and TDR. *Cold Regions Science and Technology*. 59 (1). 34-41.