

THE COMPLEX PHENOMENOLOGICAL MODEL FOR PREDICTION OF INHOMOGENEOUS DEFORMATIONS OF RAILWAY BALLAST LAYER AFTER TAMPING WORKS

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Abstract:

The given article considers the method of calculating the track geometry deformation with respect to uneven accumulation of residual deformations along the track. The technique proposes two significant changes in existing approaches to calculating the efficiency of the ballast layer. The transition from the approach of allowable stresses design in the ballast layer to the deformative approach of accumulations of track geometry deformations allows us to draw conclusions regarding the intervals of track tamping and the duration of ballast layer life cycle. The transition from the determinative to probabilistic approaches makes it possible to draw conclusions not only from the average unevenness, but also with regard to all possible facts of unevenness. The method is based on the mechanism of sudden and gradual deformations occurrence, which depends on a number of key factors: dynamic stresses on the ballast, non-uniformity of track elasticity, performance of current maintenance work. Based on the experimental studies results, the dependencies of sudden deformations and the intensity of gradual deformations on the level of stress on the ballast layer were established. The experimental results of the influence of the sub-ballast base elasticity on the intensity of accumulation of residual deformations are shown. On the basis of the developed method, the prediction of track geometry deterioration for a given structure of the track, the rolling stock and the permissible level of geometric deviations for track maintenance is presented.

Key words:

prediction of track geometry deterioration, phenomenological modelling, track unevenness, uniform subsidence, inhomogeneous subgrade, tamping works

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1. Introduction

One of the most important elements of the track, which the life cycle of the track depends on, is a ballast layer, since it most intensely accumulates irreversible deformations. The track geometry deterioration begins, first of all, due to the deformation of the ballast layer, which, in the future, leads to the increase of dynamic loads on all other elements of the track and the decrease of their service life.

The work of the ballast layer depends on the simultaneous mutual influence of the most important groups of factors, namely: factors of the track construction, rolling stock, operating conditions and maintenance factors. Based on the knowledge of ballast layer deformation mechanisms and factors influencing it, it is possible to predict the process of track geometry deformation, and accordingly, to build a strategy for current maintenance and track repair.

The phenomenological model for track geometry deformation prediction is proposed in this article. This model is based on the results of a series of experimental studies and takes into account the main mechanisms of the uneven accumulation of track deformation due to the subsidence of the ballast layer.

2. The present phenomenological models of track subsidence

At present, a large number of models, from simple phenomenological models to complex finite element and discrete element models are presented for calculating deformation processes in the ballast layer. The

complex models (Nielsen 2018, Nishiura et al. 2018, Miguel et al. 2018), as a rule, consider in detail the influence of separate factors on the basis of one element of the track in the form of a separate sleeper with the ballast, and so on. In practice, as was summarised by Gerber and Fengler (2010), the maintenance of the track, the deformation of the track are influenced by a wide range of factors, such as the construction of the superstructure of the track, rolling stock, elasticity and uneven elasticity of the subgrade of the track, method of maintenance and its criteria, atmospheric influences, etc. The analysis of theoretical, laboratory and field studies described in the literature suggests that suitable for practical use prediction of long-term processes can be obtained on the basis of simple phenomenological models. The results of experimental studies and field observations of the behaviour of the track are laid in the models.

Many present dependencies describe the process of subsidence depending on the number of load cycles and many other factors, where the most widely used are logarithmic, exponential and linear dependencies obtained during laboratory and field tests. Further, the most widely used groups of phenomenological formulas for residual subsidence of the ballast layer and, separately, the prediction formulas for changing the quality indexes of the track are considered (Table 1).

Table 1. Survey of the Phenomenological Models of Subsidence

Author	Formula	Factors of Influence
1	2	3
Popov (1955), Lysyuk (2003)	$h_{\sigma_1} = \frac{T}{b \cdot T + d} \text{ – for clean ballast layer;}$ $h_{\sigma_1} = \frac{T}{b_1 \cdot T + d_1} + a_2 \cdot T^{n_2} \text{ – for soiled ballast layer.}$	T – tonnage passed; b , d – empirical coefficients characterizing the change of characteristics in the process of multiple loading; a_2 , n_2 – coefficients characterizing the soiling, the humidity of the ballast, other factors.
Selig (1998)	$\varepsilon_N = 0.082 \cdot (100 \cdot n - 38.2) \cdot$ $\cdot (\sigma_1 - \sigma_3)^2 (1 + 0.2 \cdot \log(N))$	ε_N – residual deformations after N cycles of loading; n – initial number of pores; σ_1 – vertical tension; σ_3 – horizontal stress.
Hettler (1984)	$e_N = e_1 \cdot (1 + b \cdot \log N)$ – relation with the number of load cycles;	e_N – subsidence after N – number of load changes, mm;

