

## THE USE OF HEURISTIC ALGORITHMS TO OPTIMIZE THE TRANSPORT ISSUES ON THE EXAMPLE OF MUNICIPAL SERVICES COMPANIES

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**Abstract:** *In this article the main optimization problems in the municipal services companies were presented. These problems concern the issue of vehicle routing. The mathematical models of these problems were described. The function of criterion and the conditions on designating the vehicle routing were defined. In this paper the hybrid algorithm solving the presented problems was proposed. The hybrid algorithm consists of two heuristic algorithms: the ant and the genetic algorithm. In this paper the stages of constructing of the hybrid algorithm were presented. A structure of the data processed by the algorithm, a function of adaptation, a selection of chromosomes, a crossover, a mutation and an inversion were characterized. A structure of the data was presented as string of natural numbers. In selection process the roulette method was used and in the crossover process the operator PMX was presented. This algorithm was verified in programming language C#. The process of verification was divided into two stages. In the first stage the best parameters of the hybrid algorithm were designated. In the second stage the algorithm was started with these parameters and the result was compared with the random search algorithm. The random search algorithm generates 2000 routes and the best result is compared with the hybrid algorithm.*

**Key words:** *municipal services companies, transport, optimization, genetic algorithm, ant algorithm*

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### 1. Introduction

The main task of the municipal services companies is the waste collection from the region. The issue of the waste collection is a complex decision problem which relates to the traveling salesman problem [21]. One of the main problems in the municipal services companies is the vehicle routing [5, 3, 6]. This paper presents two problems in designating the vehicle routing in the municipal services companies where the heuristic algorithms can be helpful in solving this issues. Moreover in this article the hybrid algorithm consisting of two algorithm: the ant and genetic algorithm was proposed by the author [17]. This issue of vehicle routing is difficult to solve because of many points of the waste collection, the limit of working time of a driver, the limit of driving time of a driver, the limit of payload of the vehicle.

The first problem in designating the vehicle routing is to find the minimum route consisting of all points of loading. Complexity of this problem relies on that each point of loading is located in the different place. The second problem in designating the vehicle routing is to find the connection between the base-the first point on

loading and the point of unloading-the first point of loading. These problems were solved by using of the heuristic algorithms. The heuristic algorithms are one of popular tools of optimization widely described in the literature. The most famous algorithms often presented in optimization problems are the ant and the genetic algorithms.

Genetic algorithms are algorithms which activity is based on the mechanisms of natural selection and heredity. The main advantages of the genetic algorithms over the other methods of optimization are following: conducting the search for the optimum not from a single point in the plane of search but from several points established by the relevant population of individuals, and reliance on the information determined by the objective function and not derivatives. Basing on the values of the objective function is a valuable advantage of the genetic algorithms. The objective function provides us with the value by which a genetic algorithm finds the acceptable and satisfactory solution from the point of view of the problem. It should be noted that the genetic algorithm is a heuristics. Methods of this type give a near-optimum solution, can

find the optimal solution but often confine themselves to the optimal solution in the local area of search. Despite this inconvenience genetic algorithms are successfully used in optimization problems.

The genetic algorithms were often used to solve the transportation problems [19, 14, 12, 7, 2, 16], the scheduling problems [1, 8], the vehicle routing in the municipal services companies [20, 22].

Ant algorithms are algorithms whose principle of operation is based on the imitation of the operation and existence of ants in the wild. The main factor in choosing a route is a pheromone, a chemical secreted by the ants. Ants have a lot of routes to choose, they head for the route with the strongest concentration of the pheromone. A classic problem in which the operation of ants algorithms were tested is the problem of finding the shortest Hamiltonian cycle in a weighted graph with any number of nodes. With reference to the fact it is an attempt to solve the traveling salesman problem [9,10]. The issue of the vehicle routing in the municipal services companies is also solved by using the ant algorithm [4].

## 2. The problems in the municipal services companies

The first problem in designating the vehicle routing is to find the minimum route consisting of all points of loading. The method of designating this route proposed by author of this publication consists of two stages. The main task of this phase is to determine so – called: collective points of loading. Collective points of loading (on the Fig.1) we create by allocating individual points of loading to such sections of the road where the vehicles on these sections cannot change their route. The volume of this point cannot exceed payload of the vehicle.



