

ESTIMATION OF THE BUS DELAY AT THE STOPPING POINT ON THE BASE OF TRAFFIC PARAMETERS

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Abstract: Contemporary methods of spatial planning of urban transport systems provide for designers enough opportunities in selecting the placement of stopping points for public transport. However in every city there exist very intense sections of the road network with a small width of the roadway. In these sections there is no opportunity to allocate special lanes for public transport. If the stop pockets on such street exist, there appear traffic conflicts when buses depart from the stopping point. Authors propose theoretical model for estimation of the bus delay at the stopping point on the base of traffic parameters. Use of the proposed model allows reducing amount of field surveys while grounding the decisions about rational variant of allocation of the bus stopping points. The paper describes some experimental results obtained with the use of the proposed model while field surveys at the most loaded streets in the central part of Kharkiv (Ukraine).

Key words: traffic parameters, time delays, mathematical model, stopping point

1. Introduction

Development of methods and tools to improve traffic management and road safety is nowadays quite an actual problem. Current research of traffic intensity in the biggest cities show that for a significant number of sections the road network capacity is not enough big; this problem is particularly acute in the historic centers of the cities, where construction of buildings and design of road network were implemented without account of the current level of motorization.

One of the most pressing issues arising in the face of considerable intensity of traffic is the impact of transport flows on movements of the public transport vehicles. The most significant this impact is at the beginning of vehicle movement while leaving a stopping point, which is located in a pocket; in such a situation probability of traffic accident is quite high. Intensive and fast traffic flow urges the bus drivers to wait for an interval between vehicles in the flow that would be enough big for safe maneuvering. It leads to delays of buses while leaving stopping points and causes the increase of passengers travel time.

Interaction between vehicles is one of the most important aspects related to the traffic flows functioning. This may be the interaction between

vehicles of the same flow or interaction between two separate traffic flows. The last mentioned situation takes place, when a vehicle changes a lane, enters a traffic flow or crosses it. While vehicles interact in the flow, realization of basic maneuvers is associated with the notion of acceptable interval between vehicles. The process of leaving by vehicle of the entrance, adjoining to the highway, is studied by many researchers.

2. Literature review

Most of the works in the area of traffic parameters estimation has an empirical character, and on their basis the methods of design and operation of road constructions are developed. There were also attempts to obtain mathematical description of the process of vehicle entry to the highway; however, they are quite inaccurate due to the complex nature of the interaction between vehicles. Solving this problem is also complicated because of lack of precise criteria for selecting a suitable distance between vehicles and of logic for the process of entry to the highway.

For the first time the microscopic models (Reuschel, 1950; Pipes, 1953) were proposed in the 1950s. These models assumed that the acceleration of the n -th vehicle was determined by the state of

neighboring vehicles. In this case the basic influence is performed by the $(n-1)$ -th vehicle that moves ahead. This vehicle is often called the leading one and the whole class of microscopic models is called "follow the leader". General view of the vehicle movement is defined by a system of differential equations, and the difference between the models is defined by the type of function f :

$$\dot{v}_n(t) = f(v_n, \Delta v_n, d_n) \quad (1)$$

where:

Δv_n – difference between the speed of neighboring vehicles;

v_n – speed of the "leading" vehicle in a flow;

d_n – mean distance between vehicles in a flow.

The first "follow the leader" models predicted that each driver adapted his speed to the speed of the leading vehicle:

$$\dot{v}_n(t) = \frac{1}{\tau} \cdot [v_{n-1}(t) - v_n(t)] \quad (2)$$

A significant drawback of this model is that it does not describe the real flow properties such as instability and occurrence of congestion waves. Therefore, in Chandler et al. (1958) it was proposed to use in the left side of the equation an argument $\Delta t \approx 1,3$ that reflects the duration of drivers' reaction on change of a leading vehicle speed. Here the multiplier $1/\tau$ in (2) could be interpreted as a coefficient S of the speed of driver reaction on changes of the leader speed. The coefficient S is a dynamic variable, which depends on the leader speed and current distance to the leader. Therefore the model could be written as a differential equation with a shifted argument:

$$\dot{v}_n(t + \Delta t) = S \cdot [v_{n-1}(t) - v_n(t)] \quad (3)$$

The equations of such a type for the sufficiently high values of Δt demonstrate instability that can simulate the development of congestion waves. However, if the sensitivity coefficient S is constant, the equation does not reproduce enough properties of real traffic flow, but depends on the flow density. More adequate model could be obtained from the

assumption that with decreasing of distance to the leader the sensitivity increases. To coordinate the sensitivity coefficient S with experimental data in (Gazis et al., 1961) the following expression was proposed:

$$\frac{1}{T} = \frac{1}{T_0} \cdot \frac{[v_n(t + \Delta t)]^{m_1}}{[x_{n-1}(t) - x_n(t)]^{m_2}} \quad (4)$$

where:

m_1, m_2 – empirically selected constants.