Author	Formula	Factors of Influence
1	2	3
	$e_T = a_1 + a_0 \cdot \log\left(\frac{T}{2 \cdot 10^6}\right)$ <p>– dependence on operating load</p> $\sigma_T = b_1 + b_0 \cdot \log\left(\frac{T}{2 \cdot 10^6}\right)$	<p>b – constant (0,2 for a separate sleeper, 0,43 for rail-sleeper track panel, according to Hettler's model tests);</p> <p>e_1 – subsidence after the pass of the first change of load, <i>mm</i>; e_T – subsidence after operating load T, <i>t</i>; a_0, a_1 – coefficients of subsidence; σ_T – mean square deviation of subsidence, <i>mm</i>.</p>
Fröhling (1997)	$e_{Ni} = \left[\left[K_1 + K_2 \cdot \left(\frac{k_{2mi}}{K_3} \right) \right] \cdot \frac{P_{dyn}}{P_{ref}} \right]^w \cdot \log(N)$	<p>e_{Ni} – subsidence on the first sleeper; k_{2mi} – stiffness under the sleeper; K_i – constants; P_{ref} – referred loading; N – number of load cycles; w – coefficient of rail subsidence; P_{dyn} – calculated dynamic loading.</p>
Guerin (1999)	$\frac{d\tau}{dN} = \alpha \cdot d^\beta$	<p>τ – subsidence; N – number of loads; d – elastic deflection of a sleeper; α, β – constants.</p>
Sato (1995)	$\tau = \gamma(1 - e^{-\alpha x}) + \beta x$	<p>τ – increment of subsidence; α, β, γ – coefficients; x – passing load.</p>
Shenton (1984)	$e_N = K \cdot \frac{F_e}{10} \cdot \left[\frac{(0.69 + 0.028 \cdot h) \cdot \sqrt[5]{N} + 1}{2.7 \cdot 10^{-6} \cdot N} \right]$ $F_e = \sqrt[5]{\frac{F_1^5 \cdot N_1 + F_2^5 \cdot N_2 + F_3^5 \cdot N_3 + \dots}{N_1 + N_2 + N_3 + \dots}}$	<p>K – coefficient of rail structure; F_e – equivalent load (takes into account the effect that high axial loads are dominant for subsidence); h – raising the track at tamping; N – number of passed axles.</p>
Holtzendorff (2003)	$z_{iN} = z_{i0} + \left[I_\sigma \cdot I_{dyn} \cdot I_{entl} \cdot I_{deg} \cdot I_{Esub} \right]_i \cdot \log(N_0 + N)$	<p>z_{iN} – subsidence of the ballast layer under i- sleeper after N cycles of loading; z_{i0} – pre-set backlash; σ_{be} – equivalent vertical stress (factor I_σ); I_{dyn} – dynamics factor; I_{entl} – factor of unloading; I_{deg} – soiling factor; I_{Esub} – hardness factor of the lower structure of the track; N_0 – the number of load cycles that were passed to the selected starting point of time.</p>
Veit (2006), Holzfeind, Hummitzsch (2010), Lichtberger (2003)	$Q = Q_0 \cdot e^{bt}$ <p>the law of changing the quality of the track geometry</p>	<p>Q_0 – quality at the time $t = 0$ (corresponds to the initial quality after stabilisation of the initial subsidence); b – intensity of deformation; t – time or tonnage passed.</p>
Horvat, Kiss (2006)	$Q = Q_0 \cdot e^{\alpha m v^2}$ <p>change in the quality of the track depending on the change in the energy per train pass</p>	<p>where α – is the intensity of deformation; m – tons of passed load (transport load of the track); v – train speed.</p>
Gerber, Fengler (2010)	$s = s_1 + s_2$ $s_1 = a \cdot \text{MAX}(F_i)$ $s_2 = b \cdot s_1 \cdot (1 - e^{-c \cdot t})$	<p>s – subsidence of the ballast layer; s_1 – plastic part of subsidence; $\text{MAX}(F_i)$ – maximum load in the history of axial load; s_2 – elastic subsidence; a, b, c – constants of laboratory or natural measurements; t – time, number of load cycles, passed tonnage.</p>

In most cases, the phenomenological formulas are given by many authors with the subsidence of a separate sleeper under the impact of many factors whose influence remains unchanged throughout the life cycle of the track. Therefore, such subsidence laws give an average increase in track subsidence, which makes it difficult to estimate the change in the quality of the track, since it is influenced not by average subsidence, but by differential subsidence, which, in turn, causes the dynamic interaction of the track with the rolling stock, and it is an additional factor, which accelerates the track deformation. In addition, up to now, the presented models take into account the main groups of factors separately, without complex consideration of the processes of track maintenance.

3. The mechanism of the track geometry deformation

In general, the work of the ballast layer and the railroad is influenced by a large number of external and internal factors. In this study, the mechanism of the track geometry deformation is considered, taking into account the main factors whose influence can be determined on the basis of experimental studies, field observations, normative and reference literature. Figure 1 shows the scheme of influence of external and internal factors and their interconnections

on the development of the unevenness of the track due to the deformation of the ballast layer. Primary external factors are of three groups: factors of rolling stock and operating conditions; factors of the track superstructure and the subgrade; repair and maintenance factors.

These external factors have a separate and reciprocal influence on internal factors that directly affect the deformation of the ballast layer and lead to the development of track unevenness. The internal factors are the characteristics of the ballast layer and its tense-deformed state, among which the most important are: the sleeper pressure on the ballast and the vibration effect on it, the number of loads and the mode of the ballast layer unloading by the sleeper, the elasticity of the sub-ballast layer and the subgrade, the soiling of the ballast layer, etc.

3.1. The uniform subsidence of the ballast layer

At the repeated sleeper loading on the ballast there is a gradual accumulation of residual deformations. The results of measurements (Lichtberger, 2003) of track subsidence at two levels of axial loads of 25 and 30 tons are shown in Fig. 2. The rapid plastic deformations occur at the beginning of the load, or when the load is changed, after which there is a gradual deformation.

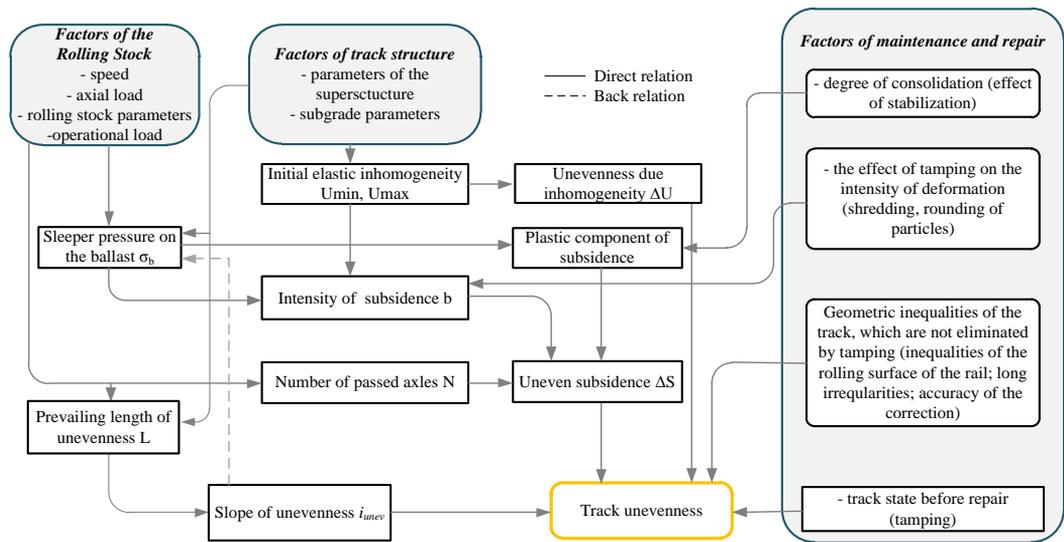


Fig. 1. Scheme of the influence of factors on the work of the ballast layer

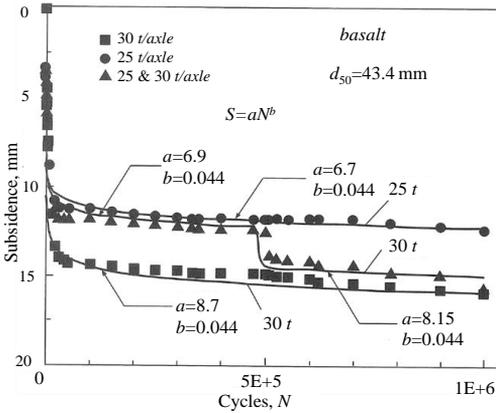


Fig. 2. Subsidence at various axial loads (every $4 \cdot 10^5$ cycles at the frequency of 15 Hz) (Lichtberger, 2003)

A number of performed laboratory and experimental studies (Popp, 2003) during the non-vibration load showed that the behaviour of the ballast layer under the sleeper during the load cycles is conditionally two-phase: the stabilization phase and the phase of uniform accumulation of deformations. In this case, the stabilization phase can occur not only at the beginning of the load cycle, but also when the level of load increases. This behaviour can be described by a simple mechanical model from the system of two connected elements: viscous and plastic. Simplified calculation scheme, according to which the maximum and minimum subsidence of the ballast layer at the prevailing wavelength is considered, which depends on the characteristics of the track, rolling stock and speed, is depicted in Fig. 3.