Fig. 1. Collective points of loading in the area of the waste collection

The main task of the second stage is to designate the minimum route consisting of all collective points of loading. Designating this route is a complex optimization problem, a decision variable appears when the vehicle commutes to the junction and then the length of all the route depends on the decision of the driver. This minimum route was designated by the hybrid algorithm.

The second problem in designating the vehicle routing is to find the best connection between the base-the first point on loading and the point of unloading-the first point of loading. These connections influence on the length of the all route.

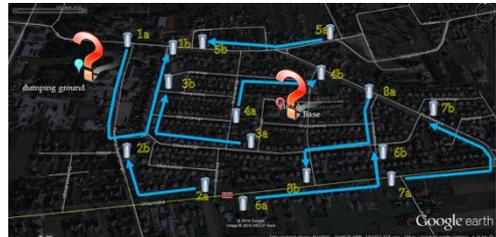


Fig. 2. The problem of selection of the first point of loading in the route

In this problem we assume that we know where the vehicle starts and ends the route of the waste collection. In this place we must define the concept of the task in the the municipal services companies. Defining of the tasks is very difficult. Designating the tasks relies on indicating the point of beginning of the task (on the Fig.2 the points a) and the point of ending of the task (on the Fig.2 the points b). The point of ending we can define as the point where the vehicle leaves the route and goes to the dumping ground. The vehicle leaves the route of loading when payload of the vehicle is exceeded. These points start and end the route of the waste collection realized by vehicles. The task is designated when we know the route of the vehicles which collect the waste (on the Fig.2 the blue arrows), the point of beginning of the route, the points of the route and the point of ending of the route. Designating the route between the point a and the point b is a complex optimization problem. This problem is a part of problem designating the minimum route consisting of all points of loading presented at the

beginning of this chapter (Fig.1). The vehicle visits points of loading and collects the waste and when payload of the vehicle is exceeded the vehicle leaves the route.

The problem of the connection between the base-the first point of loading we can interpret as the first task which will be realized as the first in the route (the mark question on the Fig.2 nearby the base). The problem of the connection between the point of unloading-the first point of loading we can interpret in the moment when the vehicle ended realization the current task, unloaded the waste on the dumping ground and we can ask which the next task will be realized in the route (the mark question on the Fig.2 nearby the dumping ground). We have three points of reference: the base, the task and the dumping ground which are the elements of a transport system. These points determine the routes of the vehicles. To the task can be reached in two ways, directly from the dumping ground or through the base.

In order to solve the problem of the connection between these elements of a transport system we have to designate the next task in the route and the minimum route between two tasks. We have to designate the minimum route which consists of all tasks. One should remember that on the route the limit of the working time of a driver, the limit of driving time of a driver, the limit of time of all tasks realization are imposed.

### 3. The mathematical models of presented problems

In order to determine the mathematical model of the first problem described in the chapter 2 the following variables were used [18]:

$W^Z$  – the set of collective points of loading,

$W^Z = \{1, \dots, i, j, \dots, \overline{W^Z}\}$ ,

$D = [d(i, j)]$  – the matrix of the distance between  $i$ -this collective point loading and  $j$  -this collective point of loading,

$XT = [xt(i, j)]$  – binary matrix of a decision variable defining the road between  $i$ -this collective point of loading and  $j$ -this collective point of loading, where,

The function of criterion minimizes the total length of the route and takes the form:

$$F(XT) = \sum_{i \in W^Z} \sum_{j \in W^Z} xt(i, j) \cdot d(i, j) \rightarrow \min \quad (1)$$

In order to determine the mathematical model of the second problem described in the chapter 2 the following variables were used:

$W^{Zp} = \{1, \dots, i, \dots, \overline{W^{Zp}}\}$  – the set of points of loading,

$W^{Zk} = \{1, \dots, j, \dots, \overline{W^{Zk}}\}$  – the set of points of loading, the points where the vehicle leave the route,

$W^{Zad} = \{1, \dots, za, \dots, \overline{W^{Zad}}\}$  – the set of the numbers of tasks,

$W^B = \{1, \dots, b, \dots, \overline{W^B}\}$  – the set of the bases,

$W^W = \{1, \dots, k, \dots, \overline{W^W}\}$  – the set of points of unloading of interpretation: the dumping ground,

$P = \{1, \dots, p, \dots, \overline{P}\}$  – the set of vehicles' numbers,

$N = \{1, \dots, n, \dots, \overline{N}\}$  – the set of drivers' numbers

$W = [w(k, i)]$  – the matrix of the distance between  $k$  -this point of unloading and  $i$  -this point of loading,