In the equation of equilibrium the dependence of speed and density has the following form:

$$v_e(\rho) = v_0 \cdot \left[1 - \left(\frac{\rho}{\rho_{\max}} \right)^{m_2-1} \right]^{\frac{1}{1-m_2}} \quad (5)$$

where:

v_0 – free flow speed, km/h;

ρ_{\max} – maximum allowable density of the traffic flow, veh./km.

One of the main disadvantages of this method is that it incorrectly describes the dynamics of a single vehicle. In the absence of the leader the vehicle acceleration is zero, while the more reasonable here would be an assumption that the driver will try to approach the speed to some desired value V_0 . In models of other type it is permitted, that for each driver there exists "safe" speed, which depends on the distance to the leader. This speed is also called optimal speed. In such models an adaptation to the optimal speed was assumed instead of the speed adaptation to the leader speed. Impact of the leader is expressed indirectly through the dependence of optimal speed from the distance. This model for the first time was proposed in Newel (1961), where an adaptation of speed with time delay was provided:

$$v_n(t + \Delta t) = v'_e(d_n(t)) = v_e(s_n(t)) \quad (6)$$

In Bando et al. (1994, 1995) it is proposed to use differential equation:

$$\dot{v}_n(t) = \frac{1}{\tau} \cdot [v'_e(d_n(t)) - v_n(t)] \quad (7)$$

where:

$v'_e(d_n)$ – safe vehicle velocity depending on the distance to the leader:

$$v'_e(d_n) = \frac{v_0}{2} \cdot [\tanh(d - d_c) + \tanh d_c] \quad (8)$$

The disadvantages of the standard model of optimal speed are that the model is very sensitive to the choice of a particular functional dependence $v'_e(d_n)$ of optimal speed from the distance and to the choice of τ as well. In the model the vehicles collisions appear for large values of τ , and for too small values there appears unrealistically large acceleration. In this case acceleration time is about five times higher than the duration of braking. Moreover, in reality, drivers can withstand greater distance and brake earlier if the speed $\Delta v_n(t)$ is too high relatively to the leader speed. To take into account these and other features of the actual behavior of drivers there were developed many variations of this model (Gipps, 1981; Krauss et al., 1996; Krauss et al., 1997; Helbing and Tilch, 1998; Bleile, 1997; Tomer et al., 2000).

Modeling of bus entry to the traffic flow belongs to the class of microscopic models. In microscopic models, in contrast to macroscopic, traffic flow modeling is carried out with precision to every vehicle in the flow. Such an approach allows describing the traffic flow more accurately.

The most common among micromodels is “Intelligent Driver Model” (IDM) (Kesting et al., 2010; Liebner, 2012). Numerical experiments with this model showed that its properties are resistant to variation of parameters. The model shows a realistic behavior during acceleration and braking of vehicles, and recreates the main studied properties of the traffic flow.

The IDM suggests the vehicle acceleration is described by a continuous function of speed v_n , “pure” distance to the leader $s_n = d_n - l_{n-1}$ and the speed Δv_n relative to the leader:

$$\dot{v}_n = a_n \cdot \left[1 - \left(\frac{v_n}{v_n^0} \right)^\delta - \left(\frac{s_n^*(v_n, \Delta v_n)}{s_n} \right)^2 \right] \quad (9)$$

The part $a_n \cdot \left[1 - \left(\frac{v_n}{v_n^0} \right)^\delta \right]$ of this equation describes

the dynamics of vehicles acceleration at the free

road. Another part $a_n \cdot \left[\left(\frac{s_n^*(v_n, \Delta v_n)}{s_n} \right)^2 \right]$ describes

the braking process caused by interaction with the leader. Parameter δ allows to calibrate the behavior associated with the acceleration process. The value $\delta = 1$ corresponds to the exponential acceleration that is typical for the most vehicle models. By increasing this parameter the acceleration does not decrease exponentially during acceleration, which better describes the behavior of drivers.