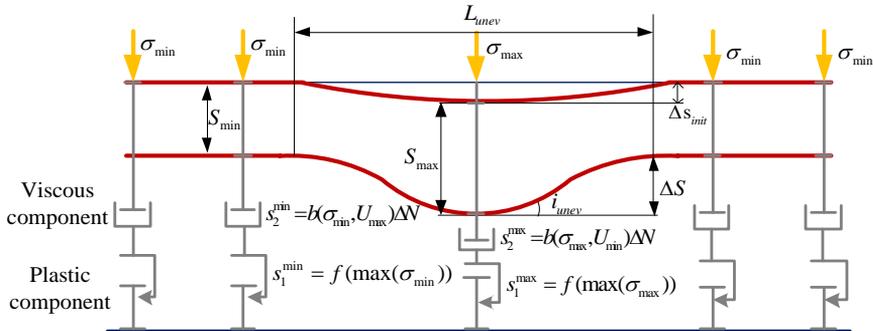


Fig. 3. Simplified calculation scheme for determining uneven subsidence along the track

The behaviour of the residual subsidence is described by the summation of the plastic part s_1 (plastic subsidence independent of time, the initial subsidence) and the viscous part s_2 (viscous subsidence independent of time, secondary subsidence):

$$s = s_1 + s_2 . \tag{1}$$

The plastic part of the subsidence s_1 occurs suddenly, and mainly, in the initial period of track stabilization. Such subsidence is calculated by the formula:

$$s_1 = s_{stab} + s_{F \max} . \tag{2}$$

The component s_{stab} is a function of degree of consolidation. It is determined on the basis of the normal measurements data of the track levelling after 1 million tons of cargo passage according to (Umanov, 2007) and can be at the average 22 mm with tamping without stabilization to 11 mm with the single-layer stabilization after tamping. This value s_{stab} is related to the degree of the initial consolidation of the ballast layer with dynamic track stabilizers. This component can be determined by the methods of ballast layer diagnostics (Sysyn et al., 2010).

The second component of the plastic subsidence $s_{F \max}$ occurs when the stress in the ballast layer under the sleeper σ_i , MPa, which value is greater than that which arose in the history of loads MAX (σ_i). It was determined by the results of the laboratory tests of the All-Russian Research Institute of Railway Transport (VNIIZHT) (Verigo et al., 1955) on the load of a separate sleeper in the ballast box with a cyclic non-vibration load. The ballast was considered as densely consolidated on the rigid sub-ballast.

At each subsequent test, the amplitude of the cyclic increased loading results in an increase of the level of initial subsidence. There was no full unloading of a sleeper. Fig. 4 shows the relationship between the initial subsidence and maximum in reached pressure, and the formula shows the approximation of the dependence by the polynomial of the second degree:

$$s_{F_{max}} = 42,953 \cdot \max(\sigma_i)^2 + 5,6844 \cdot \max(\sigma_i), \quad (3)$$

where σ_i – the calculated stress in the ballast layer under the sleeper, *MPa*.

The component of the plastic subsidence $s_{F_{max}}$ reaches its maximum value for about 1 million tons passage. To describe this, the asymptotic dependence of the ballast subsidence is used depending on the number of loads (Lysyuk, 2003):

$$s(N) = s_{F_{max}} \frac{N}{a + bN}, \quad (4)$$

where N – number of repetitive loads; $a = 10000$, $b = 1$ – coefficients that depend on the stress state and the physical properties of the crushed stone.

The viscous behaviour of the residual subsidence depends on the number of load changes. The major difference of the proposed model to the before mentioned consists in considering not the absolute value of the subsidence, but its difference; therefore, for a simplified description of the elastic component, the

linear dependence on the number of loads is approximated:

$$\Delta s_2 = b(\sigma_i, U_s) \Delta N, \quad (5)$$

where ΔN – the increase in the number of load cycles; $b(\sigma_i, U_s)$ – the intensity of accumulation of residual deformations, which depends on the stress in the ballast layer σ_i under the sleeper and the elastic modulus of the subgrade U_s .

The intensity $b(\sigma_i, U_s)$ is defined as the product of the corresponding components: $b(\sigma_i, U_s) = b(\sigma_i) \cdot k_U(U_s)$.

The component of the stresses in the ballast is determined by the empirical formula of the third degree polynomial obtained by processing the results of experimental tests of the VNIIZHT (Verigo at al., 1955) for pure crushed stone fractions of 25-60 mm (Fig.5):

$$b(\sigma_i) = 1,981\sigma_i^3 + 0,199\sigma_i^2 + 0,029\sigma_i \quad .(mm/10000 \text{ axles}) \quad (6)$$

The three lines at Fig.5 show, that the viscous behaviour of the residual subsidence also depends on the ballast quality and ballast soiling. The promising technique for in-situ ballast aggregate gradation determination is described by Guerrieri, M., Parla, G. (2013).