$BZ = [bz(b, i)]$  – the matrix of the distance between  $b$ -this base and  $i$ -this point of loading,

$WB = [wb(k, b)]$  – the matrix of the distance between  $k$ -this point of unloading and  $b$ -this base,

$ZW = [zw(j, k)]$  – the matrix of the distance between  $j$ -this point where the vehicle leave the route and  $k$ -this point of unloading,

$T1 = [t1(p, n, j, k)]$  – the matrix of travel times between  $j$ -this point where the vehicle leave the route and  $k$ -this point of unloading for  $p$ -this vehicle and  $n$ -this driver,

$T2 = [t2(p, n, b, i)]$  – the matrix of travel times between  $b$ -this base and  $i$ -this point of loading for  $p$ -this vehicle and  $n$ -this driver,

$T3 = [t3(p, n, k, b)]$  – the matrix of travel times between  $k$ -this point of unloading and  $b$ -this base for  $p$ -this vehicle and  $n$ -this driver,

$T4 = [t4(p, n, k, i)]$  – the matrix of travel times between  $k$ -this point of unloading and  $i$ -this point of loading for  $p$  -this vehicle and  $n$  -this driver,

$T5 = [t5(p, n, k)]$  – the matrix of times of unloading of a vehicle in  $k$ -this point of unloading for  $p$  -this vehicle and  $n$ -this driver,

$T6 = [t6(p, n, za)]$  – the matrix of times of loading of a vehicle in  $za$ -this task for  $p$ -this vehicle and  $n$ -this driver,

$T7 = [t7(p, n, za)]$  – the matrix of times of driving of a vehicle in  $za$ -this task for  $p$ -this vehicle and  $n$ -this driver,

$[TW_{ocz}(p, n, k); p(TW_{ocz}(p, n, k))]$ – the matrix of waiting time for unloading in  $k$ -this point of unloading for  $p$ -this vehicle and  $n$ -this driver,

$\varphi(p)$  – payload  $p$ -of this vehicle,

$\theta(za)$  – the volume of the task,

$T^{odp}$  – statutory resting time on the route,

$T^{dop1}$  – the permitted driving time,

$T^{dop}$  – the permitted working time of driver,

$\Delta T$  – the range of realization of the all tasks.

The main task is to find the following decision variables:

$\mathbf{XBZ} = [xbz(b, i)]$  – the route between  $b$ -this base and  $i$ -this point of loading,

$\mathbf{X} = [x(k, i)]$  – the route between  $k$ -this point of unloading and  $i$ -this point of loading,

$\mathbf{XWB} = [xwb(k, b)]$  – route between  $k$ -this point of unloading and  $b$ -this base,

$\mathbf{XZW} = [xzw(j, k)]$  – an auxiliary variable defining the route of the assignment between the point where the vehicle leaves the route and the point unloading

to the function of criterion determining the minimum route saved by the formulation:

$$F(xbz, x, xwb, xzw) = \sum_{bc \in W^B} \sum_{ic \in W^{Zp}} xbz(b, i) \cdot bz(b, i) + \sum_{kc \in W^W} \sum_{ic \in W^{Zp}} x(k, i) \cdot w(k, i) + \sum_{kc \in W^W} \sum_{bc \in W^B} xwb(k, b) \cdot wb(k, b) + \sum_{jc \in W^{Zk}} \sum_{kc \in W^W} xzw(j, k) \cdot zw(j, k) \rightarrow \min \quad (2)$$

will take the minimum value.

Constraints take the form of:

1) The limit of driving time of a driver: the sum of travel times to the task both from the base and directly from the point of unloading, the sum of travel times between the point where the vehicle leaves the route and the point of unloading, the sum of travel times between the point of unloading and the base, the sum of travel times of the task:

$$\begin{aligned} & \forall n \in N, p \in P \\ & \sum_{bc \in W^B} \sum_{ic \in W^{Zp}} xbz(b, i) \cdot t2(p, n(b, i)) + \\ & \sum_{bc \in W^B} \sum_{ic \in W^{Zp}} \sum_{za \in W^{Zad}} xbz(b, i) \cdot t7(p, n, za) + \\ & \sum_{kc \in W^W} \sum_{ic \in W^{Zp}} \sum_{za \in W^{Zad}} x(k, i) \cdot t7(p, n, za) + \quad (3) \\ & \sum_{kc \in W^W} \sum_{ic \in W^{Zp}} x(k, i) \cdot t4(p, n(k, i)) + \\ & \sum_{kc \in W^W} \sum_{bc \in W^B} xwb(k, b) \cdot t3(p, n(k, b)) + \\ & \sum_{jc \in W^{Zk}} \sum_{kc \in W^W} xzw(j, k) \cdot t1(p, n, (j, k)) \leq T^{dop1} \end{aligned}$$