With the development of computer technology broad application received simulation models based on cellular automata. In models of cellular automata (CA) road is divided into cells, and time is also considered as discrete. Often (but not always – Kurzhanskij et al. (2010), Sumalee et al. (2011), Hadiuzzaman and Qiu, (2013)) it is considered that in the cell can be no more than one vehicle. Often the possible vehicles speed is considered as discrete as well. The CA model assumes at every step $m \rightarrow m + 1$ that the state of all vehicles in the system is updated according to the following rules:

- acceleration (shows the tendency to move as quickly as possible, without exceeding the maximum permissible speed):

$$v_n(m+1) = \min\{v_n(m) + 1; v_{\max}\} \quad (10)$$

- braking (ensures that no clashes with a vehicle that is moving ahead will happen):

$$v_n(m+1) = \min\{v_n(m); S_{n+1}(m) - S_n(m) - d\} \quad (11)$$

where:

d – distance between the vehicles, m.

- random perturbation (takes into account possible differences in the behavior of the vehicles):

$$v_n(m+1) = \begin{cases} \max\{v_n(m) - 1; 0\}, & p, \\ v_n(m), & 1 - p; \end{cases} \quad (12)$$

- movement of the vehicle:

$$S_n(m+1) = S_n(m) + v_n(m) \quad (13)$$

The listed rules are the basic; for modeling of more complex aspects of the flow dynamics it is necessary to formulate additional rules. Numerical experiments show that the flow is stable at low density and lose stability at high density. Here the key role in the appearance of imbalance plays stochastic nature of the process, i.e. the appearance of traffic jams is conditioned by the probability of congestion p that is not equal to zero. If $p = 0$, then the traffic flow stays stable for all the values of its density (Nagel and Schreckenberg, 1992; Nagel and Herrmann, 1993). This fact could be considered as a serious theoretical disadvantage of SA models compared with IDM models, where fluctuations play a role of the initial push, and further development of traffic jams is explained by instability (which is fully determined) of the equilibrium solution.

3. Model for estimation of the bus delay while leaving a stopping point

3.1. Basic model of the process

It is established by researches that to describe the flow with relatively low intensity, which is characterized by a probability of a certain number of vehicles moving through the road section, Poisson distribution (Cascetta, 2013) may be applied. Moreover, if the vehicle appearance is characterized by the Poisson distribution, the time intervals between vehicles τ_i are distributed exponentially:

$$F(t) = 1 - e^{-\lambda t} \quad (14)$$

where:

$$F(t) \text{ – probability that } \tau_i \leq t, \\ 0 \leq F(t) \leq 1, \text{ when } t > 0 \text{ or } t \in (0; \infty),$$

λ – basic distribution parameter (traffic flow intensity), veh./sec.

In general, let us assume that the flow of vehicles passing by the stopping point is recurrent, if random variables $\tau_1, \tau_2, \dots, \tau_n$, reflecting the time interval between the neighboring vehicles in a flow, form a sequence of independent identically distributed random variables with the law $F(t)$.

Let's consider three events for the process of arrival and departure of the bus at the stopping point:

1.Event A_0 : suggests that before the time moment t there was any moving vehicle, and in the time

interval $(t; t + \tau(v))$ there was no vehicle passing near a bus stop.

2.Event A_n : suggests that before the time moment t there exactly n vehicles passed a bus stop, and in the time interval $(t; t + \tau(v))$ there was no vehicle passing near.

3.Event B : suggests that in the time interval $(t; t + \tau(v))$ there was no vehicle passing near a bus stop, i.e. bus will leave the stop without any obstacles, and the number n of vehicles that passed the stopping point, while the bus was there, can take any non-negative integer value $n = 0, 1, 2, \dots$ etc.

In this case:

$$B = \sum_{n=0}^{\infty} A_n \quad (15)$$

The probability of the event B depends on a couple of parameters t and $\tau(v)$:

$$P(t, \tau) = P(B) = P(A_0) + \sum_{n=1}^{\infty} P(A_n) \quad (16)$$

where:

$$P(A_0) = \bar{F}(t + \tau(v)),$$

$$P(A_n) = \bar{F}(t + \tau(v) - x).$$

Probability of the random event A_n could be determined, if the conditional probability of this event under condition that $t_n = x$ is defined:

$$P\{A_n | t_n = x\} = P\{\tau_{n+1} > \tau(v) + t - x\} \quad (17)$$

Let define the absolute probability of event A_n as following:

$$P(A_n) = \int_0^t P\{A_n | t_n = x\} \cdot f_n(x) dx \quad (18)$$

where:

$f_n(x)$ – density function of random event that the last vehicle passes by before the bus is going to leave the stop,
 $t_n = \tau_1 + \tau_2 + \dots + \tau_n$.