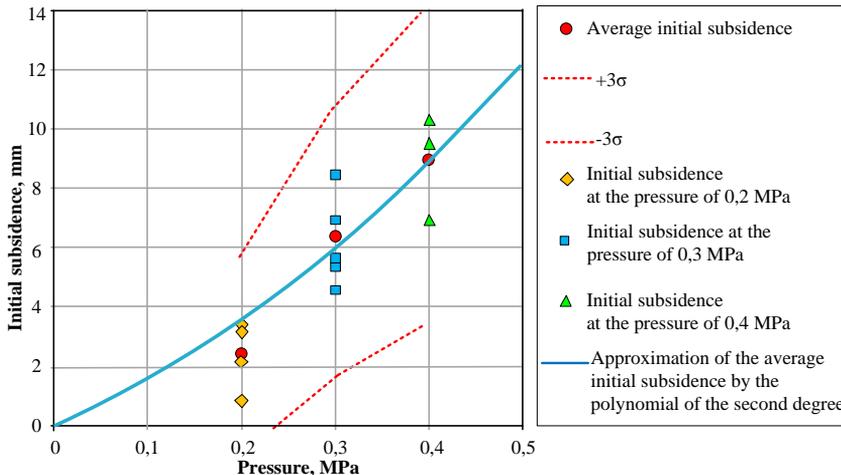


Fig. 4. Dependence of initial subsidence and maximum in loading history pressure

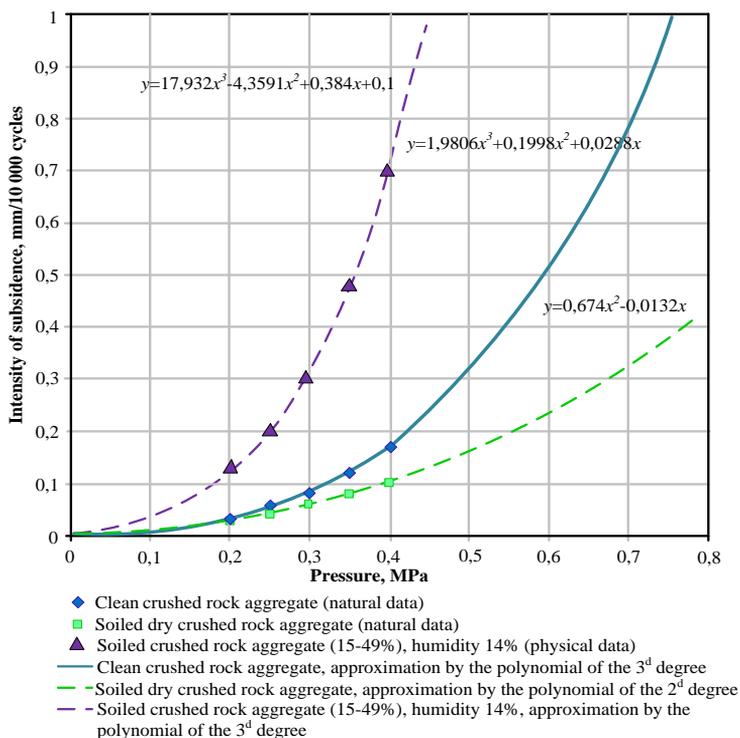


Fig. 5. Dependence of the intensity of subsidence on the stresses in the ballast under the sleeper

3.2. The influence of the subgrade elasticity on the uniform subsidence of the ballast layer

The elasticity of the subgrade has a significant effect on the ballast layer, which was studied by a number of authors on the basis of the natural measurements of the track. Holzfeind and Hummitzsch (2010) have examined the track unevenness accumulation at lines with different subgrade stiffness. Holtzendorff (2003) has taken into account the subgrade influence on subsidence intensity with amplifying parameter. Especially significant influence of subgrade stiffness was observed and estimated by Kovalchuk at al. (2018) at subgrade structures, where the severe stiffness reduction was taking place. At present, the most common methods of the subgrade elasticity estimation are based on track stiffness measurement (Drożdziel, J., Sowiński, B., 2010).

The authors of the abovementioned work at the Laboratory of Railway Transport Department of Technical University of Dresden performed laboratory researches of the influence of the elasticity of the

sub-ballast base for determining the quantitative interrelationships of the intensity of the ballast layer subsidence.

To do this, a simplified physical model of the ballast was created in the form of a cone of sand of 145 mm high and 890 mm width, to the peak of which a cyclic load was transmitted through a rigid round plate with a diameter of 380 mm. The load was carried out by a servo-hydraulic test machine ZWICK HB 160 (Fig. 6) in the form of a sinusoidal shape with a frequency of 2 Hz and different degrees of maximum amplitude values: 5 kN, 7,5 kN and 10 kN.

The elasticity of the sub-ballast base is modelled using a different number of rubber plates of 8 mm high and 600 mm x 600 mm size in 3 versions of elasticity: 0 mm (absolutely rigid base), 8 mm and 16 mm. Those are previous measurements of the elasticity of a single plate. On the basis of 9 different load configurations and elasticity, the study was carried out on the influence of the sub-ballast layer elasticity and ballast layer stresses.

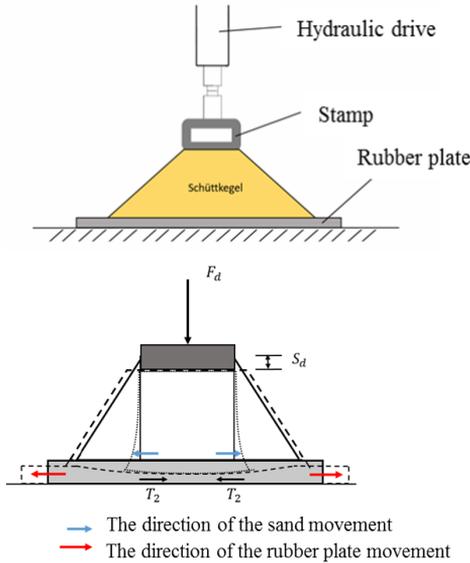


Fig. 6. Experimental research (Laboratory of Railway Transport Department of Technical University of Dresden)

As a result of experimental studies, the residual subsidence of a plate was measured during 1500 load cycles (Fig. 7). The process of accumulation of subsidence is divided into two phases. In the first phase (up to 50 cycles) there is a rapid accumulation of deformations, after which the second phase begins, which is characterized by a relatively gradual accumulation of deformations. Each increase of loading causes initially the acceleration of subsidence, which subsequently grows into a uniform growth of residual deformations.

As a general result of experimental measurements for the subsequent calculations, the relative intensity of residual subsidence is used depending on the elasticity of the sub-ballast base (Table 2). The increase in the intensity of the subsidence with an increase in the elasticity of the sub-ballast base is meant under this value relative to the intensity at an absolutely rigid basis. From the experimental measurements, it follows that the relative intensity of residual subsidence, with the number of cycles above 1000, remains practically unchanged.