2) The limit of the working time of a driver: the driving time of a driver, the sum of loading times, the sum of unloading times, the sum of expected times of unloading and resting time on the route, in order to reduce the volume of the formula the variable of the driving time of a driver takes the mark  $T^{dr}$ :

$$\begin{aligned} & \forall n \in N, p \in P \\ & T^{dr} + \\ & \sum_{bc \in W^B} \sum_{ic \in W^{Zp}} \sum_{za \in W^{Zad}} xbz(b, i) \cdot t6(p, n, za) + \\ & \sum_{kc \in W^W} \sum_{ic \in W^{Zp}} \sum_{za \in W^{Zad}} x(k, i) \cdot t6(p, n, za) + \quad (4) \\ & \sum_{jc \in W^{Zk}} \sum_{kc \in W^W} xzw(j, k) \cdot t5(p, n, k) + \\ & \sum_{jc \in W^{Zk}} \sum_{kc \in W^W} xzw(j, k) \cdot \end{aligned}$$

$$[TW_{ocz}(p, n, k); p(TW_{ocz}(p, n, k))] +$$

$$T^{odp} \leq T^{dop}$$

3) The limit of realization all tasks: the task must be realized within a given range of time,  $T^{work}$ - the working time of a driver:

$$T^{pracy} \leq \Delta T \quad (5)$$

4) The limit of payload of the vehicle: the vehicle can realize the task if payload of this vehicle is greater or equal to the size of this task:

$$\forall za \in W^{Zk}, p \in P \quad \theta(za) \leq \varphi(p) \quad (6)$$

**4. The hybrid algorithm solving the transport issues in the municipal services companies**

In order to solve the presented problems in this paper the hybrid algorithm was proposed. The hybrid algorithm consists of the following steps:

Step 1. The determining of a structure of the data processed by the algorithm.

Step 2. Working of the ant algorithm.

Step 3. The determining of a function of adaptation.

Step 4. The selection of chromosomes dependent on the function of adaptation.

Step 5. The crossover of chromosomes selected randomly out of the pool of parent.

Step 6. The mutation of chromosomes.

Step 7. The inversion.

The presented algorithm is a hybrid of the two heuristic algorithms: ant algorithm and genetic algorithm. The main algorithm is the genetic algorithm that fully takes the initiative to find a solution of these problems. The initial population for the genetic algorithm [13,23] is designated by the ant algorithm [11,15]. The steps from three to six of the genetic algorithm are repeated until the stop condition is achieved. The stop condition in this algorithm is a predetermined number of generations (iterations).

**Step 1. The determining of a structure of the data processed by the algorithm.**

In the problem of designating the minimum route consisting of all collective points of loading (Fig.1) the hybrid algorithm does not work directly on the decision variables of the function of criterion but on the encoded forms of these variables. In order to encode the variables of the function of criterion in an appropriate structure and create the chromosome as a representative of the admissible solution the problem of designating the minimum route must be defined as the appointment of the suitable permutation of collective points of loading so that their location would generate the minimum value of the function of criterion, which in our case it is the minimum length of the route. The task of the hybrid algorithm is to find the best set of collective points of loading by optimizing function of adaptation. The structure of data suitable for processing by the hybrid algorithm can be defined as a string consisting of  $k$

collective point of loading. The total length of the chromosome is  $k$  genes. Each chromosome is a representative of the transport route (Fig. 3).



Fig. 3. The Structure of the chromosome in the first transportation problem

In the problem of designating the best connection between the elements of a transport system (Fig.2) the structure of data suitable for processing by the hybrid algorithm can be defined as a string consisting of the implemented  $za$  tasks and the base. The total length of the chromosome is  $2za + 1$  genes. The maximum length of the chromosome is imposed by the situation in which a single vehicle performs only one task and returns to the base (Fig.4.a). In order to realize that situation the base must be encoded in a few genes (Fig.4.b).



Fig. 4. The Structure of the chromosome in the second transportation problem.

Each chromosome is a representative of the route. We must remember that the first and last gen of the chromosome is encoded as the gen of the base.