Thus the distribution function of the random variable $t_n = \tau_1 + \tau_2 + \dots + \tau_n$ can be found recurrently relying on the distribution function of the random variable t_{n-1} :

$$F_n(x) = \int_0^x F_{n-1} \cdot (x - y) \cdot f(y) dy \tag{19}$$

where:

$f(y)$ – density function of the random variable t_n .
To determine the density function of the random variable t_n let define the derivative of the expression (19):

$$F'_n(t) = f_n(x) = \int_0^x f_{n-1} \cdot (x - y) \cdot f(y) dy \tag{20}$$

Then we obtain:

$$\begin{aligned} \sum_{n=0}^{\infty} P(A_n) &= \\ &= P(A_0) + \int_0^t [1 - F(\tau(v) + t - x)] \cdot \sum_{n=1}^{\infty} f_n(x) dx \end{aligned} \tag{21}$$

For definition and interpretation of $\sum_{n=1}^{\infty} f_n(x)$ the

respective expression $\sum_{n=1}^{\infty} F_n(x)$ should be defined.

Let $v(x)$ is a number of vehicles that passed by the bus stop during period x . In this case

$$F_n(x) = P\{t_n \leq x\} = P\{v(x) \geq n\} \tag{22}$$

$$\begin{aligned} \sum_{n=1}^{\infty} F_n(x) &= \sum_{n=1}^{\infty} P\{v(x) \geq n\} = \\ &= \sum_{n=1}^{\infty} n \cdot P\{v(x) = n\} = \sum_{n=1}^{\infty} n \cdot p_n = \mu[v(x)] \end{aligned} \tag{23}$$

where:

$\mu[v(x)]$ – mathematical expectation of the number of vehicles that passed by the bus stop during period $[0; x]$, which we denote as $H(x)$.

It should be noted that $\sum_{n=1}^{\infty} p_n = 1$ and

$\sum_{n=1}^{\infty} n \cdot p_n = \mu[v(x)] = H(x)$. Hence we obtain:

$$\sum_{n=1}^{\infty} f(x) = \sum_{n=1}^{\infty} F'_n(x) = H'(x) = h(x) \tag{24}$$

where:

$h(x)$ – derivative of the average number of vehicles that passed by during period x .

Let express probability of event B in a form:

$$\begin{aligned} P_i\{\tau\} &= P(B) = \\ &= \bar{F}(t + \tau(v)) + \int_0^t \bar{F}(t + \tau(v) - x) \cdot h(x) dx \end{aligned} \tag{25}$$

where:

$$\bar{F}(t) = 1 - F(t) = P\{\tau_i > t\} = 1 - P\{\tau_i \leq t\}.$$

Substituting (14) into $\bar{F}(t) = 1 - F(t)$, where m_1 – average value of interval between vehicles in traffic flow, from $m_1 = \mu[\tau_i]$ we get:

$$\bar{F}(t) = 1 - (1 - e^{-\lambda t}) = e^{-\lambda t} \tag{26}$$

Let substitute obtained expression (13) in (12):

$$\begin{aligned} P_i\{\tau\} &= e^{-\lambda \cdot (t + \tau(v))} + \int_0^t e^{-\lambda \cdot (t + \tau(v) - x)} \cdot \lambda dx = \\ &= e^{-\lambda \cdot (t + \tau(v))} + e^{-\lambda \cdot (t + \tau(v))} \cdot \int_0^t e^{\lambda \cdot x} \cdot \lambda dx = \\ &= e^{-\lambda \cdot (t + \tau(v))} + e^{-\lambda \cdot (t + \tau(v))} \cdot (e^{\lambda \cdot x} \Big|_0^t) = \\ &= e^{-\lambda \cdot (t + \tau(v))} + e^{-\lambda \cdot (t + \tau(v))} \cdot (e^{\lambda t} - 1) = \\ &= e^{-\lambda \cdot (t + \tau(v))} + e^{-\lambda \cdot (t + \tau(v))} \cdot e^{\lambda t} - e^{-\lambda \cdot (t + \tau(v))} = e^{-\lambda \cdot \tau(v)} \end{aligned} \tag{27}$$