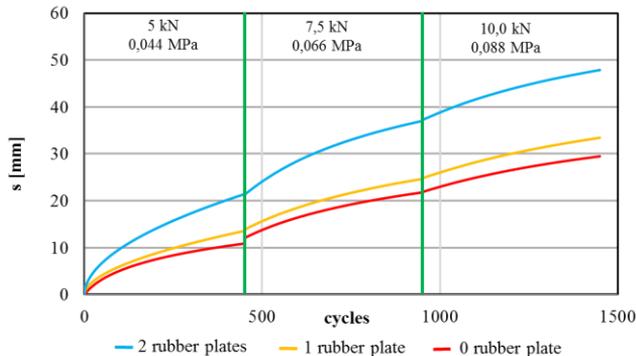


Fig. 7. Subsidence of a plate between 50 and 1500 load cycles at 9 different load and elastic configurations

Table 2. Dependence of the intensity of subsidence on the stiffness of the sub-ballast base

Test	Rubber thickness	Stiffness of the sub-ballast base, N/mm^3	Rate of subsidence, mm/1 cycle	Relative rate
0 rubber plate	0	∞	0,0153	1
1 rubber plate	8	0,176	0,0174	1,137
2 rubber plates	16	0,267	0,0215	1,405

The obtained results are well correlated with the averaged data on the influence of the track stiffness on the intensity of subsidence (Lichtberger, 2003). Taking into account these data, the component of the elasticity of the subgrade or sub-ballast layer $k_U(U_s)$ in the form of a dimensionless increasing coefficient is calculated. Formula (6) corresponds to the maximum rigid basis; therefore, if there is some actual value of the coefficient of bedding of the subgrade U_s , this intensity rapidly increases.

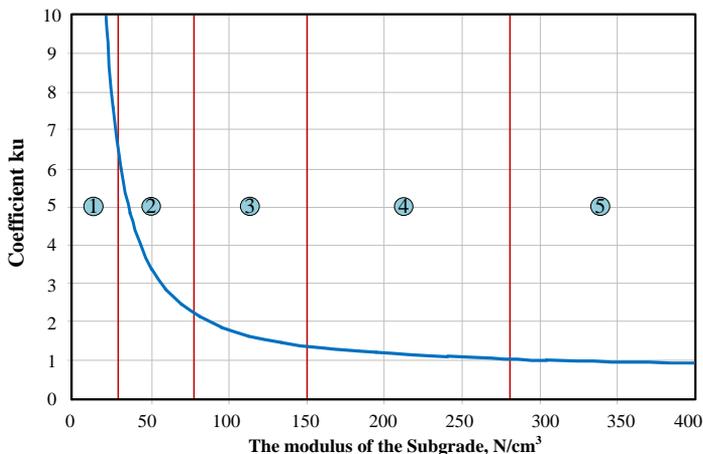
Taking into account the fact that formula (6) corresponds to the most rigid basis, the empirical relationship between the coefficient of the subgrade bedding and the coefficient k_U is formed, which is shown graphically in Fig. 8, and has the form:

$$k_U(U_s) = \frac{6000}{14U_s^{1,29} - 26} + 0,74 \quad (7)$$

A number of other factors influence the intensity of the subsidence, but due to the lack of experimental data on their influence and methods of their actual determination, they are not considered in this study.

3.3. The uneven subsidence of the track

The appearance of a separate unevenness occurs due to the uneven subsidence of the track. The origin of the unevenness arises as a result of many factors of the heterogeneity of the mechanical properties of the material of the upper structure of the track, of the unevenness of its elements and, accordingly, of the unequal dynamic effect of the wheel on the rail. The set of factors influencing the origin of unevenness form the initial quality of the track. This study takes into account the main factors of the initial quality: the initial irregular elasticity of the subgrade and the initial inhomogeneous subsidence of the ballast layer. The initial irregular elasticity of the subgrade (U_s^{min}, U_s^{max}) depends on the quality of the improvement of the subgrade and the arrangement of its protective layers.



1. Very bad subgrade (marshy soils) U up to $30 N/cm^3$; 2. Bad subgrade (viscous clay soils) $U=30-80 N/cm^3$; 3. Good subgrade $U=80-150 N/cm^3$; 4. Very good subgrade $U=150-280 N/cm^3$; 5. Rigid or rock base $U>280 N/cm^3$

Fig. 8. Relationship between the elastic modulus of the subgrade and the coefficient k_U (the classification of the subgrade state according to Lichtberger (2003))

Several studies (Holzfeind, 2010, Kovalchuk, 2018, GROSSONI et al., 2018) indicate that the irregular elasticity of the subgrade is the main cause of the origination and the character of track unevenness growth, and the studies (Lichtberger, 2003) give the maximum and minimum limits of the elastic modulus of the sub-rail foundation for various types of soils of the subgrade as well as at the presence of a stabilizing layer. In this calculation, the initial quality of the track due to the irregular elasticity is equal to the difference between the deflections Δs_U under the action of static load P_{st} on the track:

$$\Delta s_U = \left(\frac{k(U_z^{\min})}{2U_z^{\min}} - \frac{k(U_z^{\max})}{2U_z^{\max}} \right) \cdot \bar{P}, \quad (8)$$

where U_z^{\min} and U_z^{\max} – the stiffness of the sub-rail foundation, which is given for 1 meter of rails, *MPa*, calculated depending on the stiffness of sleeper fastening, ballast and subgrade U_s^{\min} , U_s^{\max} .

The studies (Selig, 1998) show the significant influence of the dynamic stabilisation of the track on the subsequent uneven subsidence of the ballast layer. In the experimental measurements (Umanov, 2007), in addition to the uniform stabilization of the track, its deviation from the mean value depending on the degree of consolidation of the ballast layer is also researched. Such uneven stabilization Δs_{stab} in the technology of single-layer stabilization of the ballast, according to (Umanov, 2007), is 10-18 % of the average subsidence s_{stab} during stabilization. In order to obtain a greater degree of stabilization, the layer-by-layer ballast stabilization is required during repair.

The initial uneven subsidence causes the action of the initial and, further, additional dynamic load. The additional effect of the dynamic load of the rolling stock wheels on the rail, in turn, increases the tension in the ballast layer, which leads to an increase in the intensity of subsidence and even greater unevenness growth. The initial uneven subsidence takes into account three components:

$$\Delta s_{init} = \Delta s_u + \Delta s_{stab} + \Delta s_{geom}. \quad (9)$$

The increase of unevenness during the life cycle occurs due to the elasticity component Δs_2 . In this study, it is assumed that the unevenness of the track is taken into account in the loading of the action of unsprung masses of the rolling stock by the formula (9) through the mean slope of the unevenness i_{unev} . The initial and mean slopes of the unevenness are determined by the total difference of subsidence $\Delta s_{unev} = s^{\max} - s^{\min} + \Delta s_{init}$ and the half of the mean wave length of the unevenness:

$$i_{unev}^{init} = \frac{2\Delta s_{init}}{L_{unev}}; \quad i_{unev} = \frac{2\Delta s_{unev}}{L_{unev}}. \quad (10)$$

Studies (Popp and Schiehlen, 2003) showed that the vertical unevenness of the subsidence at some parameters of the mechanical properties of the track has a certain predominant value approaching the length of the elastic line of the rail. The author Gudehus (1998) gives formula (11) for the determining the prevailing wavelength of the unevenness, which also takes into account the characteristics of the rolling stock:

$$L_{unev} = 2\pi \sqrt{\frac{q_k V^2}{2U_z} + \sqrt{\left(\frac{q_k V^2}{2U_z}\right)^2 + \frac{EI}{U_z}}}, \quad (11)$$

where q_k – unsprung mass of the rolling stock, *kg*; V – speed of movement, *m/s*; EI – bending stiffness of the rail; U_z – stiffness of the underrail base, which is given for 1 meter rail, *MPa*.