The route in the chromosome we can interpret as (Fig.4.c): the vehicle leaves the base (5) and goes to the task (1), leaves the task (1) and goes to the task (4), leaves the task (4) and goes to the base (one base encoded in genes 7,8) leaves the base and goes to the task (2), leaves the task (2) and goes to the base (6), leaves the base (6) and goes to the task (3), leaves the task (3) and goes to the base. The one base is encoded in genes 5,7,8,6,9.

**Step 2. Working of the ant algorithm.**

In order to implement the ant algorithm in the first problem we assume that route of the ants

consists of the all collective points of loading. The route of each ant starts from randomly choosing the collective point. In order to define the ant algorithm the set of the collective points of the route was designated:

$W^{Tp} = \{1, \dots, y, z, \dots, W^{Tp}\}$  – where  $y, z$  – another elements of set  $W^{Tp}$ ,  $y \neq z$ .

The further route of the ant and thereby the choice of the next point is selected from the specified probability (if  $z \in \Omega^{mr}$  then  $P_{yz}^{mr}(t)$ , if  $z \notin \Omega^{mr}$   $P_{yz}^{mr}(t) = 0$ ):

$$P_{yz}^{mr}(t) = \left\{ \frac{[\tau_{yz}(t)]^\alpha \cdot [\eta_{yz}(t)]^\beta}{\sum_{l \in \Omega^{mr}} [\tau_{yl}(t)]^\alpha \cdot [\eta_{yl}(t)]^\beta} \right. \quad (7)$$

where:

$\tau_{yz}(t)$  – the intensity of pheromone trail on the section between  $y$  – this point of the route and  $z$  – this point of the route,

$\eta_{yz}(t)$  – the heuristic information e.g.

$\eta_{yz}(t) = \frac{1}{w(y,z)}$ , where  $w(y,z)$  a distance between  $y$  – this point of the route and  $z$  – this point of the route,

$\alpha, \beta$  – parameters determining the effect of pheromones and the heuristic information on the behavior of ants,

$\Omega^{mr}$  – the set of vertices which hasn't been yet visited by the ant, where  $l$  – the element of set  $\Omega^{mr}$ .

Each point of the route is visited only once. The route of the ant ends after realization all the  $k$  collective points. In the final step of the iteration the pheromone trail is updated. In order to update the pheromone in the route the ant – cycle was used as the most efficient version of the ant algorithms

The pheromone trail is updated after the implementation of all the routes by ants. In the first iteration the pheromone trail is equally strong in all connections between the points, equal to  $\tau_0$ . In other iterations the pheromone trail we can calculate:

$$\tau_{yz}(t+1) = (1 - \rho)\tau_{yz}(t) + \sum_{mr=1}^{Mr} \tau_{yz}^{mr}(t) \quad (8)$$

where:

$mr$  – another ant in the population,  $mr \in MR$ ,  $MR = \{1, \dots, mr, \dots, Mr\}$  – the set of the all ants,

$\rho$  – a factor pheromone, ( $0 < \rho \leq 1$ ),

$\tau_{yz}(t+1)$  – the strengthening of the pheromone in  $t+1$  – this iteration, we assume  $\tau_0$  for the first iteration on all connections.

The partially strengthening of the pheromone in  $t$  – this iteration takes the form:

$$\Delta \tau_{yz}^{mr}(t) = \begin{cases} \frac{1}{L^{mr}(t)} \\ 0 \end{cases} \quad (9)$$

where:

$L^{mr}(t)$  – the length of the all route realized in  $t$  – this iteration by  $mr$  – this ant, if the route  $(y, z)$  was realized by  $mr$  – this ant then the value  $\Delta \tau_{yz}^{mr}(t)$  is equal to  $\frac{1}{L^{mr}(t)}$ , otherwise 0.

For the second problem the route of the ants consists of the elements such as the task and the base. The dumping ground is not treated as the separate point of route because it is the ending point of the task. Ants can make a choice and they either move to the base and then to the task, or directly to the task. The probability of selection of the route from  $y$  to  $z$  by the ant is the same as if the first problem. We must remember that the route of the ants consists of the task and the base.