To define the probability of event B (probability that the bus will leave the stopping point without obstacles) let substitute $\lambda = \frac{1}{m_1}$ in (27):

$$P(B) = e^{-\frac{\tau(v)}{m_1}} \tag{28}$$

3.2. Description of the alternative cases of the process

Let define the duration of waiting for the bus to enter the recurrent traffic flow after boarding of all the passengers (in this case the duration of the passengers boarding does not interest us). Aiming this, let denote the leave of the stopping point by the bus as $A1\{\zeta_r > \tau(v)\}$ under condition $\tau(v) = const$

(Fig. 1). Let also denote as $A\{\tau_i > \tau(v)\}$ the leave of the stopping point by the bus after a vehicle passed the bus stop (Fig. 2). Duration of departure considering possible delay τ_{ld} (Fig. 3) is determined as follows:

$$\tau_{ld} = \tau(v) \cdot \chi_{A1} + \chi_{\bar{A}1} \cdot (\zeta_r + \tau_i) \tag{29}$$

where:

τ_i – duration of departure considering possible delay after a vehicle passed the bus stop, sec.,

χ_{A1} – index of the random event A1 (event \bar{A} is opposite to the event A).

Let assume that the flow of vehicles that pass by the bus stop is simple (Poisson) with parameter λ , then due to the absence of aftereffect for the exponential interval in the simplest flow, τ_{ld} has the same distribution as τ_i , because ζ_r and τ_i both are distributed exponentially. Respectively, $\tau_{ld} = \tau_i$ (as it shown at Fig. 4):

$$\tau_i = \tau(v) \cdot \chi_{A1} + \chi_{\bar{A}1} \cdot (\tau_1 + \tau'_i) \tag{30}$$

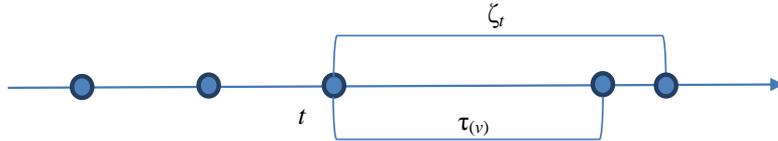
then we obtain:

$$\begin{aligned} \tau(v) \cdot \chi_{A1} + \chi_{\bar{A}1} \cdot (\xi_r + \tau_i) = \\ = \tau(v) \cdot \chi_{A1} + \chi_{\bar{A}1} \cdot (\tau_1 + \tau'_i) \end{aligned} \tag{31}$$

$$\tau_i = \begin{cases} \tau(v), & P(A1), \\ \tau_1 + \tau'_i, & P(A), \end{cases} \tag{32}$$

where:

$$\tau'_i = \tau_1 + \tau_2 + \dots + \tau_n .$$

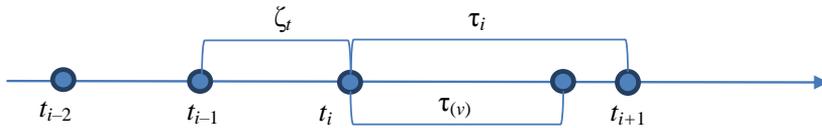


t – the moment of switching-on by the driver of turning signal in order to leave the stopping point,

ζ_r – duration of waiting for appearance of a vehicle,

$\tau(v)$ – duration of the departure of the bus from the stop

Fig. 1. Waiting of the bus to enter from the stopping point to a recurrent traffic flow



τ_i – time interval between vehicles,

$t_{i-2}, t_{i-1}, t_{i+1}$ – moments of passing the bus stop by vehicles.