The stiffness of the connecting elements of fastenings, sleepers and ballast layer is determined according to the method (Danilenko, Rybkin, 2006). In the calculation algorithm, the force of the unsprung mass of the rolling stock is corrected at each calculation cycle through the calculation of the unevenness slope i_{unev} . The wave length of the unevenness L_{unev} remains unchanged throughout the cycle load period.

3.4. Stress determination in the ballast under the sleeper

The subsidence of the ballast layer and the resulting unevenness of the track occur in the process of repeated loading of the ballast with a sleeper. The

main parameter characterizing the level of sleeper loading on the ballast is the pressure of the sleeper, or the tension in the ballast under the sleeper. However, the actual pressure of the sleeper on the ballast layer is quite an uncertain value, which varies widely across the area of the sleeper and depends on the shape of the contact between ballast grains and the degree of rail tamping. Therefore, to estimate the ballast layer performance, the value of the calculated sleeper pressure on the ballast layer is used, which is found through the equivalent force of the rail on the sleeper and depends on the factors of the track construction and the rolling stock, and calculated according to the standard method for the CIS countries (Danilenko, Rybkin, 2006). The advantage of this method is the statistical approach in calculating the maximum probable dynamic loads on the track and stresses in its elements. The disadvantage of this method is the interpretation of the elements strength in the form of permissible stresses, after which the element is considered to be inoperative. For such elements of the track, as ballast or a subgrade, the deformation occurs gradually throughout the period of operation, and the change of state depends on the accumulation of uneven deformations. In this study, it is proposed to expand the standard technique in order to take into account long-term processes in the track. The following formulae from the standard technique (Danilenko, Rybkin, 2006) are used for the calculation of the maximum probable dynamic stress in the ballast under the sleeper.

The pressure in the ballast under the sleeper depends on the value of the equivalent force of the rail on the sleeper P_{eq}^{II} :

$$\sigma_b = \frac{P_{eq}^{II}}{\Omega \cdot \alpha}, \quad (12)$$

where $\Omega \cdot \alpha = \frac{1}{2} \alpha \cdot a \cdot b$ – the effective bearing area

of a half sleeper, taking into account its bending (a , b – the length of a sleeper and the width of the bottom of its bedding (cm), α – the coefficient of a sleeper bending).

The maximum equivalent load for calculating forces in the elements of the underrail base takes into account the joint action of adjacent axles and is determined by the formula:

$$P_{eq}^{II} = P_{dyn}^{max} + \sum \bar{P}_i \cdot \eta_i. \quad (13)$$

Functions η_i are dependent on the value kx_i , where x_i corresponds to the distances from the calculated section to each wheel, which is taken into account:

$$\eta_i = e^{-kx} (\cos kx_i + \sin kx_i), \quad (14)$$

where η_i – the ordinates of the line of influence of transverse forces taken in the sections under all wheeled carriages that are considered (except for the calculated).

In practical engineering calculations, it is assumed that in the calculated section of the rails (that is, in the section where the stress state is determined) the maximum possible dynamic wheel loading is effective $P_{cal} = P_{dyn}^{max}$, and the effect of the adjoining wheels is taken as their average dynamic forces \bar{P}_{dyn_i} ,

taking into account that the maximum dynamic force of the calculated wheel doesn't coincide with the maximum forces of the neighbouring wheels.

The calculated dynamic load is considered the maximum probable value from the combination of the influence of constant static and variable dynamic forces:

$$P_{cal} = P_{max}^{prob} = \bar{P} + \lambda_f \cdot S, \quad (15)$$

where \bar{P} – the average value of dynamic force; S – mean square deviation; λ_f – the normalizing factor ($\lambda_f = 2,5$) for a given probability level of $F = 0,994$.

The average value of the dynamic load is determined by the formula:

$$\bar{P} = P_{st} + \bar{P}_p, \quad (16)$$

where P_{st} – the wheel static pressure on the rail; \bar{P}_p – average value of the inertia force caused by oscillations of the over suspension part of the carriage.

It is conventionally assumed that the average value of this inertia force is three quarters of the maximum value:

$$\bar{P}_p = 0,75 P_p^{max}, \quad (17)$$

where P_p^{\max} – the maximum value of the inertia force caused by oscillations of the over suspension part of the carriage.

Dynamic load of a wheel on a rail P_p^{\max} with the use of empirical dependencies of dynamic deflections of spring suspension z_{\max} with respect to the speeds of motion is determined by the formula:

$$P_p^{\max} = G_p z_{\max}, \quad (18)$$

where G_p – stiffness of spring suspension of a bogie, reduced to one wheel, kN/m ; z_{\max} – maximum dynamic deflection of suspension.

The total average deviation of the dynamic vertical load of a wheel on a rail \bar{S} , is determined by the formula of the composition of the probability distribution of its components:

$$\bar{S} = \sqrt{S_p^2 + S_{pu}^2 + 0,05S_{ium}^2 + 0,95S_{cus}^2}. \quad (19)$$

Due to specific information, the average percentage of axles with an isolated smooth unevenness is taken 5 %; correspondingly, the average percentage of axles with a continuous unevenness is equal to 95 %. The mean square deviation of the dynamic load of the wheel on the rail S_p from the vertical fluctuations of over suspension part of the carriage P_p^{\max} is determined by the formula:

$$S_p = 0,08 P_p^{\max}. \quad (20)$$

The mean square deviation of the dynamic load of a wheel on a rail S_{pu} , kN , from the forces of inertia of the unsprung masses P_{pu}^{\max} that arise at the presence of the isolated unevenness on the rail line, is determined by the formulas:

$$S_{pu} = 0,707 P_{pu}^{\max}, \quad (21)$$

$$P_{pu}^{\max} = \frac{\alpha_0 i_{unev} V \pi^2}{\sqrt{2g}} \sqrt{\frac{U_z q_k}{k}}, \quad (22)$$

where V – speed of movement, km/h ; \bar{P} – average dynamic load; l_{sl} – distance between the sleeper

axes, m ; U_z – stiffness of the underrail base, which is reduced to 1 meter rails, MPa ; q_k – the value of the unsprung mass of the carriage, kN , which falls on 1 wheel; i_{unev} – the average slope of the isolated unevenness; α_0 – the coefficient that takes into account the effect of the inertia of the rail masses, which oscillate, on the value of the dynamic force; k – the coefficient of relative stiffness of the base and the rail, m^{-1} , determined by the formula:

$$k = \sqrt[4]{\frac{U_z}{4EI}}. \quad (23)$$

The calculated distances l_s between the sleeper axes are taken at 0,50, 0,543, 0,60 and 0,625 m respectively for 2000, 1840, 1680 and 1600 sleepers per 1 km .