### Step 3. The determining of a function of adaptation.

The hybrid algorithm works on the encoded forms of the decision variables. In order to designate the value of the  $l$  – this chromosome the function of adaptation  $F_l$  was determined and it takes the form:

$$F_l = C_{max} - F_l(\mathbf{XT}) \quad (10)$$

$C_{max}$  – the value larger than the value of chromosome. The maximum value of the function of adaptation  $F_l$  is equal to the minimum value of the function of criterion,

$F_l(\mathbf{XT})$  – the real value of the function of adaptation for  $l$  – this chromosome which is equal to the function of criterion for each route.

### Step 4. The selection of chromosomes dependent on the function of adaptation.

The operation of reproduction (selection) consists of duplication of chromosomes depending on the

function of adaptation. The chromosomes with the higher function of adaptation are more likely to introduce their own copy to the next generation. In selection process the roulette method was used based on the selection of a new population according to the probability distribution defined on the values of the function of adaptation.

The selection process consists of the following stages:

- The calculation of the function of adaptation for a single chromosome,
- The calculation of the total population of adaptation,
- The calculation of the probability of the selection  $l$  of the chromosome,
- The calculation of the distribution  $l$  of the chromosome.

Choosing the chromosome to the next generation consists of the random selection of the number  $r$  from the range of  $[0,1]$ . We choose  $l$ - the chromosome with the value of distribution  $q_l$  while the relationship  $q_{l-1} < r \leq q_l$  is fulfilled.

**Step 5. The crossover of chromosomes selected randomly out of the pool of parent.**

In the crossover operation an operator which works on the structure of the numeric strings was used, which is called PMX (partially matched crossover). The PMX crossover is shown in Fig. 5.

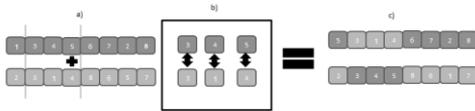


Fig. 5. The PMX crossover a) the chromosomes used to the crossover b) a swap of genes c) the chromosomes after crossover

The PMX crossover is the random selection of the two chromosomes in pairs, random selection of the two points of crossover and exchange of genes shown by the series created from these points of crossover. In the process of crossover genes from one chromosome are assigned to genes from the another. The crossover occurs with probability  $p^k$ .

**Step 6. The mutation of chromosomes.**

A mutation is the swap of place of two randomly selected tasks. The mutation occurs with probability  $p^m$ .The principle of the mutation is shown in Fig. 6.



Fig. 6. The use of a mutation operator

**Step 7. The inversion.**

The initial stage of inversion is a random selection of two points in the chromosome. These points create the string which needs to be reversed. The inversion occurs with probability  $p^{in}$ .The principle of the inversion is shown in Fig. 7.

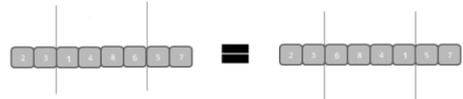


Fig. 7. The principle of the inversion

**5. Verification of the hybrid algorithm**

The algorithm was verified using programming language C #. Verification of the algorithm takes place in the phase of designating the minimum route which consists of collective points of loading in the first problem and the tasks and the base in the second problem.Verification relies on comparing the result of the hybrid algorithm with the result of the random search algorithm. The random search algorithm generates 2000 routes and the best result is compared with the hybrid algorithm. It was used 92 collective points of loading in the first problem and 30 tasks in the second problem. The ant algorithm which generated the initial population was started 50 times, so the result is the average of all starts. The number of iterations is equal to 200, the population is 100. The parameters of the algorithm take the values:  $\alpha = 1,2,5,10,20$ ,  $\beta = 0,5; 1,0; 5$ ,  $\rho = 0,2; 0,4; 0,6; 0,8$ .

60 combinations of these parameters were checked and the best combination of parameters, where the algorithm gave the best solution, was found. Combination of parameters was shown in Tab. 1.