Fig. 2. Waiting of the bus to leave the stopping point under condition that a vehicle passes by the bus stop

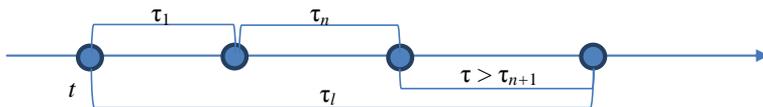


Fig. 3. Waiting of the bus to leave the stopping point taking into account possible delay

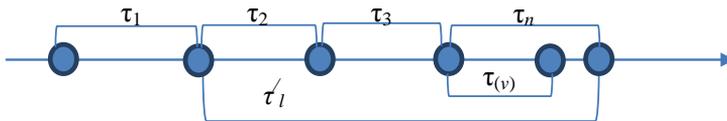


Fig. 4. Waiting of the bus to leave the stopping point with account of a certain number of vehicles passing the bus stop

Let use the Laplace transformation in order to determine duration $\tau(v)$ of the delayed departure of the bus from the stopping point. Then the probability that during random time $\tau(v)$ some random event will not take place in the simplest (Poisson) flow, the intensity of which occurrence is equal to S , could be defined in the following form:

$$\varphi_{\tau(v)}(S) = \mu \left[e^{-S \cdot \tau(v)} \right] \tag{33}$$

Since $\tau(v)$ is constant, then $\mu \left[e^{-S \cdot \tau(v)} \right] = e^{-S \cdot \tau(v)}$ is a constant as well. In this expression $a(s) = \mu \left[e^{-S \cdot \tau_i} \right]$ is searched value.

Let use the Laplace transformation for the duration of the stop leaving by the bus with account of the bus delay:

$$a(s) = e^{-S \cdot \tau_i} \cdot P(A) + b(s) \cdot a(s) \tag{34}$$

where:

$e^{-S \cdot \tau_i} \cdot P(A)$ – probability that an additional event will not appear, if the bus leaves a stopping point without delay during the first time interval ($P(A)$ – probability of the event that $\tau_i = \tau(v)$ for $e^{-S \cdot \tau(v)}$, where $\tau(v)$ is constant),

$b(s) \cdot a(s)$ – probability that an additional event will not appear, if the bus does not leave a stopping point from the first try (not during the first time interval),

$b(s)$ – probability that an additional event will not appear during the time interval, which duration is less than $\tau(v)$, i.e. $\tau_i < \tau(v)$.

Then the Laplace transformation for the duration of departure, if during the first interval τ_i bus did not leave the stop, will take a form:

$$\begin{aligned} b(s) &= \mu \left[e^{-S \cdot (\tau_i | \tau_i < \tau(v))} \right] \cdot P \{ \tau_i < \tau(v) \} = \\ &= \mu \left[e^{-S \cdot \tau_i} \cdot \chi_{\{ \tau_i < \tau(v) \}} \right] \end{aligned} \tag{35}$$

The distribution function for the interval, for which the bus did not departed, could be in this case defined as follows:

$$\begin{aligned} F_{\tau(v)}(x) &= \int_0^x f_{\tau(v)}(t) dt = \\ &= \begin{cases} 0, & x < 0, \\ \frac{1 - e^{-\lambda \cdot x}}{1 - e^{-\lambda \cdot \tau(v)}}, & 0 \leq x \leq \tau(v), \\ 1, & x > \tau(v). \end{cases} \end{aligned} \tag{36}$$

To determine the distribution density of the vehicles interval in a traffic flow let derivate obtained expression in the range $0 \leq x \leq \tau(v)$:

$$f_{\tau(v)}(x) = \begin{cases} \frac{\lambda \cdot e^{-\lambda \cdot x}}{1 - e^{-\lambda \cdot \tau(v)}}, & 0 \leq x \leq \tau(v), \\ 0, & x \notin [0; \tau(v)]. \end{cases} \tag{37}$$

Then the mathematical expectation of the random variable τ_i with parameter $\tau_i < \tau_i$ is defined as follows:

$$\begin{aligned} \mu \left[e^{-S \cdot (\tau_i | \tau_i < \tau(v))} \right] &= \int_0^{\tau(v)} e^{-S \cdot x} \cdot \frac{\lambda \cdot e^{-\lambda \cdot x}}{1 - e^{-\lambda \cdot \tau(v)}} dx = \\ &= \frac{\lambda}{\lambda + S} \cdot \frac{1 - e^{-(\lambda + S) \cdot \tau(v)}}{1 - e^{-\lambda \cdot \tau(v)}} \end{aligned} \tag{38}$$

Let substitute the obtained expression for $b(s)$ in (33):

$$b(s) = \mu \left[e^{s \cdot \tau} \right] \cdot \chi_{\{\tau_i < \tau(v)\}} = \frac{\lambda}{\lambda + S} \cdot \left(1 - e^{-s \cdot (\lambda + S)} \right) \quad (39)$$

flow, which moves with the speed v_n , could be calculated as the following ratio:

Substituting the obtained expression for $a(s)$ in (32), we obtain:

$$\tau(v) = \frac{v_n}{a} \quad (47)$$

$$\begin{aligned} a(s) &= e^{-S \cdot \tau(v)} \cdot e^{-\lambda \cdot \tau(v)} + a(s) \cdot \frac{\lambda}{\lambda + S} \cdot \left(1 - e^{-\tau(v) \cdot (\lambda + S)} \right) = \\ &= \frac{(\lambda + S) \cdot e^{-(\lambda + S) \cdot \tau(v)}}{S + \lambda \cdot e^{-\tau(v) \cdot (\lambda + S)}} \end{aligned} \quad (40)$$

where:
 a – acceleration of the bus, m/sec².

Since

$$\begin{aligned} \left(\mu \left[e^{-s \cdot x} \right] \right)'_s &= \mu \left[e^{-s \cdot x} \cdot (-x) \right] \Big|_{s=0} = \\ &= \mu[-x] = -\mu[x] \end{aligned} \quad (41)$$

then similarly we could define the mathematical expectation for the duration of waiting by the bus to leave a stopping point, differentiating the expression (38) in the zero point:

$$a'(s) \Big|_{s=0} = \frac{1}{\lambda} \cdot \left(1 - e^{\tau(v) \cdot \lambda} \right) \quad (42)$$

$$\begin{aligned} \mu[\tau_i] &= -a'(0) = -\frac{1}{\lambda} \cdot \left(1 - e^{\tau(v) \cdot \lambda} \right) = \\ &= \frac{1}{\lambda} \cdot \left(e^{\tau(v) \cdot \lambda} - 1 \right) \end{aligned} \quad (43)$$

Hence the average duration of the bus departure $\bar{T}_i(v)$ is defined as follows:

$$\bar{T}_i(v) = \frac{1}{\lambda} \cdot \left(e^{\tau(v) \cdot \lambda} - 1 \right) \quad (44)$$

Since

$$\bar{T}_i(v) = \tau(v) + \bar{T}_e \quad (45)$$

then the average delay duration of the bus departure from a stopping point could be calculated as follows:

$$\bar{T}_e = \bar{T}_i(v) - \tau(v) = \frac{1}{\lambda} \cdot \left(e^{\tau(v) \cdot \lambda} - 1 \right) - \tau(v) \quad (46)$$

In this case the duration of leaving by the bus of a stopping point through a “free window” in the traffic

4. Experimental results

As experimental base for the proposed mathematical model we used the most loaded streets in the central part of Kharkiv city. For implementation of calculations it is necessary to determine the speed of the traffic flows and acceleration of the bus. Observations for estimation of these two parameters cannot be implemented in a regular way – with the use of stopwatch, speedometer or odometer. Therefore, to solve this problem, more complex methods should be used.

As the technical tool to measure the parameters of movement of vehicles en route in the center of the Kharkiv city the GPS-navigator was used. This device allows to record simultaneously the current time, speed and location of the vehicle. To achieve this, just the current state of the navigator screen should be captured.

The information obtained during such observations could be digitized, which allows to calculate all the necessary parameters of the vehicles movement en route. The observations have been conducted at different periods of the working day in June 2015. To conduct the measurement, the checker, being inside the bus, had captured the navigator screen at the bus stops and while movements through the intersections. In total 12 measurements were made – 3 in each direction of Pushkinskaya and Sumskaya streets (Fig. 5).

The central part of Kharkiv is characterized by the low speed of buses: average speed during the working day is 11,2 km/h for Sumskaya street and 14,7 km/h for Pushkinskaya street.

On the basis of these results the regression model for the bus acceleration when leaving the stops was obtained. Measurements of the bus state since the moment, when it started the movement from the stopping point, were carried out as frequently as the device allows – about each 2 seconds. Among all the observations on the movements were selected only those which were not characterized by a slowdown

caused, as a rule, by the obstacles from other vehicles. The results of observations are presented graphically at Fig. 6, where the theoretical line, describing the acceleration of the bus, is also shown. Theoretical dependence of the bus continuous acceleration was obtained with the use of regression analysis. Statistical characteristics of the resulting model prove its significance (Tab. 1), which leads to the conclusion that it could be used in simulations of traffic flows.