The mean square deviation of the dynamic load of a wheel on a rail of inertia forces of the unsprung mass P_{ium} arising from the presence on the rolling surface of isolated unevenness is determined by the formula:

$$S_{ium} = 0,25 P_{ium}^{\max}. \quad (24)$$

The mean square deviation of the dynamic load of a wheel on a rail S_{cus} , kN from inertia forces of the unsprung mass P_{cus}^{\max} during the wheel movement with smooth continuous unevenness on the rolling surface is determined by the formula:

$$S_{cus} = 0,225 \cdot P_{cus}^{\max} = \frac{0,052 \cdot 10^{-4} \alpha_0 U_z V^2 \sqrt{q_k}}{d^2 \sqrt{10^{-1} \cdot k U_z - 3,26 k^2 q_k \cdot 10^{-2}}}, \quad (25)$$

where d – wheel diameter, m .

After determining the mean square deviations of all variables of the inertia forces, it is possible to determine the total mean deviation S from the formula (19) and then determine the maximum probable value of the dynamic load $P_{cal} = P_{max}^{prob}$ from formula (15).

3.5. The impact of track maintenance on ballast layer deformation

When performing track tamping, the geometric state of the track is improved. However, such an improvement does not occur to the initial state, besides, the intensity of ballast layer deformation after tamping increases due to further crushing of crushed stone particles and the change of their shape. Thus, with multiple tamping, a significant deterioration of the ballast layer may occur (Fig. 9). The degree of improvement depends on the geometric state of the track before tamping. In order to take into account the degree of improvement at tamping, the results of the research are used from (Horvat, 2006, Holzfeind, 2010, Veit, 2006, Soleimanmeigouni et al. 2016). It is taken into account the fact that with the help of tamping it is impossible to obtain better quality than the initial quality of the track, i.e. the unevenness conditioned by uneven elasticity of the subgrade Δs_U and uneven subsidence with a single-layer stabilization of the track Δs_{stab} cannot ensure better quality.

The author suggests calculating the improvement achieved with the help of tamping by the following expression:

$$\Delta s_{tamp} = k_{tamp} \cdot (\Delta s_{unev} - \Delta s_U - \Delta s_{geom} - \Delta s_{stab}), \quad (26)$$

where k_{tamp} – the coefficient of linear dependence is assumed equal to 0,5÷0,8.

The degree of improvement according to the expression (26) in percentage is shown in Fig. 9 for $k_{tamp} = 0,648$ and the initial uneven plasticity of the subgrade and uneven stabilization $\Delta s_U + \Delta s_{stab} = 1,3 \text{ mm}$. It is assumed that after passing the load to the first cycle of tamping, the ballast becomes fully stabilized, and for further tamping the expression (26) takes the form:

$$\Delta s_{tamp} = k_{tamp} \cdot (\Delta s_{unev} - \Delta s_U - \Delta s_{geom}). \quad (27)$$

The growth of the intensity of deformation after the performance of tamping depends on the degree of damage of particles of crushed stone by the working elements of tamping machines. The intensity of deformation is influenced by the mechanical properties of particles' resistance to crashing, which, to a large extent, depend on the type of ballast material (Fischer, 2017).

Therefore, studies have taken different values of intensity increase. Thus, sources (Holzfeind, 2010) show the increase in the intensity of uneven subsidence for 70-90 % of the intensity of tamping, other sources show the value of the intensity increase 20 to 50 %. The railways of east Europe use predominantly crushed rock aggregate made of granite material, which has high resistance to crushing; therefore the intensity increase for 20 % is roughly taken for calculation.

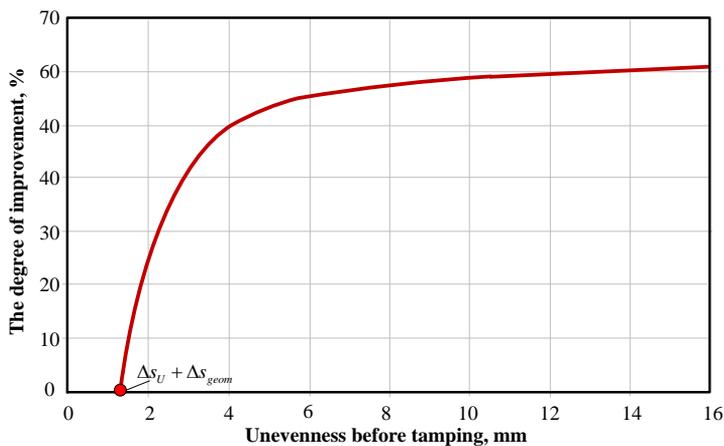


Fig. 9. Dependence of the degree of improvement due to tamping on the geometric state of the track before repair and the given initial quality of the track

4. The calculation results and discussion

The following example of calculation of track geometry deformation for the given operating conditions, the given construction of the track superstructure, the quality of the subgrade and the value of the permissible maximum-probable unevenness for track tamping works is given. The factor of the initial quality of the track depends on the non-uniform stiffness of the railway subgrade and the quality of the ballast layer consolidation.

To simplify the problem, the passenger or the freight traffic is considered separately with rolling stock that is a passenger car on the bogie KVZ-TsNII with the speed of 160 km/h and loaded four-wheeled freight car on the bogie TsNII-X3-0 with a speed of 80 km/h. The locomotive impact is not taken into account.

The static load from the wheel on the rail for the freight car is 116,2 kN, for the passenger car is 71,25 kN, the unsprung mass is respectively 7,1 kN and 9,95 kN. The stiffness of the suspension is determined by the static deflection of a spring in 48 mm for a freight car and 155,0 mm for a passenger car.