Table 1. Combination of parameters in the ant algorithm

Lp.	$\alpha$	$\beta$	$\rho$	Lp.	$\alpha$	$\beta$	$\rho$	Lp.	$\alpha$	$\beta$	$\rho$
1	1	0,5	0,2	21	1	1	0,2	41	1	5	0,2
2	1	0,5	0,4	22	1	1	0,4	42	1	5	0,4
3	1	0,5	0,6	23	1	1	0,6	43	1	5	0,6
4	1	0,5	0,8	24	1	1	0,8	44	1	5	0,8
5	3	0,5	0,2	25	3	1	0,2	45	3	5	0,2
6	3	0,5	0,4	26	3	1	0,4	46	3	5	0,4
7	3	0,5	0,6	27	3	1	0,6	47	3	5	0,6
8	3	0,5	0,8	28	3	1	0,8	48	3	5	0,8
9	5	0,5	0,2	29	5	1	0,2	49	5	5	0,2
10	5	0,5	0,4	30	5	1	0,4	50	5	5	0,4
11	5	0,5	0,6	31	5	1	0,6	51	5	5	0,6
12	5	0,5	0,8	32	5	1	0,8	52	5	5	0,8
13	10	0,5	0,2	33	10	1	0,2	53	10	5	0,2
14	10	0,5	0,4	34	10	1	0,4	54	10	5	0,4
15	10	0,5	0,6	35	10	1	0,6	55	10	5	0,6
16	10	0,5	0,8	36	10	1	0,8	56	10	5	0,8
17	20	0,5	0,2	37	20	1	0,2	57	20	5	0,2
18	20	0,5	0,4	38	20	1	0,4	58	20	5	0,4
19	20	0,5	0,6	39	20	1	0,6	59	20	5	0,6
20	20	0,5	0,8	40	20	1	0,8	60	20	5	0,8

In the first problem described in this paper the best result was achieved by the set of parameters:  $\alpha = 3, \beta = 1, \rho = 0,2$  (Fig. 8). The initial population for the hybrid algorithm was generated using the set of this parameters.

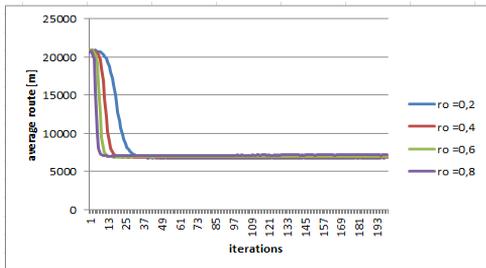


Fig. 8. The best result generated by the ant algorithm in the first transportation problem

In the second problem best result was achieved by the set of parameters:  $\alpha = 1, \beta = 1, \rho = 0,2$  (Fig. 9). The initial population for the hybrid algorithm was generated using this set of this parameters.

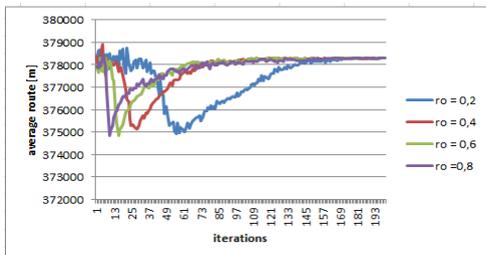


Fig. 9. The best result generated by the ant algorithm in the second transportation problem

The hybrid algorithm was started 50 times so the result is the average of all starts. The number of iterations is equal to 200 and 400, the population is 100. The parameters of the algorithm take the values:  $p_k = 0,2; 0,4; 0,6; 0,8; 1,$

$$p_m = 0,01; 0,03; 0,05, \quad p_{in} = 0; 0,4; 0,6; 0,8; 1.$$

75 combinations of these parameters were checked and the best combination of parameters, where the algorithm gave the best solution, was found. Combination of parameters was shown in Tab. 2.

In the first problem described in this paper the best result was achieved by the set of parameters:  $p_{krzyz} = 0,8, p_{mut} = 0,01$  (Fig. 10).