Table 1. The regression analysis results

Regression model parameters	Value
Multiple R	0,9157
R Square	0,839
Adjusted R Square	0,832
Standard Error	0,865
Observations	27
Coefficient of regression (for the bus acceleration, m/sec. ²)	0,342

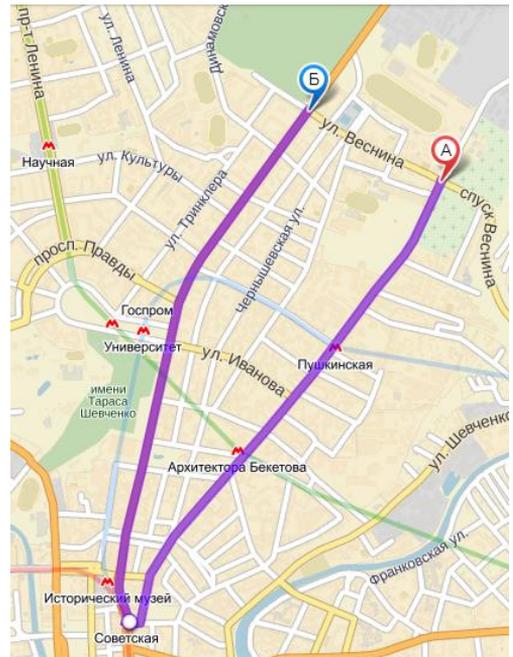


Fig. 5. The objects, where observations were conducted: A – Pushkinskaya str., B – Sumskayastr.

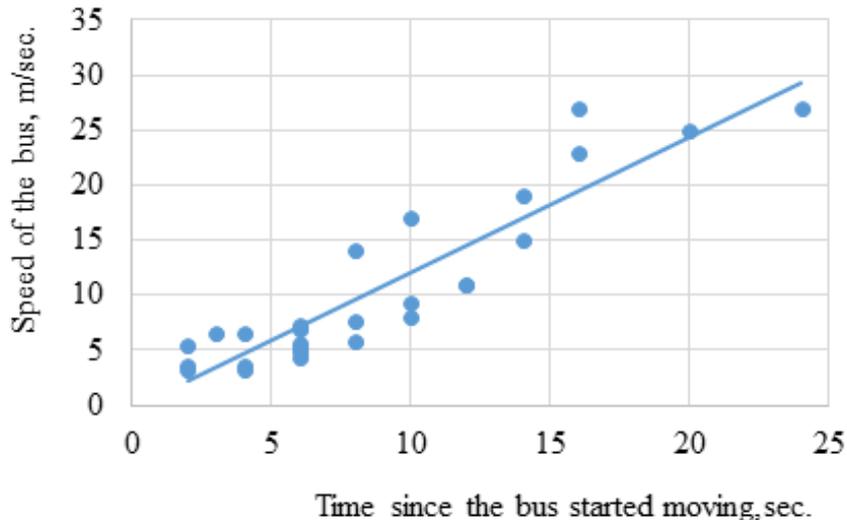


Fig. 6. The bus speed since it started moving at the stopping point

Estimation of the average intensity in the selected streets of Kharkiv also was implemented on the base of observations. To achieve this, the observers conducted the video records of traffic at Sumskaia and Pushkinskaia streets. In total 40 observations were conducted in different parts of these streets. The observations were conducted during one hour at the streets sites that were selected at random. The obtained value of average intensity of traffic is about 990 veh./h. After substituting of the obtained values into the formula (33) it was determined that the delay of the bus departure from the stopping points on the Sumskaia street in average equals 32 sec., and on the Pushkinskaia street is 81 sec. The results of analysis of the traffic speed impact on the duration of the bus departure delay at the stopping points show that the growth rate of the function is proportional to the value achieved at the point.

5. Conclusions

The proposed method of estimation of the time delay while departure of the buses from the stopping points allows to evaluate the appropriateness of the bus stops location at links of the transport network and can be used for reduction of a number of observations of the traffic flow parameters while solving the problems of the interaction between the buses and other vehicles in the flow. Use of the proposed model allows reducing amount of field surveys while grounding the decisions about rational variant of allocation of the bus stopping points. Data on acceleration and speed of route vehicles, obtained as a result of observations, have statistical significance, which means that obtained regression model could be used for assessment of the characteristics of route vehicles movement in a traffic flow.

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