In the calculation, the design of the track, which is typical for the conditions of the CIS railways, is adopted: namely, the long-welded rails of R65 type, reinforced concrete sleepers, the fastenings of KB type, sleepers' layout is 1840 pcs./km in straight and curved lines. The elasticity of the subgrade varies from $U_{z_{sg \min}} = 140 \text{ N/cm}^3$ to $U_{z_{sg \max}} = 196 \text{ N/cm}^3$,

which corresponds to the second category of base quality according to (Prokudin, 2005). Accordingly, the maximum and minimum value of stiffness of the underrail base, which is given for 1 meter rail: $U_{z \min} = 59,4 \text{ MPa}$ and $U_{z \max} = 67,7 \text{ MPa}$. Operational load at the section is 20 megatons per year.

Calculated by the formula (11), the wavelength is, for example, 4,22 m at the passenger car on the bogie KVZ-TsNII and at the speed of 160 km/h and 3,82 m for the freight car on the bogie TsNII-X3-0 at the speed of 80 km/h. The calculated initial slope of unevenness i depends on irregular stiffness of the subgrade and corresponds 0,073 ‰ for the freight and 0,038 ‰ for the passenger rolling stock. The total initial slope of the unevenness with the initial uneven subsidence of the ballast layer is 0,544 ‰ and 0,493 ‰.

The calculation of dynamic loads on rails, stresses on the ballast layer and the development of track unevenness is performed in the form of statistical maximum-probable value at the probability level of 0,994. A certain distribution of a number of isolated vertical irregularities of the track depending on the state of the track according to the system (Danilenko, Rybkin, 2006), which is shown in Fig. 10, corresponds to these maximum probable depths of unevenness assumed in the current calculation of the railroad strength. This drawing shows the statistical distribution of subsidence probabilities, where the depth of isolated irregularities do not exceed indexes indicated in the work (Sysyn, 2008) for different track states and for irregularities greater than 5 mm deep. The intersection with the horizontal line shows the limit value of the unevenness with the probability level of 0,994. It follows from the drawing that even excellent condition of the track cannot exclude depths appearance in more than 8 mm in 6 of 1000 cases.

Thus, on the basis of the probabilistic approach, it becomes possible to draw conclusions regarding the number of irregularities of a certain range of depth. For example, at the maximum probable depth of subsidence for 20 mm (corresponding to the value of deviation for 10 mm), the number of subsidence of the II degree of deviation is 88,3 %, the number of subsidence of the III degree of deviation and above is 11,7 % at the probability level of 0,994. This result does not contradict the current criteria for tamping repair, which is the ratio of the number of deviations (pcs/km) of the second degree and deviations of the III degree and above.

The calculation of the development of the maximum probable depth of unevenness, depending on the tonnage passed for passenger and freight traffic is shown in the diagram (Fig. 11). Upon reaching this value, which corresponds to the admissible criterion of maintenance, the implementation of the track tamping is taken into account. At the same time, the depth of the subsidence is drastically decreasing, but does not return to the initial value through life cycle. Besides, the intensity of subsidence at each subsequent tamping increases significantly, which leads to further reduction of the tamping intervals.

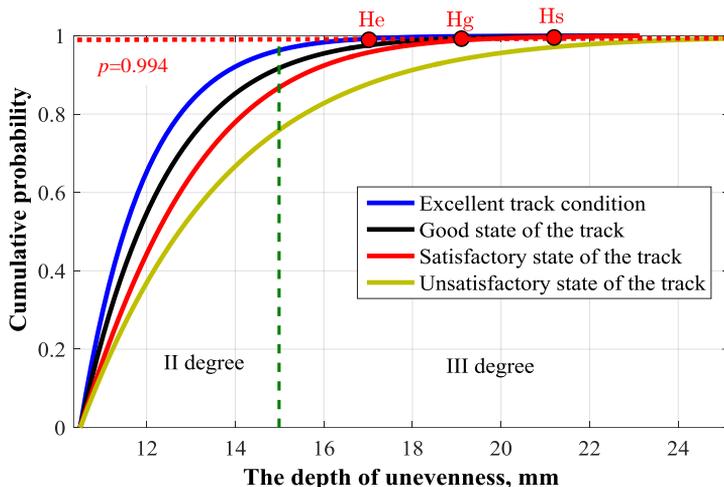


Fig. 10. The subsidence distribution with the probability level of 0,994 at different track states (Sysyn, 2008)

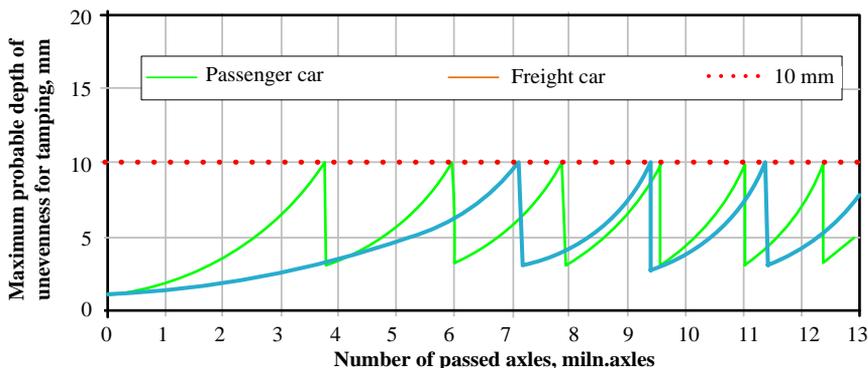


Fig. 11. The comparison of the development of subsidence for passenger and freight cars depending on the number of passed axles

It is evident from Fig. 11 that the unevenness under the influence of a freight car develops much faster, and the first tamping is already necessary after the passage of 3.76 million axles, while the unevenness under the influence of a passenger car reaches the depth of 10 mm after passing 7.16 million axles.

5. The conclusion and subsequent studies

This study presents the method for assessment of track geometry deformation, which allows considering the main factors of the track construction, rolling stock, operating conditions and maintenance factors in full measure. The proposed method is based on

the standard normative rules of calculation, which allows using it for practically all possible variants of factors. At the same time, the proposed improvements by the authors allow eliminating the main disadvantage of the standard method – the approach of permissible stresses in the ballast layer, which are not related to the duration of the life cycle. It is proposed to assess the development of track subsidence together with track maintenance effect instead of probable stresses. The application of the probabilistic approach makes it possible to draw conclusions regarding the number of unevenness. The developed methods could be used for improvement of strategic

maintenance planning of railway infrastructure (Wieczorek at al. 2018).

The purpose of further research is:

- 1) to determine the optimal allowable values of the vertical track unevenness for track tamping from the point of view its frequency reduction and ensuring the maximum of the ballast layer life cycle.
- 2) the development of the optimal strategy for current maintenance which assumes corresponding value of the maintenance criterion and aims at ensuring the maximum possible life cycle of the ballast layer.

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