Table 2. Combination of parameters in the hybrid algorithm

Lp.	$p^k$	$p^m$	$p^{in}$	Lp.	$p^k$	$p^m$	$p^{in}$	Lp.	$p^k$	$p^m$	$p^{in}$
1	0,2	0,01	0	26	0,4	0,05	0	51	0,8	0,03	0
2	0,2	0,01	0,4	27	0,4	0,05	0,4	52	0,8	0,03	0,4
3	0,2	0,01	0,6	28	0,4	0,05	0,6	53	0,8	0,03	0,6
4	0,2	0,01	0,8	29	0,4	0,05	0,8	54	0,8	0,03	0,8
5	0,2	0,01	1	30	0,4	0,05	1	55	0,8	0,03	1
6	0,2	0,03	0	31	0,6	0,01	0	56	0,8	0,05	0
7	0,2	0,03	0,4	32	0,6	0,01	0,4	57	0,8	0,05	0,4
8	0,2	0,03	0,6	33	0,6	0,01	0,6	58	0,8	0,05	0,6
9	0,2	0,03	0,8	34	0,6	0,01	0,8	59	0,8	0,05	0,8
10	0,2	0,03	1	35	0,6	0,01	1	60	0,8	0,05	1
11	0,2	0,05	0	36	0,6	0,03	0	61	1	0,01	0
12	0,2	0,05	0,4	37	0,6	0,03	0,4	62	1	0,01	0,4
13	0,2	0,05	0,6	38	0,6	0,03	0,6	63	1	0,01	0,6
14	0,2	0,05	0,8	39	0,6	0,03	0,8	64	1	0,01	0,8
15	0,2	0,05	1	40	0,6	0,03	1	65	1	0,01	1
16	0,4	0,01	0	41	0,6	0,05	0	66	1	0,03	0
17	0,4	0,01	0,4	42	0,6	0,05	0,4	67	1	0,03	0,4
18	0,4	0,01	0,6	43	0,6	0,05	0,6	68	1	0,03	0,6
19	0,4	0,01	0,8	44	0,6	0,05	0,8	69	1	0,03	0,8
20	0,4	0,01	1	45	0,6	0,05	1	70	1	0,03	1
21	0,4	0,03	0	46	0,8	0,01	0	71	1	0,05	0
22	0,4	0,03	0,4	47	0,8	0,01	0,4	72	1	0,05	0,4
23	0,4	0,03	0,6	48	0,8	0,01	0,6	73	1	0,05	0,6
24	0,4	0,03	0,8	49	0,8	0,01	0,8	74	1	0,05	0,8
25	0,4	0,03	1	50	0,8	0,01	1	75	1	0,05	1

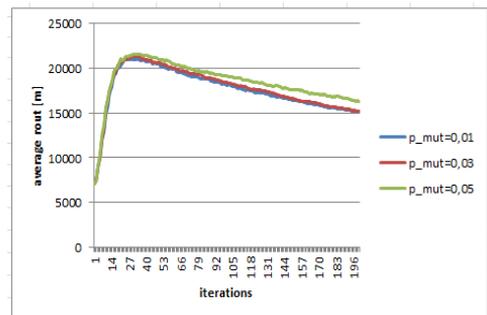


Fig. 10. The best result generated by the hybrid algorithm in the first transportation problem

In the second problem described in this paper the best result was achieved by the set of parameters:  $p_{krzyz} = 1,0, p_{mut} = 0,01$  (Fig. 11).

Comparison of algorithms for the first problem was presented on the Fig. 12, comparison of algorithms for the second problem was presented on the Fig. 13. The hybrid algorithm found the best results than the random search algorithm.

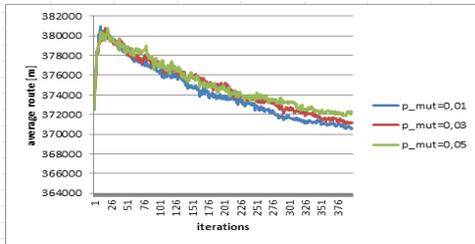


Fig. 11. The best result generated by the hybrid algorithm in the second transportation problem

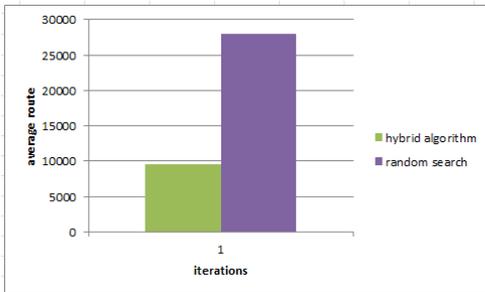


Fig.12. Comparison of algorithms in the first transportation problem

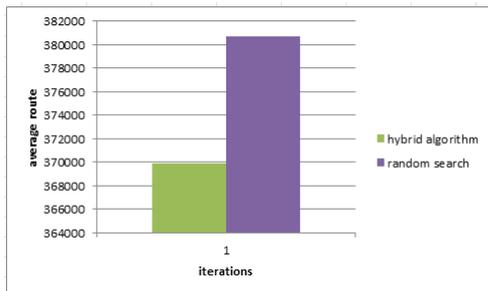


Fig.13. Comparison of algorithms in the second transportation problem

### 6. Conclusion

The heuristic algorithms are the tools which we can use in the optimization problems in the municipal services companies. The algorithms of this type give a near-optimum solution, can find

the optimal solution but often confine themselves to the optimal solution in the local area of search but their results we can accept. The problems described in this papers were solved by using the hybrid algorithm. The most important stage of the verification is the stage when we designate the parameters.

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